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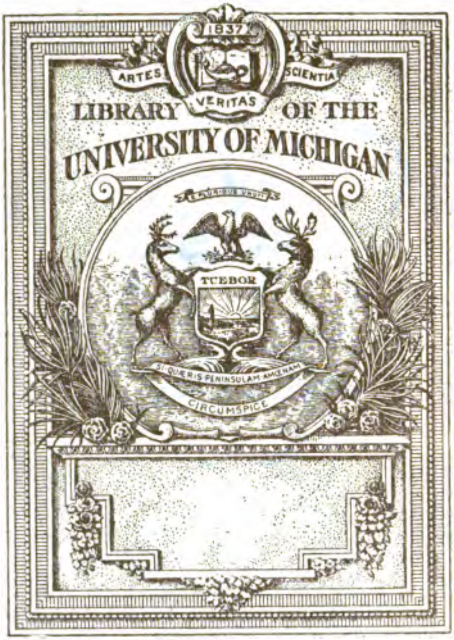

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AND

Some Occaſional REMARKS.

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VOL. II.

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PART III. continued.

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CHAP. VIII.

Of the Manner *wherein* Light *is* reflected.

THE *Refraction* of Light has been conſidered, and explained: its *Reflection* is now to be inquired into (a). And firſt for the Manner wherein it is performed.

When a Ray of Light falls upon a Body not tranſparent, *Part* or *all* of it *is reflected*; if any enters, it is ſuffocated and loſt within the Body: When it falls upon a *tranſparent* Body, *Part* of it *is reflected*, and *Part* enters; of what enters ſome is alſo ſuffocated and loſt in the Body; the reſt, when it arrives at the *other Side*, ſome of it is reflected there, the Remainder going out and leaving the Body, unleſs its Inclination to *that Side* exceeds a *certain* Degree; which if it does, it is *all reflected* there. And the Power whereby a Ray is reflected at *this other Side* of a Body (which for Diſtinction ſake I ſhall hereafter call *the ſecond Surface*) is ſtronger than that by which it would be reflected by the ſame Surface, were it about to enter the Body there with an equal Degree of Obliquity.

(a) The Subject of this *Inquiry* is diſtinguiſhed from that of the former, by the Name of *Catoptrics*, as tending to explain the Manner in which Objects appear, when ſeen by *reflected* Light.

The *Degree of Inclination* neceſſary to cauſe a *total Reflection* of a Ray at the ſecond Surface of a *Medium*, is that which requires that the refracted Angle (was the Ray ſuppoſed to paſs out there) ſhould be equal to, or greater than a right one; and conſequently it depends on the refractive Power of the *Medium* through which the Ray paſſes; and is therefore different in different *Media*. When a Ray paſſes through Glaſs ſurrounded with Air, and is inclined to its ſecond Surface under an Angle of 42 *Degrees or more*, it will be *wholly reflected* there *(b)*. For as 11 is to 17,

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(b) From hence it follows, that when a Ray of Light arrives at the ſecond Surface of a tranſparent Subſtance with as great or greater Degree of Obliquity, than that which is neceſſary to make a total Reflection, it will there be all returned back to the firſt; and if it proceeds towards that with as great an Obliquity as it did towards the other (which it will do if the Surfaces of the *Medium* be parallel to each other) it will there be all reflected again, &c. and will therefore never get out, but paſs from Side to Side, till it be wholly ſuffocated and loſt within the Body.

From hence may ariſe an obvious Inquiry, how it comes to paſs that Light, falling very obliquely upon a Glaſs-Window from without, ſhould be tranſmitted into the Room? In Anſwer to this, it muſt be conſidered, that however obliquely a Ray falls upon the firſt Surface of any *Medium* whoſe Sides are parallel (as thoſe of the Glaſs in a Window are) it will ſuffer ſuch a Degree of Refraction in entering there, that it ſhall fall upon the ſecond with a leſs Obliquity than that which is neceſſary to cauſe a total Reflection. For Inſtance let the *Medium* be Glaſs, as ſuppoſed in the preſent Caſe, then as 17 is to 11 (the *Ratio* of Refraction out of Air into Glaſs) ſo is the Sine of the largeſt Angle of Incidence with which a Ray can fall upon any Surface, to the Sine of a leſs Angle than that of total Reflection. And therefore, if the Sides of a Glaſs be parallel, the Obliquity, with which a Ray falls upon the firſt Surface, cannot be ſo great, but that it ſhall paſs the ſecond without ſuffering a total Reflection there.

(the *Ratio* of Refraction out of Glaſs into Air) ſo is the Sine of an Angle of 42 Degrees to a fourth Number, that will exceed the Sine of a right Angle.

When Light paſſes out of a denſer into a rarer *Medium*, the nearer the ſecond *Medium* approaches the firſt in Denſity (or more properly in its refractive Power) the leſs of it will be reflected in paſſing from one to the other: and when their refracting Powers are equal, all of it will paſs into the ſecond *Medium*.

Whether Light be reflected from the firſt or ſecond Surface of a Body, the Law it obſerves is this, viz. *That the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame.*

By the *Angle of Reflection* is meant the Angle comprehended between a Perpendicular to the Surface at the Point where the Reflection is made, and the reflected Ray.

Theſe are all the Circumſtances attending the Reflection of Light neceſſary to be taken Notice of at preſent: There are others, but they reſpect the *Doctrine of Light and Colours* not yet explained; we ſhall therefore paſs them by till we treat of that Subject, and in the mean Time proceed to conſider the Reflection of Light from plain and ſpherical Surfaces.

CHAP. IX.

Of the Reflection *of* Light *from* plain *and* ſpherical Surfaces.

IT was obſerved in the foregoing Chapter, that the Law of Reflection is ſuch, that the Angle of Reflection of each Ray ſhall be equal to the Angle of Incidence of the ſame. From whence the ſeven following Propoſitions relating to the Reflection of Light from plain and ſpherical Surfaces may be deduced.

I. Rays of Light reflected from a plain Surface have the ſame Degree of Inclination to each other that their reſpective incident ones have.

For the Angle of Reflection of each Ray being equal to that of its reſpective incident one, it is evident that each reflected Ray will have the ſame Degree of Inclination to that Portion of the Surface from whence it is reflected, that its incident one has; but it is here ſuppoſed that all thoſe Portions of Surface from whence the Rays are reflected are ſituated in the ſame Plain; conſequently the reflected Rays will have the ſame Degree of Inclination to each other that their incident ones have, from whatever Part of the Surface they are reflected.

See this and the following Propoſitions expreſſed more determinately, and demonſtrated in the Note below (a).

II. Parallel Rays, reflected from a concave Surface, are render'd converging.

(a) Propoſition I.

Of the Reflection of Rays from a plain Surface.

When Rays fall upon a plain Surface, if they diverge, the Focus *of the reflected Rays will be at the ſame Diſtance behind the Surface, that the* Radiant *Point is before it: if they converge, it will be at the ſame Diſtance before the Surface, that the imaginary* Focus *of the incident Rays is behind it.*

This Propoſition admits of two Caſes.

Caſe I. *Of diverging Rays.*

Dem. Let AB, AC (Fig. 45.) be two diverging Rays incident in the plain Surface DE, the one perpendicularly, the other obliquely; the perpendicular one AB will be reflected to A proceeding as from ſome Point in the Line AB produced; the oblique one AC will be reflected into ſome Line as CF ſuch, that the Point G, where the Line FC produced interſects the Line AB produced alſo, ſhall be at an equal Diſtance from the Surface DE with the *Radiant* A. For the Perpendicular CH being drawn, ACH and HCF will be the Angles of Incidence and Reflection, which being equal, their Complements ACB and FCE are ſo too: but the Angle BCG is equal to FCE as being vertical to it; therefore in the Triangles ABC and GBC the Angles at C are equal, the Side BC is common, and the Angles at B are alſo equal to each other, as being right ones; therefore the Lines AB and BG, which reſpect the equal Angles at C, are alſo equal, and conſequently the Point G, the *Focus* of the incident Rays AB, AC, is at the ſame Diſtance behind the Surface, that the Point A is before it Q. E. D.

Caſe II. *Of converging Rays.*

This is the Converſe of the former Caſe. For ſuppoſing FC and AB to be two converging incident Rays, CA and BA will be the reflected ones (the Angles of Incidence in the former Caſe

To illuſtrate this, let AF, CD, EB, (Fig. 43.) repreſent three parallel Rays falling upon the concave Surface FB, whoſe Center is C. To the Points F and B draw the Lines CF, CB; theſe being drawn from the Center will be

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Caſe being now the Angles of Reflection, and *vice verſâ*) having the Point A for their *Focus*; but this, from what was demonſtrated above, is at an equal Diſtance from the reflecting Surface with the Point G, which in this Caſe is the imaginary *Focus* of the incident Rays, FC, and AB.

Obſervat. I. *It is not here, as in the Refraction of Rays in paſſing through a plain Surface, where ſome of the refracted Rays proceed as from one Point, and ſome as from another* (*See Obſervat.* I. *Chap.* 3. *in the Notes*) *but they all proceed after Reflection as from one and the ſame Point, however obliquely they may fall upon the Surface; for what is here demonſtrated of the Ray AC holds equally of any other, as AI, AK, &c.*

The Caſe of parallel Rays incident on a plain Surface, is included in this Propoſition; for in that Caſe we are to ſuppoſe the *Radiant* to be at an infinite Diſtance from the Surface, and then by the Propoſition, the *Focus* of the reflected Rays will be ſo too; that is, *the Rays will be parallel after Reflection, as they were before.*

Propoſition II.

Of the Reflection of parallel Rays from a ſpherical Surface.

When parallel Rays are incident upon a ſpherical Surface, the Focus *of the reflected Rays will be the middle Point between the Center of Convexity and the Surface.*

This Propoſition admits of two Caſes.

Caſe I. *Of parallel Rays falling upon a convex Surface.*

Dem. Let AB, DH, (Fig. 46) repreſent two parallel Rays incident on the convex Surface BH, the one perpendicularly, the other obliquely; and let C be the Center of Convexity; ſuppoſe HE to be the reflected Ray of the oblique Incident one DH proceeding as from F a Point in the Line AB produced. Through the Point H draw the Line CI, which will be perpendicular to the Surface at that Point, and the Angles DHI and IHF, *being the Angles and Incidence and Reflection*, will be equal.

To

be perpendicular to the Surface at thoſe Points. The incident Ray CD alſo paſſing through the Center will be perpendicular to the Surface, and therefore will return after Reflection in the ſame Line; but the oblique Rays AF

To the former of theſe, the Angle HCF is equal, *the Lines AC and DH being parallel*, and to the latter the Angle CHF *as being vertical*; wherefore the Triangle CFH is *Iſoſceles*; and conſequently the Sides CF and FH are equal: But ſuppoſing BH to *vaniſh*, FH is equal to FB, and therefore upon this Suppoſition FC and FB are equal, that is, the *Focus* of the reflected Rays is the middle Point between the Center of Convexity and the Surface. Q. E. D.

Caſe II. *Of parallel Rays falling upon a concave Surface.*

Dem. Let AB, DH (Fig. 47.) be two parallel Rays incident, the one perpendicularly, the other obliquely, on the concave Surface BH, whoſe Center of Concavity is C. Let BF and HF be the reflected Rays meeting each other in F: this will be the middle Point between B and C. For drawing through C the Perpendicular CH, the Angles DHC and FHC, *being the Angles of Incidence and Reflection*, will be equal, to the former of which the Angle HCF is equal, *as alternate*; and therefore the Triangle CFH is *Iſoſceles*. Wherefore CF and FH are equal: but if we ſuppoſe BH to *vaniſh*, FB and FH are alſo equal, and therefore CF is equal to FB; that is, the focal Diſtance of the reflected Rays is the middle Point between the Center and the Surface. Q. E. D.

Obſervat. II. *It is here obſervable, that the farther the Line DH, either in Figure the* 46 *or* 47, *is taken from AB, the nearer the Point F falls to the Surface. For the farther the Point H recedes from B, the larger the Triangle CFH will become; and conſequently ſince it is always an* Iſoſceles *one, and the Baſe CH, being the* Radius, *is every where of the ſame Length, the equal Legs CF and FH will lengthen; but CF cannot grow longer unleſs the Point F approach towards the Surface. And the farther H is removed from B, the faſter F approaches to it.*

This is the Reaſon, that whenever parallel Rays are conſidered, as reflected from a ſpherical Surface, the Diſtance of the oblique one from the perpendicular one is taken ſo ſmall with reſpect to the focal Diſtance of that Surface, that without any phyſical *Error it may be ſuppoſed to* vaniſh.

and EB will be reflected into the Lines FM and BM ſituated on the contrary ſide their reſpective Perpendiculars CF and CB. They will therefore proceed converging after Reflection towards ſome Point as M, in the Line CD: which Point, by what is demonſtrated in the Note laſt referred to, will be in the Middle between C and D.

III.

From hence it follows, that if a Number of parallel Rays as AB, CD, EG, &c. fall upon a convex Surface, as expreſſed Figure the 48, and if BA, DK, the reflected Rays of the incident ones AB, CD, proceed as from the Point F, thoſe of the incident ones CD, EG, viz. *DK, GL, will proceed as from N, thoſe of the incident ones EG, HI, as from O, &c. becauſe the farther the incident ones CD, EG &c. are from AB, the nearer to the Surface are the Points F, f, f, in the Line BF, from which they proceed after Reflection; ſo that properly the* Foci *of the reflected Rays BA, DK, GL, &c. are not in the Line AB produced, but in a curve Line paſſing through the Points F, N, O, &c.*

The ſame is applicable to the Caſe of parallel Rays reflected from a concave Surface, as expreſſed by the pricked Lines on the other half of the Figure, where PQ, RS, TV, are the incident Rays; QF Sf, Vf, the reflected Rays interſecting each other in the Points X, Y, and F; ſo that the Foci *of thoſe Rays are not in the Line FB, but in a Curve paſſing through thoſe Points.*

Had the Surface BH in Figure 46, or 47, been form'd by the Revolution of a Parabola *about its* Axis *having its* Focus *in the Point F, all the Rays reflected from the convex Surface would have proceeded as from the Point F, and thoſe reflected from the concave would have fallen upon it, however diſtant their incident ones AB, DH, might have been from each other. For in the* Parabola, *all Lines drawn parallel to the* Axis *make Angles with the Tangents to the Points where they cut the* Parabola *(that is, with the Surface of the* Parabola) *equal to thoſe which are made with the ſame Tangents by Lines drawn from thence to the* Focus. *(De* L'Hoſpital *Sections Coniques. Liv.* I. *Prop.* 5.) *Therefore, if the incident Rays deſcribe thoſe parallel Lines, the reflected ones will neceſſarily deſcribe theſe other, and ſo will all proceed as from, or meet in, the ſame Point.*

Pre-

III. Converging Rays falling on the like Surface are made to converge more.

For every thing remaining as above, let GF, HB, be the incident Rays. Now becauſe theſe Rays have larger Angles of Incidence than the parallel ones AF and EB in the foregoing Caſe, their Angles of Reflection will alſo be larger than theirs; they will therefore converge after Reflection, ſuppoſe in the Lines FN, and BN, having their Point of Con-

Propoſition III.

Of the Reflection of diverging and converging Rays from a ſpherical Surface.

When Rays fall upon any ſpherical Surface, if they diverge, the Diſtance of the Focus *of the reflected Rays from the Surface is to the Diſtance of the* Radiant *Point from the ſame (or, if they converge, to that of the imaginary* Focus *of the incident Rays) as the Diſtance of the* Focus *of the reflected Rays from the Center is to the Diſtance of the* Radiant *Point (or imaginary* Focus *of the incident Rays) from the ſame.*

This Propoſition admits of ten Caſes.

Caſe I. *Of diverging Rays falling upon a convex Surface.*

Dem. Let RB, RD (Fig. 49) repreſent two diverging Rays flowing from the Point R as from a *Radiant*, and falling the one perpendicularly, the other obliquely, on the convex Surface BD, whoſe Center is C. Let DE be the reflected Ray of the incident one RD, produce ED to F, and through R draw the Line RH parallel to FE till it meets CD produced in H. Then will the Angle RHD be equal to EDH the Angle of Reflection, *as being alternate to it*, and therefore equal alſo to RDH which is the Angle of Incidence; wherefore the Triangle DRH is *Iſoſceles*, and conſequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are ſimilar, (or to expreſs it in *Euclid*'s Way, the Sides of the Triangle RHC are cut proportionably. 2. Elem. 6.) and therefore FD is to

Concourſe N farther from C than the Point M; that to which the parallel Rays AF and EB converged to in the foregoing Caſe: and their preciſe Degree of Convergency as determined in the Note, will be greater than that wherein they converged before Reflection.

IV. Diverging Rays, falling upon the like Surface, are after Reflection parallel, diverging, or converging. If they diverge from the *Focus* of parallel Rays, they then become parallel; if

from

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to RH, or its equal RD, as CF to CR; but BD *vaniſhing*, FD and RD differ not from FB and RB, wherefore FB is to RB alſo, as CF to CR; that is, the Diſtance of the *Focus* from the Surface is to the Diſtance of the *Radiant* Point from the ſame, as the Diſtance of the *Focus* from the Center is to the Diſtance of the *Radiant* from thence. Q. E. D.

Caſe II. *Of converging Rays falling upon a concave Surface.*

Dem. Let KD and CB be the converging incident Rays having their imaginary *Focus* in the Point R, which was the *Radiant* in the foregoing Caſe. Then as RD was in that Caſe reflected into DE, KD will in this be reflected into DF; for, ſince the Angles of Incidence in both Caſes are equal, *as they are by being vertical*, the Angles of Reflection will be ſo too; ſo that F will be the *Focus* of the reflected Rays: but it was there demonſtrated that FB is to RB as CF to CR, that is, the Diſtance of the *Focus* from the Surface is to the Diſtance (in this Caſe) of the imaginary *Focus* of the incident Rays, as the Diſtance of the *Focus* from the Center is to the Diſtance of the imaginary *Focus* of the incident Rays from the ſame. Q. E. D.

Caſe III. *Of converging Rays falling upon a convex Surface, and tending to a Point between the* Focus *of Parallel Rays and the Center.*

Dem. Let BD (Fig. 50.) repreſent a convex Surface whoſe Center is C, and *Focus* of parallel Rays is P; and let AB, KD, be two converging Rays incident upon it, and having their imaginary *Focus* at R, a Point between P and C. Now becauſe KD

tends

from a Point nearer to the Surface than that, they will diverge, but in a leſs Degree than before Reflection; if from a Point between that and the Center, they will converge after Reflection and that to ſome Point on the contrary Side the Center, but ſituated farther from it than the Point they diverged from: if the incident Rays diverge from a Point beyond the Center, the reflected ones will converge to one on the other Side of it, but nearer to it than

the

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tends to a Point between the *Focus* of parallel Rays and the Center, the reflected Ray DE will diverge from ſome Point on the other Side the Center, ſuppoſe F; as explained above in the Text under Propoſition the VIIth. Through D draw the Perpendicular CD and produce it to H, then will KDH and HDE be the Angles of Incidence and Reflection, which being equal, their vertical ones RDC and CDF will be ſo too, and therefore the *Vertex* of the Triangle RDF is biſected by the Line DC: wherefore (3 El. 6.) FD and DR, or, BD vaniſhing, FB and BR are to each other as FC to CR; that is, the Diſtance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident ones, as the Diſtance of the former from the Center is to the Diſtance of the latter from the ſame. Q. E. D.

Caſe IV. *Of diverging Rays falling upon a concave Surface and proceeding from a Point between the* Focus *of parallel Rays and the Center.*

Dem. Let RB, RD, be the diverging Rays incident upon the concave Surface BD, having their *Radiant* in the Point R, the imaginary *Focus* of the incident Rays in the foregoing Caſe. Then as KD was in that Caſe reflected into DE, RD will now be reflected into DF. But it was there demonſtrated that FB and RB are to each other as CF to CR; that is, the Diſtance of the *Focus* is to that of the *Radiant*, as the Diſtance of the former from the Center is to the Diſtance of the latter from the ſame. Q. E. D.

The Angles of Incidence and Reflection being equal, it is evident, that if in any Caſe the reflected Ray be made the incident one, the Point they diverged from; and if they diverge from the Center, they will be reflected thither again.

1. Let them diverge in the Lines MF, MB, proceeding from M the *Focus* of parallel Rays; then as the parallel Rays AF and EB were reflected into the Lines FM and BM (Propoſit 2.) theſe Rays will now on the contrary be reflected into them.

2. Let them diverge from N a Point nearer to the Surface than the *Focus* of parallel Rays, they will then be reflected into the diverging Lines

the incident will become the reflected one: and therefore the four following Caſes may be conſidered reſpectively as the Converſe of the four foregoing; for in each of them the incident Rays are ſuppoſed to coincide with the reflected ones in the other. Or they may be demonſtrated independently of them as follows.

Caſe V. *Of converging Rays falling upon a convex Surface, and tending to a Point nearer the Surface than the* Focus *of parallel Rays.*

Dem. Let ED, RB (Fig. 49.) be the converging Rays incident upon the convex Surface BD whoſe Center is C, and *Focus* of parallel Rays is at P; and let the imaginary *Focus* of the incident Rays be at F, a Point between P and B, and let DR be the reflected Ray. From C and R draw the Lines CH, RH, the one paſſing through D, the other parallel to FE. Then will the Angle RHD be equal to HDE the Angle of Incidence, *as alternate to it*, and therefore equal to HDR, the Angle of Reflection; wherefore the Triangle HDR is *Iſoſceles*, and conſequently DR is equal to RH. Now the Lines FD and RH being parallel, the Triangles FDC and RHC are ſimilar, and therefore RH, or RD, is to FD as CR to CF; but BD vaniſhing, RD and FD coincide with RB and FB, wherefore RB is to FB as CR to CF; that is, the Diſtance of the *Focus* from the Surface is to the Diſtance of the imaginary *Focus* of the incident Rays, as the Diſtance of the *Focus* from the Center is to the Diſtance of the imaginary *Focus* of the incident Rays from the ſame. Q. E. D.

Caſe

Lines FG and BH which the incident Rays GF and HB deſcribed, that were ſhewn to be reflected into them in the foregoing Propoſition: but the Degree wherein they diverge, as demonſtrated in the Note, will be leſs than that wherein they diverged before Reflection.

3. Let them proceed diverging from X a Point between the *Focus* of parallel Rays and the Center, they then make leſs Angles of Incidence than the Rays MF and MB which became parallel by Reflection, they will conſequently have leſs Angles of Reflection, and proceed therefore converging towards ſome Point as Y; which Point will always fall on the contrary Side the Center, becauſe a re-

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Caſe VI. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point between the* Focus *of parallel Rays and the Surface.*

Dem. Let FD and FB repreſent two diverging Rays flowing from the Point F as a *Radiant*, which was the imaginary *Focus* of the incident Rays in the foregoing Caſe. Then as ED was in that Caſe reflected into DR, FD will be reflected into DK, (for the Reaſon mention'd in Caſe the ſecond) ſo that the reflected Ray will proceed as from the Point R: but it was demonſtrated in the Caſe immediately before-going, that RB is to FB as CR to CF; that is, the Diſtance of the *Focus* from the Surface is to that of the *Radiant* from the ſame, as the Diſtance of the former from the Center is to that of the latter from the ſame. Q. E. D.

Caſe VII. *Of converging Rays falling upon a convex Surface, and tending towards a Point beyond the Center.*

Dem. Let AB, ED, (Fig. 50.) be the incident Rays tending to F, a Point beyond the Center C, and let DK be the reflected Ray of the incident one ED. Then becauſe the incident Ray flected Ray always falls on the contrary Side the Perpendicular with reſpect to that on which its incident one falls; and, as demonſtrated in the Note, it will, be farther diſtant from the Center than X.

4. If the incident ones diverge from Y, they will after Reflection converge to X, thoſe which were the incident Rays in the former Caſe being the reflected ones in this.

And laſtly, if the incident Rays proceed from the Center, they fall in with their reſpective Perpendiculars, and for that Reaſon are reflected thither again.

V. Parallel Rays reflected from a convex Surface are rendered diverging.

To ſhew this, let AB, GD, EF, (Fig. 44.) be three parallel Rays falling upon the convex Sur-

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ED tends to a Point beyond the Center, the reflected Ray DK will proceed as from one on the contrary Side, ſuppoſe R; as explained in the Text under Propoſition the VIIth. Through D draw the Perpendicular CD and produce it to H. Then will EDH and HDK be the Angles of Incidence and Reflection, which being equal, their vertical ones CDF and CDR will be ſo too: conſequently the *Vertex* of the Triangle FDR is biſected by the Line CD: wherefore (3 Elem. 6.) RD is to DF or BD vaniſhing. RB is to BF as RC to CF; that is, the Diſtance of the *Focus* of the reflected Rays is to that of the imaginary *Focus* of the incident Rays, as the Diſtance of the former from the Center is to the Diſtance of the latter from the ſame. Q. E. D.

Caſe VIII. *Of diverging Rays falling upon a concave Surface, and proceeding from a Point beyond the Center.*

Dem. Let FB, FD, be the incident Rays having their *Radiant* in F, the imaginary *Focus* of the incident Rays in the foregoing Caſe. Then as ED was in that Caſe reflected into DK, FD will now be reflected into DR; ſo that R will be the *Focus* of the re-

Surface BF whoſe Center of Convexity is C, and let one of them, *viz.* GD, be perpendicular to the Surface: through B, D, and F, the Points of Reflection, draw the Lines CV, CG, and CT, which becauſe they paſs through the Center will be perpendicular to the Surface at thoſe Points. The incident Ray GD being perpendicular to the Surface will return after Reflection in the ſame Line, but the oblique ones AB and EF in the Lines BK and FL ſituated on the contrary Side their reſpective Perpendiculars BV and FT. They will therefore diverge after Reflection as from ſome Point M in the Line GD produced; which Point, as demonſtrated in the Note, will be in the middle between D and C.

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reflected Rays. But it was demonſtrated in the foregoing Caſe, that RB is to FB as RC to CF; that is, the Diſtance of the *Focus* of the reflected Rays from the Surface is to the Diſtance of the *Radiant* from the ſame, as the Diſtance of the *Focus* of the reflected Rays from the Center is to the Diſtance of the *Radiant* from thence. Q. E. D.

The two remaining Caſes may be conſidered, as the Converſe of thoſe under Propoſition the ſecond of this Note, becauſe the incident Rays in theſe are the reflected ones in them; or they may be demonſtrated in the ſame Manner with the foregoing, as follows.

Caſe IX. *Converging Rays falling upon a convex Surface, and tending to the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let ED, RB, (Fig. 49.) repreſent two converging Rays incident on the convex Surface BD, and tending towards F, which we will *now* ſuppoſe to be the *Focus* of parallel Rays; and let DR be the reflected Ray, and C the Center of Convexity of the reflecting Surface. Through C draw the Line CD, and produce

VI. Diverging Rays reflected from the like Surface are rendered more diverging.

For, every thing remaining as above, let GB, GF, be the incident Rays. Theſe having larger Angles of Incidence than the parallel ones AB and EF in the preceding Caſe, their Angles of Reflection will alſo be larger than theirs; they will therefore diverge after Reflection, ſuppoſe in the Lines BP and FQ, as from ſome Point N farther from C than the Point M; and the Degree wherein they will diverge, as determined in the Note, will exceed that wherein they diverged before Reflection.

it to H, drawing RH parallel to ED produced to F. Now it has been demonſtrated (Caſe 5.) where the incident Rays are ſuppoſed to tend to the Point F, that RB is to FB as RC to CF; but F in this Caſe being ſuppoſed to be the *Focus* of parallel Rays, it is the middle Point between C and B (by Propoſition 2d) and therefore FB and FC are equal, and conſequently the two other Terms in the Proportion, *viz.* RB and RC, muſt be ſo too; which can only be upon a Suppoſition that R is at an infinite Diſtance from B; that is, that the reflected Rays BR and DR be parallel. Q. E. D.

Caſe X. *Diverging Rays falling upon a concave Surface, and proceeding from the* Focus *of parallel Rays, become parallel after Reflection.*

Dem. Let RD, RB (Fig. 50) be two diverging Rays incident upon the concave Surface BD, as ſuppoſed in Caſe the fourth: where it was demonſtrated that FB is to RB as CF to CR. But in the preſent Caſe RB and CR are equal, becauſe R is ſuppoſed to be the *Focus* of parallel Rays; therefore FB and FC are ſo too. Which cannot be unleſs F be taken at an infinite Diſtance from B; that is, unleſs the reflected Rays BF and DF be parallel. Q. E. D.

VII. Converging Rays reflected from the like Surface, are parallel, converging or diverging. If they tend towards the *Focus* of parallel Rays, they then become parallel; if to a Point nearer the Surface than that, they converge, but in a leſs Degree than before Reflection; if to a Point between that and the Center, they ſhall diverge after Reflection, as from ſome Point on the contrary Side the Center, but ſituated farther from it than the Point they converged to; if the incident Rays converge to a Point beyond the Center, the reflected ones will diverge as from one on the contrary Side of it, but nearer to it than the Point the incident ones converged to: and if the incident Rays converge towards the Center, the reflected ones will proceed as from thence.

1. Let

Obſervat 3. *It is here obſervable, that in the Caſe of diverging Rays falling upon a convex Surface (ſee Fig. 49.) the further the Point D is taken from B, the nearer the Point F, the* Focus *of the reflected Rays, approaches to B, while the* Radiant *R remains the ſame. For it is evident from the Curvature of a Circle that the Point D (See Fig. 51.) may be taken ſo far from B, that the reflected Ray DE ſhall proceed as from F, G, H, or even from B, or from any Point between B and R, and the farther it is taken from B, the* faſter *the Point, from which it proceeds, approaches towards R: as will eaſily appear if we draw ſeveral incident Rays with their reſpective reflected ones, in ſuch Manner that the Angles of Reflection may be all equal to their reſpective Angles of Incidence, as is done in the Figure. The like is applicable to any of the other*

Caſes

1. Let them converge in the Lines KB and LF tending towards M the *Focus* of parallel Rays; then as the parallel Rays AB, EF were reflected into the Lines BK and FL (*Propoſit.* 5.) thoſe Rays will now on the contrary be reflected into them.

2. Let them converge in the Lines PB, QF, tending towards N a Point nearer the Surface than the *Focus* of parallel Rays, they will then be reflected into the converging Lines BG and FG, in which the Rays GB, GF, proceeded that were ſhewn to be reflected into them

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Caſes of diverging or converging Rays incident upon a ſpherical Surface. This is the Reaſon that, when Rays are conſidered as reflected from a ſpherical Surface, the Diſtance of the oblique Rays from the perpendicular one is taken ſo ſmall, that it may be ſuppoſed to vaniſh.

From hence it follows, that if a Number of diverging Rays are incident upon the convex Surface BD at the ſeveral Points B, D, D, &c. *they ſhall not proceed after Reflection as from any one Point in the Line RB produced, but as from a curve Line paſſing through the ſeveral Points F, f, f,* &c. *The ſame is applicable in all the other Caſes.*

Had the Curvature BD (Fig. 49.*) been* Hyperbolical *having its* Foci *in R and F, then R being the* Radiant *(or the imaginary* Focus *of incident Rays) F would have been the* Focus *of the reflected ones, and* vice verſâ, *however diſtant the Points B and D might be taken from each other. In like Manner had the Curve BD (Fig.* 50.*) been* Elliptical *having its* Foci *in F and R, the one of theſe being made the* Radiant *(or imaginary* Focus *of incident Rays) the other would have been the* Focus *of the reflected ones, and* vice verſâ. *For both in the* Hyperbola *and* Ellipſis, *Lines drawn from each of their* Foci *through any Point make equal Angles with the Tangent to that Point; (De* L'Hoſpital *Sections Coniques, Liv.* II. *Prop.* 8. & *Liv.* III. *Prop.* 11.*) Therefore, if the incident Rays proceed to or from one of their* Foci, *the reflected ones* them in the Propoſition immediately foregoing: but the Degree wherein they will converge, as demonſtrated in the Note, will be leſs than that wherein they converged before Reflection.

3. Let them converge in the Lines RB and SF proceeding towards X, a Point between the *Focus* of parallel Rays and the Center; their Angles of Incidence will then be leſs than thoſe of the Rays KB and LF which became parallel after Reflection, their Angles of

ones will all proceed, as from or to the other. So that in order that diverging or converging Rays may be accurately reflected to or from a Point, the reflecting Surface muſt be formed by the Revolution of an Hyperbola *about its longer* Axis, *when the incident Rays are ſuch that their* Radiant, *or imaginary* Focus *of incident Rays, ſhall fall on one Side the Surface, and the* Focus *of the reflected ones on the other: When they are both to fall on the* ſame *Side, it muſt be formed by the Revolution of an* Ellipſis *about its longer* Axis. *However upon Account of the great Facility with which ſpherical Surfaces are formed in Compariſon of that with which Surfaces, formed by the Revolution of any of the* Conic *Sections about their* Axes, *are made, the latter are very rarely uſed. Add to this another Inconvenience,* viz. *that, the* Foci *of theſe Curves being Mathematical Points, it is but one Point of the Surface of an Object that can be placed in any of them at a Time, ſo that it is only* in Theory *that Surfaces formed by the Revolution of theſe Curves about their* Axes *render Reflection perfect.*

Now becauſe the focal Diſtance of Rays reflected from a ſpherical Surface cannot be found by the *Analogy* laid down in the third Propoſition of this Note, without making uſe of the Quantity ſought; I ſhall here give an Inſtance whereby the Method of doing it in all others will readily appear.

Prob. Let it be required to find the focal Diſtance of diverging Rays incident upon a convex Surface, whoſe *Radius* of Convexity is of Reflection will therefore be leſs, on which Account they muſt neceſſarily diverge, ſuppoſe in the Lines BH and FI, from ſome Point as Y; which Point for the Reaſon given under Propoſition the fourth will fall on the contrary Side the Center with reſpect to X, and, as demonſtrated in the Note, will be farther from it than that.

4. If the incident Rays tend towards Y, the reflected ones will diverge as from X, thoſe, which

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is 5 Parts, and the Diſtance of the *Radiant* from the Surface is 20.

Sol. Call the focal Diſtance ſought x, then will the Diſtance of the *Focus* from the Center be $5-x$, and that of the *Radiant* from the ſame 25; therefore by Propoſition the third, we have the following Proportion, *viz.* $x : 20 :: 5 - x : 25$, and, multiplying Extreams together and Means together, we have $25x, = 100 - 20x$, which after due Reduction gives $x. = \frac{100}{45}$.

If in any Caſe it ſhould happen, that the Value of x ſhould be a negative Quantity, the focal Point muſt then be taken on the contrary Side the Surface to that on which it was ſuppoſed that it would fall in ſtating the *Problem*.

If Letters inſtead of Figures had been made uſe of in the foregoing Solution a general *Theorem* might have been raiſed, to have determined the focal Diſtance of reflected Rays in all Caſes whatever. See this done by *Dr. Brown* in his Supplement to *Gregory*'s Optics, pag. 112. Edit. Second.

Becauſe it was obſerved (Chap. III. in the Notes) that different incident Rays, though tending to or from one Point, would after *Refraction* proceed to or from different Points, a Method was there inſerted of determining the *diſtinct* Point, which each ſeparate Ray entring a ſpherical Surface converges to or diverges from after *Refraction*: the ſame has been obſerved here with regard to Rays *reflected* from a ſpherical Surface, (See *Obſervat.* 2 & 3.) but which were the incident ones in one Caſe, being the reflected ones in the other.

And laſtly, if the incident Rays converge towards the Center, they fall in with their reſpective Perpendiculars; on which Account they proceed after Reflection, as from thence.

but the Method of determining the *diſtinct* Point, to or from which any given incident Ray proceeds after *Reflection*, is much more ſimple. It is only neceſſary to draw the reflected Ray ſuch, that the Angle of Reflection may be equal to the Angle of Incidence, which will determine the Point it proceeds to or from in any Caſe whatever.

CHAP. X.

Of the Appearance *of Bodies ſeen by* Light *reflected from plain and ſpherical Surfaces.*

IN the Beginning of the ſeventh Chapter, in which was explained the Appearance of Bodies ſeen through *refracting* Subſtances of various Forms, we laid down ſome Obſervations reſpecting the apparent Situation of Bodies ſeen by *refracted* Light: all which equally reſpect the apparent Situation of Bodies ſeen by *Reflection*; to them therefore we refer the Reader. But beſides thoſe, there is one peculiar to the Subject of this Chapter, *viz.* *That each Point in the Repreſentation of an Object made by Reflection appears ſituated ſomewhere in an infinite right Line that paſſes through its correſpondent Point in the Object, and is perpendicular to the reflecting Surface.*

The Truth of this appears ſufficiently from the ſeveral Propoſitions laid down in the foregoing Chapter, in each of which Rays flowing from any *Radiant* are ſhewn to proceed after Reflection to or from ſome Point in a Line that paſſes through the ſaid *Radiant*, and is perpendicular to the reflecting Surface. For Inſtance (Fig. 43.) Rays flowing from Y are collected in X, a Point in the Perpendicular CD, that produced paſſes through Y; again (Fig. 44.) Rays flowing from G proceed after Reflection, as from N, a Point in the Perpendicular CD, that produced paſſes through G; and ſo for the reſt [a].

I. When an Object is ſeen by Reflection from a *plain* Surface, the Image of it appears at the *ſame Diſtance* behind the Surface that the Object is placed before it, of the *ſame Magnitude* therewith, and directly *oppoſite* to it.

To explain this, let AB (Fig 52.) repreſent an Object ſeen by Reflection from the plain Surface SV, and let the Rays AF, AG be ſo inclined to the Surface that they ſhall enter an Eye at H after Reflection; and let AE be perpendicular to the Surface: then by the Obſervation juſt laid down, the Point A will appear in ſome Part of the Line AE produced, ſuppoſe I; that is, the oblique Rays AF and AG will proceed after Reflection as from that Point, and further, becauſe the reflected Rays FH, GK, will have the ſame Degree of Inclination to each other, that their incident ones have (as was ſhewn in Propoſition the firſt of the foregoing Chapter) that Point muſt neceſſarily be at the ſame Diſtance from the Surface that the Point A is; the Repreſentation therefore of the Point A, will be at the ſame Diſtance behind the Surface, that the Point itſelf is before it, and directly oppoſite to it: conſequently ſince the like may be ſhewn of the Point B, or any other, the whole Image IM will appear at the *ſame Diſtance* behind the Surface that the Object is before it, and directly *oppoſite* to it; and becauſe the Lines AI, BM, which are perpendicular to the plain Surface, are for that Reaſon parallel to each other, it will alſo be of the *ſame Magnitude* therewith. As was to be ſhewn [b].

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[a] This Obſervation, except where an Object is ſeen by Reflection from a plain Surface, relates only to thoſe Caſes where the Repreſentation is made by means of ſuch Rays, as fall upon the reflecting Surface with a very ſmall Degree of Obliquity; becauſe ſuch, as fall at a conſiderable Diſtance from the Perpendicular, proceed not after Reflection as from any Point in that Perpendicular (See the ſecond and third Obſervations in the Note to the foregoing Chapter) but as from other Points ſituated in a certain Curve, as there explained; upon which Account theſe Rays are neglected as forming a confuſed and deformed Repreſentation. And therefore it is to be remembered, that however the Situation of the Eye, with reſpect to the Object and reflecting Surface, may be repreſented in the following Figures, it is to be ſuppoſed as ſituated in ſuch Manner with reſpect to the Object, that Rays, flowing from thence and entring it after Reflection, may be ſuch only as fall with a very ſmall Degree of Obliquity upon the Surface: that is, the Eye muſt be ſuppoſed to be placed almoſt directly behind the Object, or between it and the reflecting Surface. The Reaſon why it is not always ſo placed, is only to avoid Confuſion in the Figures.

II. When an Object is ſeen by Reflection from a *convex* Surface, its Image appears *nearer* to the Surface, and *leſs* than the Object.

Let

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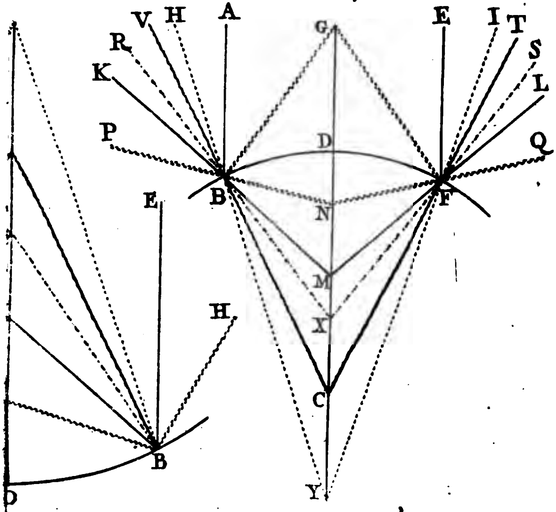
[b] If the Object be placed before a common Looking-glaſs, and viewed obliquely, three, four, or more Images of it will appear behind the Glaſs.

To

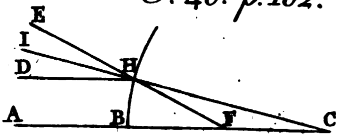
Part III. Plate VII. p. 120.

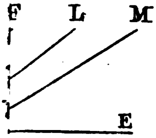
p. 102.

F. 44. p. 110.

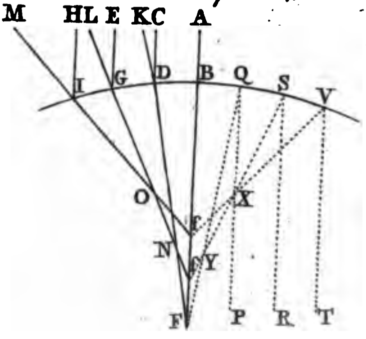


F. 46. p. 102.





F. 48. p. 104.



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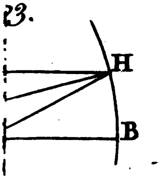

Let AB (Fig. 54.) repreſent the Object, SV a reflecting Surface, whoſe Center of Convexity is C: and let the Rays AF, AG, be ſo inclined to the Surface, that after Reflection thereat, they ſhall enter the Eye at H: and let AE be perpendicular to the Surface: then will the oblique Rays AF, AG, proceed after Reflection as from ſome Point in the Line AE produced, (by the Obſervation laid down at the Beginning of this Chapter) ſuppoſe from I; which Point, becauſe the reflected Rays will diverge more than

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To explain this, let ABCD (Fig. 53.) repreſent the Glaſs, and let EF be the *Axis* of a Pencil of Rays flowing from E, a Point in an Object ſituated there. The Rays of this Pencil will in Part be reflected at F, ſuppoſe into the Line FG, (ſee the Manner in which Light is reflected Chap. 18.) What remains will (after Refraction at F which we don't conſider here) paſs on to H; from whence (on Account of the Quickſilver which is ſpread over the ſecond Surface of Glaſſes of this Kind to prevent any of the Rays from being tranſmitted there) they will be ſtrongly reflected to K, where Part of them will emerge and enter an Eye at L: by this means one Repreſentation of the ſaid Point will be formed in the Line LK produced, ſuppoſe in M. Again, another Pencil whoſe *Axis* is EN, firſt reflected at N, then at O, and afterwards at P, will form a ſecond Repreſentation of the ſame Point at Q. And thirdly, another Pencil whoſe *Axis* is ER, after Reflection at the ſeveral Points R, S, H, T, V, ſucceſſively, will exhibit a third Repreſentation of the ſame Point at X; and ſo on *in infinitum*. The ſame being true of each Point in the Object, the whole will be repreſented in the like Manner; but the Repreſentations will be faint, in Proportion to the Number of Reflections the Rays ſuffer and the Length of their Progreſs within the Glaſs. We may add to theſe another Repreſentation of the ſame Object in the Line LO produced, made by ſuch of the Rays as fall upon O, and are from thence reflected to the Eye at L.

This may be tried by placing a Candle before the Glaſs as at E, and viewing it obliquely, as from L.

than the incident ones (Prop. the ſixth of the foregoing Chapter) muſt be nearer to the Surface than the Point A. And ſince the ſame is true alſo of the Rays which flow from B, or any other Point, the Repreſentation IM will be *nearer* to the Surface than the Object; and becauſe it is terminated by the Perpendiculars AE and BF which incline to each other, as concurring at the Center, it will alſo appear *leſs*.

III. When an Object is ſeen by Reflection from a *concave* Surface, the Repreſentation of it is *various*, both with regard to its Magnitude and Situation, according as the Diſtance of the Object from the reflecting Surface is *greater* or *leſs*.

1. When the Object is nearer to the Surface, than its *Focus* of parallel Rays, the Image falls on the *oppoſite* Side the Surface, is more *diſtant* from it, and *larger* than the Object.

Thus, let AB, (Fig. 55.) be the Object, SV the reflecting Surface, F the *Focus* of parallel Rays, C its Center. Through A and B the Extremities of the Object draw the Lines, CE, CR, which will be perpendicular to the Surface, and let the Rays AR, AG, be incident upon ſuch Points of it that they ſhall be reflected into an Eye at H. Now becauſe the *Radiant* Points A and B are nearer the Surface than F the *Focus* of parallel Rays, the reflected Rays will diverge (Chap. IX. Prop. 4.) and will therefore proceed as from ſome Points on the oppoſite Side the Surface; which Points, by the Obſervation laid down at the Beginning of this Chapter, will be in the Perpendiculars AE, BR, produced, ſuppoſe in I and M: but they will diverge in a leſs Degree than their incident ones (ſee the Propoſition juſt referred to) and therefore the ſaid Points will be farther from the Surface than the Points A and B. The Image therefore will be on the *oppoſite* Side the Surface with reſpect to the Object, it will be *more diſtant* than it, and conſequently, being terminated by the Perpendiculars CI and CM, it will alſo be *larger*.

2. When the Object is placed in the *Focus* of parallel Rays, the reflected Rays enter the Eye parallel (Chap. IX. Prop. 4.) in which Caſe the Image ought to appear at an *infinite* Diſtance behind the reflecting Surface; but the Repreſentation of it, for the like Reaſons that were given in the foregoing Caſe, being *large* and *diſtinct*, we judge it not much farther from the Surface than the Image [c].

3. When the Object is placed between the *Focus* of parallel Rays and the Center, the Image falls on the *oppoſite* Side the Center, is *larger* than the Object, and in an *inverted Poſition*.

Thus let AB (Fig. 56.) repreſent the Object, SV the reflecting Surface, F its *Focus* of parallel Rays, and C its Center. Through A and B the Extremities of the Object draw the Lines CE and CN which will be perpendicular to the Surface; and let AR, AG, be a Pencil of Rays flowing from A. Theſe Rays proceeding from a Point beyond the *Focus* of parallel Rays will after Reflection converge towards ſome Point on the oppoſite Side the Center (Chap. IX. Prop. 4. Caſe 3.) which will fall upon the Perpendicular EC produced; but at a greater Diſtance from C than the Radiant A from which they diverged, (by the Propoſition and Caſe juſt referred to.) For the ſame Reaſon, Rays flowing from B will converge to a Point in the Perpendicular NC produced, which ſhall be farther from C than the Point B; from whence it is evident, that the Image IM is *larger* than the Object AB, that it falls on the *contrary* Side the Center, and that their Poſitions are *inverted* with reſpect to each other.

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[c] See what has been ſaid concerning the apparent Situation of Objects ſeen by parallel Rays, in Chapter VII.

4. If the Object be placed beyond the Center of Convexity the Image is then formed *between the Center and the* Focus *of parallel Rays*, is *leſs* than the Object, and its Poſition is *inverted.*

This Propoſition is the Converſe of the foregoing: for as in that Caſe Rays proceeding from A were reflected to I, and from B to M; ſo Rays flowing from I and M will be reflected to A and B; if therefore an

Object be ſuppoſed to be ſituated beyond the Center in IM, the Image of it will be formed in AB, *between that and the* Focus *of parallel Rays*, will be *leſs* than the Object, and *inverted.*

5. If the Middle of the Object be placed in the Center of Convexity of the reflecting Surface, the Object and its Image will be *coincident*; but the Image will be *inverted* with reſpect to the Object.

That the Place of the Image and the Object ſhould be the ſame in this Caſe needs little Explication; for the Middle of the Object being in the Center, Rays flowing from thence will fall perpendicularly upon the Surface, and therefore neceſſarily return thither again; ſo that the Middle of the Image will be *coincident* with the Middle of the Object. But that the Image ſhould be inverted is perhaps not ſo clear. To explain this, let AB (Fig. 57.) be the Object having its middle Point C in the Center of the reflecting Surface SV; through the Center and the Point R draw the Line CR which will be perpendicular to the reflecting Surface, join the Points AR and BR, and let AR repreſent a Ray flowing from A, this will be reflected into RB, for C being the middle Point between A and B the Angles ARC and CRB are equal; and a Ray from B will likewiſe be reflected to A; and therefore the Poſi-

tion

tion of the Image will be *inverted* with reſpect to that of the Object [d].

6. If in any of the three laſt Caſes, in each of which the Image is formed on the ſame Side the reflecting Surface with the Object, the Eye be ſituated farther from the Surface than the Place where the Image falls, the Rays of each Pencil, croſſing each other in the ſeveral Points of the Image, will enter the Eye as from a real Object ſituated there; ſo that the Image will appear *pendulous* in the Air between the Eye and the reflecting Surface, and in the Poſition wherein it is formed, *viz. inverted* with reſpect to the Object, in the ſame Manner that an Image formed by *refracted* Light appears to an Eye placed beyond it; which was fully explained under the fourth Propoſition of the ſeventh Chapter, and therefore needs not be repeated here.

But as to what relates to the Appearance of the Object when the Eye is placed nearer to the Surface than the Image, that was not there fully inquired into. That Point ſhall therefore now be more ſtrictly examined under the following Caſe, which equally relates to *refracted* and *reflected* Light.

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[d] In this Propoſition it is to be ſuppoſed that the Object AB is ſo ſituated with reſpect to the reflecting Surface, that the Angle ACR may be right; for otherwiſe the Angles ARC and BRC will not be equal, and Part of the Image will therefore fall upon the Object and part off.

7. If the Eye be ſituated between the reflecting Surface and the Place of the Image, the Object is then ſeen *beyond* the Surface; and the farther the Eye recedes from the Surface towards the Place of the Image, the *more confuſed*, *larger*, and *nearer* the Object appears.

To explain this, let AB (Fig. 58.) repreſent the Object, IM its Image, one of whoſe Points, M, is formed by the Concurrence of the reflected Rays DM, EM, &c. which before Reflection came from B; the other, I, by the Concurrence of DI, EI, &c. which came from A: and let *ab* be the Pupil of an Eye ſituated between the Surface DP and the Image. This Pupil will admit the Rays Ha, Kb, which, becauſe they are tending towards I, are ſuch as came from A, and therefore the Point A will appear diffuſed over the Space RS. In like Manner the Pupil will alſo receive into it the reflected Rays Ka and Lb, which, becauſe they are tending towards M, by Suppoſition came from B; and therefore the Point B will be ſeen ſpread as it were over the Space TV, and the Object will ſeem to fill the Space RV; but the Repreſentation of it will be *confuſed*, becauſe the intermediate Points of the Object, being equally inlarged in Appearance, there will not be Room for them between the Points S and T, but they will coincide in part one with another; for Inſtance, the Appearance of that Point in the Object, whoſe Repreſentation falls upon *c* in the Image, will fill the Space *mn*, and ſo of the reſt. Now if the ſame Pupil be removed into the Situation *ef*, the reflected Rays E*e* and G*f* will then enter the Eye, and therefore one Extremity of the Object will appear to cover the Space XY; and becauſe the Rays O*f* and L*e* will alſo enter it in their Progreſs towards M, the Point B, from whence they came, will appear to cover ZV; the Object therefore will appear *larger* and *more confuſed* than before. And when the Eye recedes quite to the Image, it ſees but one ſingle Point of the Object, and that appears diffuſed all over the reflecting Surface: for Inſtance, if the Eye recedes to the Point M, then Rays flowing from the Point B enter it upon whatever Part of the Surface they fall: and ſo for the reſt. The Object alſo appears *nearer* to the Surface, the farther the Eye recedes from it towards the Place of the Image, probably becauſe as the Appearance of the Object becomes more and more confuſed, its Place is not ſo eaſily diſtinguiſhed from that of the reflecting Surface itſelf, till at laſt when it is quite confuſed (as it is when the Eye is arrived at M) they both appear as one, the Surface aſſuming the Colour of the Object[e].

As

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[e] As to the preciſe apparent Magnitude of an Object ſeen after this Manner, it is ſuch that the Angle it appears under ſhall be equal to that which the Image of the ſame Object would appear under, were we to ſuppoſe it ſeen from the ſame Place: that is, the apparent Object (for ſuch I muſt call it to

diſtinguiſh

Part III. Plate VIII. p. 128.

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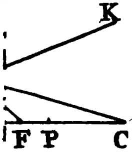
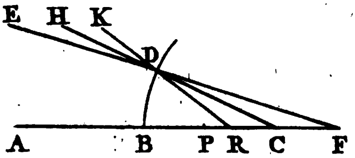
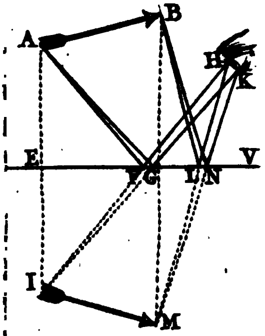


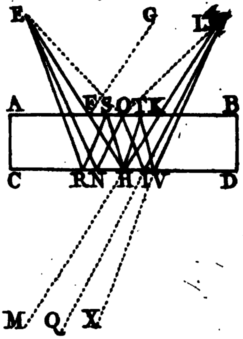
F. 50. p. 106.



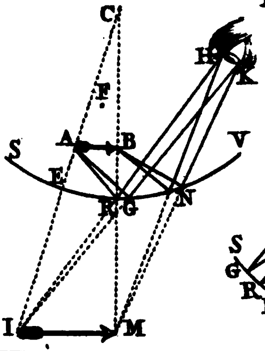
F. 52. p. 119.



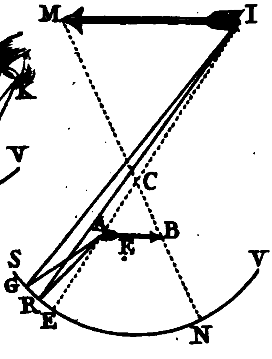
F. 53. p. 120.



F. 55. p. 122



F. 56. p. 123



As each Point in the Repreſentation of an Object made by Reflection is ſituated ſomewhere in a right Line that paſſes through its correſpondent Point in the Object, and is perpendicular to the reflecting Surface, as was ſhewn in the Beginning of this Chapter; we may from hence deduce a moſt eaſy and expeditious Method of determining both the

diſtinguiſh it from the Image of the ſame Object) and the Image ſubtend equal Angles at the Eye.

Dem. Here we muſt ſuppoſe the Pupil of the Eye to be a Point only, becauſe the Magnitude of that cauſes ſome ſmall Alteration in the apparent Magnitude of the Object; as we ſhall ſee by and by. Let then the Point a repreſent the Pupil, then will the extreme Rays that can enter it be Ha and Ka, the Object therefore will appear under the Angle HaK, which is equal to its vertical one MaI under which the Image IM would appear, were it to be ſeen from a. Again if the Eye be placed in f, the Object appears under the Angle GfO equal to IfM which the Image ſubtends at the ſame Place, and therefore the apparent Object and Image of it ſubtend equal Angles at the Eye. Q. E. D.

Now if we ſuppoſe the Pupil to have any ſenſible Magnitude, ſuch ſuppoſe that its Diameter may be ab, then the Object ſeen by the Eye in that Situation will appear under the Angle HxL, which is larger than the Angle HaK under which it appear'd before; becauſe the Angle at x is nearer than the Angle at a, to the Line IM, which is a Subtenſe common to them both.

From this Propoſition it follows, that, were the Eye cloſe to the Surface at K, the real and apparent Object would be ſeen under equal Angles (for the real Object appears from that Place under the ſame Angle that the Image does, as will be ſhewn at the End of this Chapter) therefore when the Eye is nearer to the Image than that Point, the Image will ſubtend a larger Angle at it than the Object does; and conſequently ſince the Image and apparent Object ſubtend equal Angles at the Eye, the apparent Object muſt neceſſarily be ſeen under a *larger* Angle than the Object itſelf, wherever the Eye be placed between the Surface and the Image.

Magnitude and Situation of the Image in all Caſes whatever. Thus,

Through the Extremities of the Object AB, and the Center C, (Fig. 59, 60, or 61.) draw the Lines AC, BC, and produce them as the Caſe requires; theſe Lines will be perpendicular to the reflecting Surface, and therefore the Extremities of the Image will fall upon them. Through F the middle Point of the Object and the Center, draw the Line FC and produce it till it paſſes through the reflecting Surface, this will alſo be perpendicular to the Surface. Through G, the Point where this Line cuts the Surface, draw the Lines AG and BG and produce them this way or that, till they croſs the former Perpendiculars; and where they croſs, there I and M the Extremities of the Image will fall. For ſuppoſing AG to be a Ray proceeding from the Point A and falling upon G, it will be reflected to B; becauſe FA is equal to FB, and FG is perpendicular to the reflecting Surface; and therefore the Repreſentation of the Point A will be in BG produced as well as in AC, conſequently it will fall on the Point I where they croſs each other. Likewiſe the Ray BG will for the ſame Reaſon be reflected to A, and therefore the Repreſentation of the Point B will be in AG produced as well as in ſome Part of BC, that is, in M where they croſs. From whence the Propoſition is clear.

If it happens that the Lines will not croſs which way ſoever they are produced, as in (Fig. 62.) then is the Object in the *Focus* of parallel Rays of that Surface, and has no Image formed in any Place whatever. For in this Caſe the Rays AH, AG, flowing from the Point A, become parallel after Reflection in the Lines HC, GB, and therefore do not flow as to or from any Point: in like Manner Rays flowing from B are reflected into the parallel Lines KB and GA; ſo that no Repreſentation can be formed by ſuch Reflection.

From hence we learn another Circumſtance relating to the Magnitude of the Image made by Reflection; *viz.* that it ſubtends the ſame Angle at the *Vertex* of the reflecting Surface that the Object does. This appears by Inſpection of the 59, 60, or 61ſt Figure, in each of which the Angle IGM, which the Image ſubtends at G the *Vertex* of the reflecting Surface, is equal to the Angle AGB, which the Object ſubtends at the ſame Place; for in the two firſt of thoſe Figures they are vertical, in the third they are the ſame. And

Farther, the Angle ICM, which the Image ſubtends at the Center, is alſo equal to the Angle ACB which the Object ſubtends at the ſame Place; for in the two firſt Figures they are the ſame, in the laſt they are vertical to each other.

From whence it is evident, that the Object and its Image are to each other in Diameter, either as their reſpective Diſtances from the *Vertex* of the reflecting Surface, or as their Diſtances from the Center of the ſame.

IV. As Objects are multiplied by being ſeen through tranſparent *Media*, whoſe Surfaces are properly diſpoſed, as was explained (Chap. VII. Prop. 4.) ſo they may alſo by reflecting Surfaces. Thus,

1. If two reflecting Surfaces be diſpoſed at right Angles, as the Surfaces AB, BC, (Fig. 63.) an Object at D may be ſeen by an Eye at E, after one Reflection at F, in the Line EF produced; after two Reflections, the firſt at G, the ſecond at H, in the Line EH produced; and alſo after one Reflection made at A, in the Line EA produced.

2. If the Surfaces be parallel, as AB, CD, (Fig. 64.) and the Object be placed at E and the Eye at F, the Object will appear multiplied an *infinite* Number of Times: Thus, it may be ſeen in the Line FG produced, after one Reflection at G; in the Line FH produced, after two Reflections, the firſt at I, the ſecond at H; and alſo in FP produced, after ſeveral ſucceſſive Reflections of the Ray EL, at the Points L, M, N, O, and P: and ſo on *in infinitum*. But the greater the Number of Re-

flections are, the weaker the Repreſentation will be.

There are reflecting Surfaces made, ſome cylindrically convex, others cylindrically concave; both which Kinds are deſigned to render the Image of an Object deformed, or the contrary, by augmenting or diminiſhing it in one Dimenſion and not in another; for the former Sort are convex one way, *viz.* round their *Axes*, and plain the other, *viz.* lengthwiſe; the other are concave round their *Axes*, and plain the other way: upon which Account Surfaces of theſe Kinds neceſſarily have ſuch Effects.

When a very deformed Picture, or rather no Picture at all, but a ſeemingly irregular and accidental Poſition of Colours is placed before one of theſe Surfaces, and ſeen by Reflection from thence, a beautiful and well proportion'd Image ſhall appear therein. But to explain *Phænomena* of this Kind is not the Intent of this *Compendium.* Enough has been ſaid to explain the Principles of *Catoptrics.* Paſs we on now to the third and laſt Diviſion of *Optics*, viz. *the Doctrine of Colours.*

CHAP. XI.

Of the different Refrangibility *in the* Rays *of* Light; *of the* Colours *the* diſtinct Species *of them are diſpoſed to excite; and of the Cauſe of that* Variety *of* Colours *which is obſervable in Bodies.*

IN treating of the Refraction of Light in the Beginning of this Part, we ſuppoſed that all Light, in paſſing out of one *Medium* into another of different Denſity, is equally refracted in the ſame or like Circumſtances. This is the Notion the Philoſophers before Sir *Iſaac Newton*'s Time had of it; but that indefatigable and circumſpect Author has diſcovered that it is not ſo, but that *there are different* Species *of Light; and that each* Species *is diſpoſed both to ſuffer a different Degree of Refrangibility in paſſing out of one* Medium *into another, and to excite in us the* Idea *of a different Colour from the reſt; and that Bodies appear of that Colour which ariſes from the Compoſition of the Colours the ſeveral* Species *they reflect are diſpoſed to excite.*

There are Abundance of Experiments made by Sir *Iſaac Newton* and others for the Confirmation of this Doctrine; I ſhall only ſelect the following ones, which will ſufficiently illuſtrate the Propoſition, and evince the Truth of it. And

Firſt, There are different *Species* of Light, and each *Species* is diſpoſed to ſuffer a different Degree of Refrangibility, and to excite the *Idea* of a different Colour.

To ſhew this, let a Room be darkened, and the Sun permitted to ſhine into it through a ſmall Hole in the Window-ſhutter, and be made to fall upon a Glaſs *Priſm* (by which is meant a Piece of Glaſs of a triangular Form, ſuch as is repreſented in Figure 65.) then will the Sun's Light in paſſing through this *Priſm* ſuffer different Degrees of Refraction, and by that means be parted into different Rays, which Rays being received upon a clean white Paper will exhibit the following Colours, *viz. Red, Orange, Yellow, Green, Blue, Indigo*, and a *Violet Purple.* Thus, let AB (Fig. 66.) repreſent the Window-ſhutter, C the Hole in it, DEF the Priſm, ZY a Ray of Light coming from the Sun, which paſſes through the Hole and falls upon the Priſm at Y, and if the *Priſm* were removed would go on to X, but in entring its firſt Surface EF ſhall be refracted into the Courſe YW, falling upon the ſecond in W, where in going out into the Air it ſhall be refracted again. Let the Light now, after it has paſſed the *Priſm*, be received upon a

Sheet of white Paper GHIK held at a proper Diſtance, and it will exhibit upon the Paper a Picture or Image at LM of an oblong Figure, whoſe Ends are ſemicircular and Sides ſtrait. And it ſhall be variegated with Colours after the following Manner. From the Extremity M to ſome Length ſuppoſe to the Line *no*, it ſhall be of an intenſe *Red*; from *no* to *pg* it ſhall be of an *Orange* Colour; from *pg* to *rs* it ſhall be *Yellow*; from thence to *tu* it ſhall be *Green*; from thence to *wx Blue*; from thence to yz *Indigo*; and from thence to the End *Violet*. And if the whole Image be divided lengthwiſe into 360 equal Parts, the *Red* ſhall take up 45 of them, the *Orange* 27, the *Yellow* 48, the *Green* 60, the *Blue* 60, the *Indigo* 40, and the *Violet* 80[f].

To

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[f] Sir *Iſaac Newton* in his *Opitics* has ſhewn, how from the Refraction of the moſt refrangible and leaſt refrangible Rays, to find the Refraction of all the intermediate ones. His Rule is this, if the Sine of Incidence be to the Sine of Refraction in the leaſt refrangible Rays as AV to BC (Fig.67.) and to the Sine of Refraction in the moſt refrangible as AV to BD; and if CE be taken equal to CD, and then ED be ſo divided in F, G, H, I, K, L, that ED, EF, EG, EH, EI, EK, EL, EC, may be proportional to the eight Lengths of muſical Chords, which ſhall ſound the Notes in an Octave, ED being the Length of the *Key*, EF the Length of the *Tone* above that Key, EG the Length of the *leſſer Third*, EH of the *Fourth*, EI of the *Fifth*, EK of the *greater Sixth*, EL of the *Seventh*, and EC of the *Octave* above that Key; that is, If the Lines ED, EF, EG, EH, EI, EK, EL, and EC bear the ſame Proportion to each other as the Numbers

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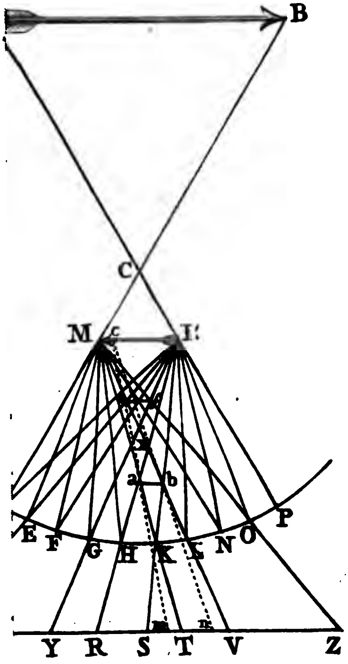
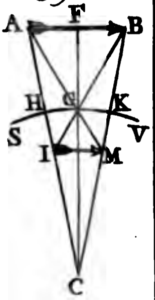
Part III. Plate IX. p. 136.

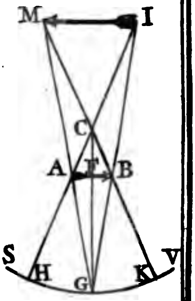
F. 58. p. 127.



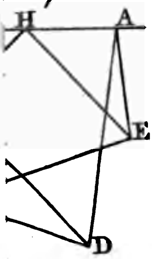
F. 59.



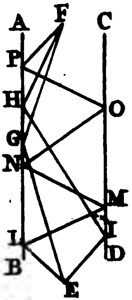
F. 61.



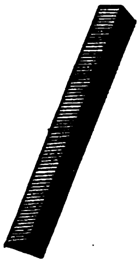
63. p. 132.



F. 64.



F. 65.



To render this Proof complete, we muſt now ſhew, that theſe Diſpoſitions of the Rays of Light, to produce ſome one Colour, and ſome another, which manifeſt themſelves after being refracted, are not wrought by any Action of the Priſm upon them, but are originally inherent in thoſe Rays; and that the Priſm only affords each *Species* an Occaſion of ſhewing its diſtinct Quality, by ſeparating them one from the other, which before, while they were blended together in the unfracted Light of the Sun, lay concealed.

This will be proved by the following Experiment, Things remaining as in the foregoing one,

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1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$, reſpectively, then ſhall BD and BF be the Limits of the Sines of Refraction of the *violet* Rays; that is, the *violet* coloured Rays ſhall not all of them have preciſely the ſame Sine of Refraction, but none of them ſhall have a greater Sine than BD nor a leſs than BF, though there be *violet* coloured Rays which anſwer to any Sine of Refraction that can be taken between theſe two. In the ſame manner BF and BG are the Limits of the Sines of Refraction of the *Indigo*; BG and BH are the Limits belonging to the *Blue*; BH and BI, the Limits pertaining to the *Green*; BI and BK, the Limits for the *Yellow*; BK and BL, the Limits for the *orange* coloured Rays; and laſtly BL and BC, thoſe of the Sines of Refraction belonging to the *Red*.

And particularly, when Light paſſes out of Glaſs into Air, if the Sine of its Angle of Incidence be 50, the Sine of the Angle of Refraction of the *Red* will be between 77 and 77$\frac{1}{8}$, of the *Orange* coloured between 77$\frac{1}{8}$ and 77$\frac{1}{5}$, of the *Yellow* between 77$\frac{1}{5}$ and 77$\frac{1}{3}$, of the *Green* between 77$\frac{1}{3}$, and 77$\frac{1}{2}$, of the *Blue* between 77$\frac{1}{2}$, and 77$\frac{2}{3}$, of the *Indigo* between 77$\frac{2}{3}$ and 77$\frac{7}{9}$ and of the *violet* coloured Rays between 77$\frac{7}{9}$ and 78.

let another Priſm, as NO (Fig. 68.) be placed either cloſe to, or at ſome Diſtance from the firſt, in a perpendicular Situation with reſpect to the former, ſo that it may refract the Rays iſſuing from the firſt ſideways. Now, if this Priſm could ſeparate the Light which falls upon it into coloured Rays, as the other did, it would divide the Image breadthwiſe into Colours, as before it was divided lengthwiſe; but no ſuch Thing is obſervable: For the Image ſhall only be thrown out of the perpendicular Situation LM into the oblique one PQ; the upper Parts, which were more refracted in the former Caſe, being more refracted in this, and therefore made to recede farther ſideways from their former Situation L, than the lower ones are from M. And farther, each Colour ſhall be uniform from Side to Side in the oblique Image, as well as in the perpendicular one.

If there be any Objection againſt the Sufficiency of this Proof, if muſt be, that the Rays, when they fall upon the ſecond Priſm, are not all in like Circumſtances, with regard to their Inclination to its Surface; I ſhall therefore, to obviate that Objection, add one more Experiment which ſeems to be peculiarly adapted to that Purpoſe. It is as follows:

Two boards AB, CD, (Fig. 69.) being erected in a darkened Room at a proper Diſtance, one of them AB, being near the Window-ſhutter EF; A Space being only left for the

Priſm GHI to be placed between them; ſo that Part of the Rays, which enter the Hole M, may, after paſſing through the Priſm, be tranſmitted through a ſmaller Hole K made in the Board AB, and paſſing on from thence go out at another Hole L, made in the Board CD, of the ſame Size as the Hole K, and ſmall enough to tranſmit the Rays of one Colour only at a Time: Let another Priſm PQR be placed behind the Board CD to receive the Rays paſſing through the Holes K and L, and after Refraction by that Priſm, let the Rays fall upon the white Surface ST. Suppoſe, firſt, the *violet* Light to paſs through the Holes, and to be refracted by the Priſm PQR to *s*, which, if that Priſm were not there, would have paſſed on to W. If the Priſm GHI be turned about ſlowly, ſo that the incident Ray ZY may fall more obliquely upon it, while the Boards and the other Priſm remain fixed, in a little Time another Colour, ſuppoſe *Indigo*, which we may ſuppoſe before to have proceeded to *i*, will paſs through the Holes K and L, and, if the Priſm PQR were away, would proceed like the former Rays to the ſame Point W. Now the Refraction of this Priſm will not carry theſe Rays to *s* as it did the other, but to ſome Place leſs diſtant from W, as to *t*. But it is manifeſt that the Holes K and L being in the ſame Situation in each Caſe, both Sorts of Rays enter the Priſm PQR under the ſame

Circumſtances, for they are equally inclined to its Surface RP, and enter it at the ſame Point thereof; which ſhews that the one *Species* is more diverted out of its Courſe by Refraction than the other is, when the Circumſtances of Incidence are the ſame in each. Farther, if the Priſm GHI be turned about till the Rays which exhibit *Blue* paſs through the Hole L, theſe will fall upon the Surface ST below *t*, as at *u*, and therefore are ſubject to a leſs Degree of Refraction than ſuch as produce *Indigo*. And thus by proceeding it will be found that the *Green* is leſs refracted than the *Blue*, and ſo of the remaining Colours, according to the Order in which they are repreſented in an Image formed by a ſingle Priſm [g].

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[g] There are Abundance of Experiments made by the Author of this Doctrine and others for the Confirmation of it, as was obſerved above. To give them all at full Length would tire the Reader. As that if a Body be painted one half Red, and the other Blue, and then viewed through a Priſm; the apparent Place of the one half ſhall be different from that of the other: and if it be painted with a Mixture of theſe two Colours it ſhall appear confuſed and deformed. Both which evidently ſhew that the Rays, which each of theſe Colours reflect, ſuffer different Degrees of Refraction in paſſing through the Priſm. And if two Bodies be painted, the one Red and the other Blue, and the Rays which flow from them be made to paſs through a convex *Lens*, the *Focus*, made by the Concurrence of the Rays which flow from that which is painted with Blue, ſhall fall at a leſs Diſtance behind the *Lens*, than that which is made by thoſe which come from the Red one. See the Experiment made by Dr. *Deſaguliers* in a very accurate Manner (Philoſoph. Tranſ. N°. 426.) in Oppoſition to *Signr. Ritzetti*, who diſputes the Concluſiveneſs of Sir *Iſaac Newton*'s Experiments.

And alſo each *Species* of Rays is diſpoſed to excite in us the *Idea* of a different Colour.

This is ſufficiently clear from what has been aleady ſaid, and is farther confirmed by what follows, *viz.* That whatever Species of Rays are thrown upon any Body, they make that Body appear of their own Colour. Thus *Minium* in *red* Light appears of its own Colour; but in *yellow* Light it appears yellow; and in *green* Light it appears green; in *blue*, blue; and in *violet-purple* coloured Light it appears of a purple Colour: in like manner Verdigreaſe will put on the Appearance of that Colour in which it is placed. But each of theſe Bodies appears moſt luminous and bright when enlightned with its own Colour, and dimmeſt in ſuch as are moſt remote from that. 'Tis certain therefore each Ray is diſpoſed to excite its own Colour, which is neither to be alter'd by Refraction nor Reflection.

Thus much in Confirmation of the firſt Part of the Propoſition, *viz.* That there are different *Species* of Light, that each *Species* is diſpoſed to ſuffer a different Degree of Refrangibility, and to excite in us the *Idea* of a different Colour. We proceed now to the ſecond Part of the Propoſition, *viz.*

2. That Bodies appear of that Colour, which reſults from a Compoſition of thoſe Colours,

which the ſeveral *Species* they reflect are diſpoſed to excite.

We have juſt now ſeen that each Ray, whatever be the Colour of the Body it is reflected from, is able to excite no other *Idea* than that of its own Colour, and that coloured Bodies reflect not all the different Sorts of Rays that fall upon them in equal Plenty; but ſome Sorts, *viz.* thoſe of their own Colour, much more copiouſly than others. We will now proceed to ſhew, that the other Colours may be produced from a Mixture of thoſe ſeven, which Rays of Light when ſeparated by a Priſm are diſpoſed to exhibit. From whence it will be rational to conclude, that Bodies appear of that Colour which ariſes from the Mixture of thoſe which they reflect.

1, All the *priſmatic* Colours (*viz.* thoſe which are made by the Priſm) mixed together appear white a little inclining to Yellow, ſuch as is that of the Sun's Light.

To ſhew this, let a convex *Lens* be placed between the Priſm and the Paper which receives the Image, in order that the Rays ſeparated by it may be collected into a *Focus*; and let the *Focus* fall upon the Paper, then will the Spot where it fall appears white. And that the Whiteneſs of this focal Point is owing to the Union of thoſe Colours appears from hence, that if we remove the Paper from the focal Point, and ſuffer the Rays to croſs each other in the *Focus*, and if when they have proceeded to ſome Diſtance beyond, they be then received upon the Paper, the ſame coloured Image will be exhibited, but inverted, becauſe the Rays croſs each other in the *Focus*; an evident Proof that the Whiteneſs of the Spot was owing to nothing but the Mixture of the Rays conſtituting the ſeveral Colours of the Image. But if the Rays of any particular Colour be intercepted before they are collected in the ſaid Spot, it then appears not only of a different Colour from what it did before, but different from any of the *priſmatic* Colours taken ſeparately.

Or if the Circumference of a Wheel be painted with the *priſmatic* Colours taken in the ſame Proportion with reſpect to each other in which they are exhibited in the Image made by the Priſm, and the Wheel be turned ſwiftly about, the Circumference of that Wheel ſhall appear *white*: if they are taken in other Proportions, the Colour of the Wheel when turned about will vary accordingly. From whence this Part of the Propoſition is alſo abundantly clear.

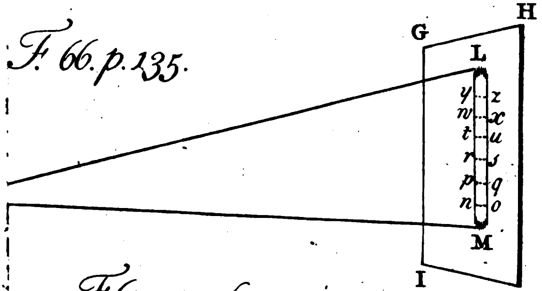
No Compoſition of theſe Colours will produce Black: That being no Colour, but the Defect, or Abſence of all Colour whatever.

That *Species* of Light, which is diſpoſed to ſuffer a greater Degree of Refraction, requires proportionally leſs Obliquity at the ſecond Surface of any *Medium* to occaſion a *total Reflection* of it there; ſo that it is poſſible that a Ray of Light may paſs through a *Medium* with ſuch Obliquity, that only that Part of it which is diſpoſed to exhibit a *violet* Colour ſhall be reflected at the ſecond Surface, and all the reſt tranſmitted there. This indeed is a neceſſary Conſequence of what was obſerved concerning the Reflection of Light at the ſecond Surface of any *Medium*; (Chap.8.) *viz.* that the Reflection of a Ray is total, when the Obliquity of the incident Ray is ſuch, that the Angle of Refraction ought to be equal to, or to exceed a right one. I ſay this is a Conſequence of that, becauſe the Angle of the Refraction of the *violet* coloured Light is larger than the Angle of Refraction of any other, though their Angles of Incidence be equal. And accordingly thus it happens, as appears by the following Experiment.

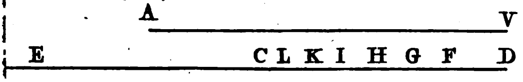
Let AB (Fig. 70.) repreſent the Window-ſhutter of a darkened Room; C an Hole to let in a Ray of the Sun; DEF, GHI, two Priſms ſo applied together that the Sides EF and GI be contiguous, and the Sides DF and GH parallel: In this Situation Light will paſs through them without any Separation into Colours; for the oppoſite Sides being parallel, if the Rays are refracted one Way where they

Part III. Plate X. p. 144.

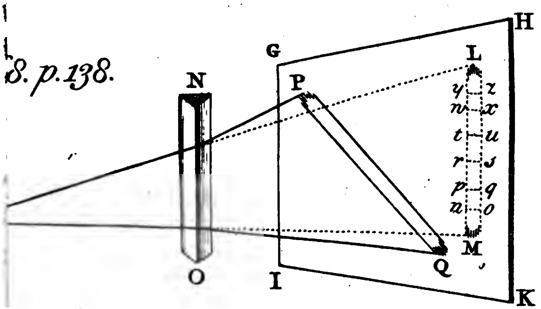
F. 66. p. 135.



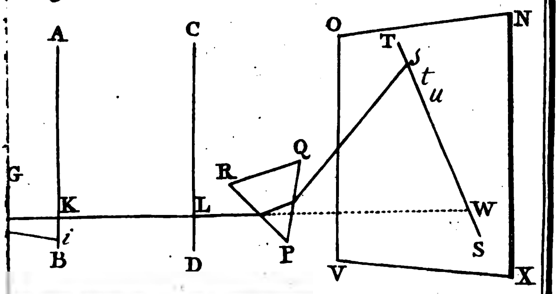
F. 67. p. 136.



8. p. 138.



p. 138.



go in, they will be as much refracted the contrary Way where they go out (See Page 51.) But if it be afterwards received by a third Priſm KLM, it will be divided ſo as to form upon any white Body NOYU the uſual Colours, *Violet* at *l*, *Indigo* at *m*, *Blue* at *n*, and *Red* at *r*. Now let it be ſuppoſed that the Surfaces EF and GI are not quite cloſe together, but that the Rays, in paſſing from one to the other, paſs through a *Medium* (*viz.* the Air) of different Denſity from that of the Priſms: and that the Ray ZC is not ſo much inclined to the ſecond Surface of the firſt Priſm as to cauſe a total Reflection of any one *Species* there; then will Part only of each *Species* be reflected and Part tranſmitted, agreeably to what was obſerved (Chapter 8.) concerning the Manner of Reflection. Let now the reflected Rays be received by a fourth Priſm TXV; theſe, after paſſing through it, will paint upon a white Surface RS the Colours of the Priſm, *viz.* *Red* at *s*, *Orange* at *t*, *Yellow* at *v*, and *Violet* at *z*. Let now the Priſms DEF, GHI, be ſlowly turned about, keeping ſtill the ſame Situation with reſpect to each other, until the Obliquity of the Rays ZC to the Surface EF be ſo far increaſed, that there ſhall begin to be a total Reflection of them there. In which Caſe it is obſervable, that firſt of all the *violet* Light will be *totally* reflected, and will therefore diſappear at *l*, appearing inſtead thereof at *z*, and increaſing the *violet* Light which fell there before. And when the Rays ZC become more oblique by the Priſms being turned farther about, the *Indigo* ſhall be totally reflected, diſappearing at *m*, but falling upon *y*, and making the *violet* there more intenſe. And by turning the Priſms ſtill farther about, all the remaining Colours will be ſucceſſively removed from the Surface PQ to RS.

CHAP. XII.

Of the Qualifications *in Bodies, which diſpoſe them to reflect the Rays of* different Colours.

WE are now to inquire what it is that gives Bodies this Power of reflecting, ſome one ſort of Rays moſt copiouſly, and ſome another. And this is probably no other than the different Magnitude of the Particles whereof they are compoſed; as will appear from the following Obſervations.

If Water be prepared with Soap ſo as to render it ſufficiently tenacious, and then blown up into a Bubble; it is obſervable, that as the Bubble grows thinner, and thinner (as it will do by reaſon of the Water's continually running down from the Top of it, till it breaks) different Colours will ariſe one after another at the Top of the Bubble, ſpreading themſelves into Rings, and deſcending till they vaniſh at the Bottom in the ſame Order they aroſe at the Top. Thus, in an Experiment of this Kind, tried by Sir *Iſaac Newton*, the Colours aroſe in this Order; firſt *Red*, then *Blue*; to which ſucceeded *Red* a ſecond Time, and

Blue immediately followed; after that *Red* a third Time, ſucceeded by *Blue*; to which followed a fourth *Red*, but ſucceeded by *Green*; after this a more numerous Order of Colours, firſt *Red*, then *Yellow*, next *Green* and after that *Blue*, and at laſt *Purple*; then again, *Red*, *Yellow*, *Green*, *Blue*, *Violet* followed each other; and the laſt Order of Colours that aroſe was *Red*, *Yellow*, *White*, *Blue*; to which ſucceeded a *dark* Spot that afforded ſcarce any Light, though it was obſerved to cauſe ſome very obſcure Reflection, for the Image of the Sun or Candle might be faintly diſcerned in it; and this laſt Spot ſpread itſelf more and more, till the Bubble broke.

Now 'tis apparent that the only Reaſon, why thoſe different Colours ſucceeded each other at the Top of the Bubble in the abovementioned Manner, was becauſe its Thickneſs in that Part continually varied, till it broke. It remained therefore to examine what was the Thickneſs of the Bubble at the Top, at the Time it exhibited each particular Colour. And this was effected by the following Contrivance, *viz.* by taking the Object-Glaſs of a long Teleſcope, ſuch having but a very ſmall Degree of Convexity, and placing it upon a flat Glaſs: theſe Glaſſes by reaſon of the Convexity of the former would touch but in one Point, and the Diſtance between them, where they did not touch, would be exceedingly ſmall, but larger the farther we conſider it from the Point of Contact. Now Water being put between theſe Glaſſes, the ſame Colours appeared as in the Bubble, in the Form of Circles or Rings ſurrounding the Point where the Glaſſes touched, which Point appeared black, like the Top of the Bubble when it was thinneſt. Next to this Spot lay a blue Circle, and next without that a white one, and ſo on in the ſame, but contrary Order to that in which the Colours aroſe on the Top of the Bubble.

Now the Diſtance between the Glaſſes, that is, the Thickneſs of the Body of Water between them, where it exhibited any one Colour of a particular Order, was equal to the Thickneſs of the Bubble at the Time the ſame Colour appeared upon it. For though the *Medium* the Light muſt paſs through to come at the Water is in one caſe, Glaſs, and in the other, Air; that makes no Difference in the *Species* of the Colour reflected from the Water: for Pieces of *Muſcovy* Glaſs, made thin enough to appear coloured, would have their Colours faded, but not the *Species* of them altered by being made wet with Water. But it was found that tranſparent Bodies of different Denſity would not, under the ſame Thickneſſes, exhibit the ſame Colours: for if the forementioned Glaſſes were laid upon each other without any Water between them, the Air between them would then afford the ſame Colours as the Water, but more expanded, ſo that each Ring had a larger Diameter, though they bore all the ſame Proportion to each other; ſo that the Thickneſs of the Air proper to reflect each Colour was in the ſame Proportion larger, than the Thickneſs of the Water adapted to reflect the ſame.

Farther, all the Light which is not reflected by the thin Subſtances, whether of Air or Water contained between the Glaſſes, is tranſmitted through them; for when viewed from the other Side, they exhibit alſo coloured Rings as before, but in a contrary Order; for the middle Spot, which in the other View appears black for Want of reflected Light, now looks perfectly white; next without this Spot the Light appears tinged with a yellowiſh *Red*; where the *White* appeared before, it now ſeems *Black*; and ſo of the reſt.

It is farther obſervable, that the forementioned thin Plates, whether of Air or Water, do not appear of the ſame Colour when viewed obliquely, as when ſeen direct: for if the Rings and Colours between a convex and plain Glaſs be viewed firſt in a direct Manner, and then under different Degrees of Obliquity, the Rings will be obſerved to dilate themſelves as the Obliquity is increaſed. But a Plate of Air between the Glaſſes alters its Colour much ſooner than the Water in the Bubble which is ſurrounded with Air. For in the Water when viewed obliquely the ſame Colour might be ſeen at more than twelve Times the Thickneſs it appeared at under a direct View; but when the Air was viewed under ſuch an Obliquity that the Thickneſs of the Plate, where it was obſerved, was but half as much again as when it was viewed directly, a different Colour appeared.

Laſtly, the ſame Colour reflected from a denſer Subſtance reduced to a thin Plate, and ſurrounded by a rarer, will be more briſk, than the ſame Colour, when reflected from a thin Plate formed of the rarer Subſtance, and ſurrounded by the denſer, as was found by blowing Glaſs very thin, which exhibited in the open Air more vivid Colours, than the Air does between two Glaſſes.

As to the Thickneſs of the Plate of Air by which the ſeveral Colours were reflected, it was found by carefully meaſuring the Diſtances of the Rings from the Point where the Glaſſes touched, that the Diſtance between the Glaſſes where the firſt Order of Colours was reflected, was from $\frac{1}{178000}$ to $\frac{2}{178000}$ Part of an Inch; that where the ſecond, was from $\frac{3}{178000}$ to $\frac{4}{178000}$; that where the third, from $\frac{5}{178000}$ to $\frac{6}{178000}$, and ſo on in a *Series* of the odd Numbers; and that the Diſtance of the Glaſſes, where the firſt Order of Colours that was tranſmitted paſſed through, was from 0 to $\frac{1}{178000}$ Part of an an Inch; that where the ſecond, was from $\frac{2}{178000}$ to $\frac{3}{178000}$; that where the third, from $\frac{4}{178000}$ to $\frac{5}{178000}$, and ſo on in a *Series* of the even Numbers. And the Thickneſs of a Plate of Water, where it reflected or tranſmitted the ſame Colours, was $\frac{3}{4}$ of the Thickneſs of the Plate of Air.

Now we learn from Experiments made with the Microſcope, that the leaſt Parts of almoſt all Bodies are tranſparent; or the ſame may be experienced in the following Manner: Take a very thin Plate of the opakeſt Body, and the Room being darkened apply it to a ſmall Hole in the Window-ſhutter, and it will ſufficiently diſcover its Tranſparency. This Experiment cannot be ſo well performed with a white Body, becauſe of the ſtrong reflective Power in ſuch; but even thoſe, when diſſolved in *Aqua Fortis* or other proper *Menſtruum*, do alſo become tranſparent. Wherefore if we ſhould ſuppoſe any Body reduced to a Thinneſs proper to produce any particular Colour and then broken into Fragments, in all Probability each Fragment would exhibit that Colour, and an Heap of ſuch Fragments would conſtitute a Body of that Colour: ſo that the Cauſe, why ſome Bodies reflect one Sort of Rays moſt copiouſly, and ſome another, is probably no other than the different Magnitude of their conſtituent Particles [h]. This will be farther confirmed by Particulars. The Colours in the ſame Part of a Peacock's Tail vary as the Tail changes its Poſture, with reſpect to the Eye; juſt ſo the thin Plates of Air or Water appear of a different Colour in the ſame Plate when view'd directly, from what they do when ſeen obliquely, as was obſerved above. The Colours of Silks, Cloths, and other Subſtances, which Water or Oil can intimately penetrate, become faint and dull by being wet with ſuch Fluids, and recover their Brightneſs when dry; juſt as we obſerved, that Plates of *Muſcovy* Glaſs grew faint and dim by wetting. All which Particulars, and many more that might be produced, give abundant Proof of the preſent Point [i].

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[h] This Sir *Iſaac Newton* thinks a probable Ground for making Conjecture concerning the Magnitude of the conſtituent Particles of Bodies. The Green of Vegetables he takes to be of the third Order, as likewiſe the Blue of Syrup of Violets. The azure Colour of the Sky, he thinks is of the firſt Order, as alſo the moſt intenſe and luminous White; but if it is leſs ſtrong, he then conjectures it to be a Mixture of the Colours of all Orders. Of the latter Sort he takes the Colour of Linen, Paper, and ſuch like Subſtances to be; but white Metals to be of the former Sort. For producing *Black*, the Particles muſt be ſmaller than for exhibiting any of the Colours, *viz.* of a Size anſwering to the Thickneſs of the Bubble where it reflected little or no Light, and for that Reaſon appeared colourleſs.

[i] See Sir *Iſaac Newton*'s Optics, *paſſim.*

CHAP. XIII.

Of the Cauſe of Opacity *and* Tranſparency *in Bodies.*

WHEN two *Mediums* or tranſparent Subſtances of equal Denſity are contiguous, or as near to each other as the Glaſſes were, where the Light was wholly tranſmitted, in the Experiment made with the Object Glaſs mentioned in the foregoing Chapter, a Ray of Light will paſs from one to the other without ſuffering either Reflection or Refraction; but if they differ in Denſity, the Light will undergo both; Part of it being reflected and Part refracted. Juſt ſo it is with a Ray of Light in paſſing through the different Particles of the ſame Body. For Inſtance, if when the Ray has paſſed through any one Particle of a Body, it finds another contiguous to it, it will enter that Particle without Interruption; but if at its Emerſion out of that Particle, it enters a Pore ſufficiently large, Part of it will be tranſmitted and Part reflected. Thus will the Light every time it enters a Pore, unleſs it be an exceeding ſmall one, be in part reflected: So that nothing more ſeems neceſſary to render a Body opake, than that the Particles, of which it is compoſed, touch but in few Points, and that the

Pores of it be very numerous and large; ſo that the Light, which enters it, may by numerous Refractions and Reflections be ſtifled and loſt within it. On the contrary, if the Particles of a Body touch each other in many Points, and its Pores be few and ſmall, or filled with a Subſtance of nearly an equal Denſity with the Particles of the Body, that Body will be tranſparent.

In confirmation of this, we may obſerve, that opake Bodies become tranſparent by filling their Pores with a Subſtance nearly of the ſame Denſity with that of their ſolid Parts: As when Paper is made wet with Water or Oil; when Linen Cloth is dipped in Water, oiled, or varniſhed, or the *Oculus Mundi* Stone ſteeped in Water. Beſides, as filling the Pores of an opake Body renders it tranſparent, ſo on the other hand evacuating the Pores of a Body that is tranſparent, or ſeparating the Parts of it from one another, makes it opake; as Salt or wet Paper by being dried, and Glaſs by being powder'd, loſe their Tranſparency, or Water beat up into Froth.

Beſides which Inſtances, abundance more might be brought in Confirmation of what is here laid down; but theſe are ſufficient.

But becauſe it may be Matter of Surprize, that Bodies ſhould be ſufficiently porous to tranſmit Light in that plenty we obſerve they do, and yet be hard or ſolid; I ſhall conclude the Subject of this Chapter and of the Doctrine of Light and Colours, with ſhewing the Conſiſtency of ſuch a Suppoſition. In order to which, let us imagine a Body whoſe conſtituent Particles are of ſuch a Form that, when laid together, the Vacancies between them may be equal in Bigneſs to the Particles; how this may be done, and yet the Body be hard, is eaſy to comprehend. Now the ſolid Parts of a Body thus formed will be but half its Bulk; and if we ſuppoſe each conſtituent Particle of this Body to be formed of leſs Particles with Vacuities between them, equal to each Particle as before, the ſolid Parts of this Body will then be but a fourth Part of its Bulk; and if each of theſe leſſer Particles again be formed in the ſame Manner, the ſolid Parts of the Body ſhall be but one Eighth of its Bulk: and thus if the Compoſition be continued according to the ſame Rule, the ſolid Parts of the Body may be made to bear as ſmall a Proportion to its whole Magnitude as ſhall be deſired, notwithſtanding which, the Body, by means of the Contiguity of the Parts, ſhall be capable of being hard in any Degree. Thus, Matter bebeing infinitely diviſible, any of the leaſt Portion of it may be ſuppoſed to be wrought into a Body of any deſigned Dimenſions how great ſoever, and yet the Pores of that Body none of them greater than the ſmalleſt Magnitude propoſed at Pleaſure; and yet the Parts of the Body ſhall ſo touch, that the Body itſelf ſhall be hard or ſolid. Which ſhews that the whole Globe of Earth, nay, all the known Bodies of the Univerſe, for any thing that appears to us to the contrary, may be compoſed of no greater Quantity of Matter than what might be reduced into a Globe of an Inch Diameter, or into a *Nut-ſhell.*

DISSERTATION II.

Of the Cauſe *of* Reflection *of* Light.

THE Opinions of Philoſophers relating to the Cauſe of this difficult *Phænomenon,* are principally four, which I ſhall here lay down and examine particularly; after which I ſhall give my own Thoughts, concerning it. And,

1. It was the Opinion of Philoſophers before Sir *Iſaac Newton* diſcovered the contrary, that Light is reflected by impinging upon the ſolid Parts of Bodies; but that it is not ſo, is clear from the following Reaſons.

And firſt, it is not reflected at the firſt Surface of a Body by impinging againſt it.

For it is evident, that in order to the due and regular Reflection of Light, that is, that the reflected Rays ſhould not be diſperſed and ſcattered one from another, there ought to be no Raſures or Unevenneſs in the reflecting Surface large enough to bear a ſenſible Proportion to the Magnitude of a Ray of Light: becauſe if the Surface abounds with ſuch, the reflected Rays will rather be ſcattered like a Parcel of Pebbles thrown upon a rough Pavement, than reflected with that Regularity with which Light is obſerved to be from a well poliſhed Surface. Now thoſe Surfaces which to our Senſes appear perfectly ſmooth and well poliſhed, are far from being ſo; for to poliſh is no other than to grind off the larger Eminences and Protuberances of the Metal with the rough and ſharp Particles of Sand, Emery, or Putty, which muſt of Neceſſity leave behind them an Infinity of Raſures and Scratches, which though inconſiderable with regard to the former Roughneſſes, and too minute to be diſcerned by us, muſt nevertheleſs bear a large Proportion to, if not vaſtly exceed the Magnitude of the Particles of Light.

Secondly, it is not reflected at the ſecond Surface, by impinging againſt any ſolid Particles.

That it is not reflected by impinging upon the ſolid Particles which conſtitute this ſecond Surface, is ſufficiently clear from the foregoing Argument; the ſecond Surfaces of Bodies being as uncapable of a perfect Poliſh as the firſt; and it is farther confirmed from hence, *viz.* that the Quantity of Light reflected differs according to the different Denſity of the *Medium* behind the Body: And that it is not reflected by impinging upon the Particles which conſtitute the Surface of the *Medium* behind it, is evident, becauſe the ſtrongeſt Reflection of all at the ſecond Surface of a Body, is when there is a *Vacuum* behind it. This therefore wants no farther Proof. (See the Manner in which Light is reflected in Chapter the 8th.)

II. It has been thought by ſome[k]; that it is reflected at the firſt Surface of a Body, by a repulſive Force equally diffuſed over it, and at the ſecond, by an attractive Force.

1. If there be a repulſive Force diffuſed over the Surface of Bodies, that repels Rays of Light at all Times, then, ſince by increaſing the Obliquity of a Ray we diminiſh its perpendicular Force (which is that only, whereby it muſt make its Way through this repulſive Force) however weakly that Force may be ſuppoſed to act, Rays of Light may be made to fall with ſo great a Degree of Obliquity on the reflecting Surface, that there ſhall be a *total* Reflection of them there[l], and not one Particle of Light be able to make its Way through, which is contrary to Obſervation; the Reflection of Light at the firſt Surface of a tranſparent Body being never total in any

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[k] See *Muſchenbroek*, Element. Phyſic. Cap. 35.

[l] *Dem.* Let AB (Fig. 71.) repreſent the reflecting Surface, ABCD the repellent Power diffuſed over it, EF a Ray of Light incident upon it at the Point F, and let the Line EF by its Length expreſs the Force with which the Ray moves. This Force is reſolvable into the Forces EG and EH, or, which is the ſame thing HF and GF, which latter is the ſole Force by which the Ray endeavours to pierce through the repulſive Power. But this Force may be diminiſhed *in infinitum* by augmenting the Obliquity of the Ray EF, and therefore it may be made leſs than that of the repelling Power, in which Caſe the Ray will neceſſarily be reflected: and ſince the

Obliquity whatever. The *Hypotheſi* therefore in this Particular muſt be falſe [s].

2. As to the Reflection at the ſecond Surface by the attractive Force of the Body; this may be conſidered in two Reſpects; firſt, when the Reflection is *total*; ſecondly, when it is *partial*.

And firſt, in Caſes where the Reflection is *total*, the Cauſe of it is undoubtedly that ſame attractive Force by which Light would be refracted in paſſing out of the ſame Body. This is manifeſt from that Analogy which is obſervable between the Reflection of Light at this ſecond Surface, and its Refraction there. For otherwiſe, what can be the Reaſon that the total Reflection ſhould begin juſt when the Obliquity of the incident Ray, at its Ar-

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the ſame is demonſtrable of any Ray, (let it move with a greater or leſs Force than the Ray EF) the Obliquity of the Rays may be ſo great, that there ſhall be a total Reflection of them. Q. E. D.

[s] And yet from an Experiment made by Sir *Iſaac Newton*, (See his Optics Book III) it appears that there is a repulſive Force between Light and ſome Bodies. The Experiment is as follows. If over an Hole in a Window-ſhutter be fixed a thin Piece of Lead or the like, in which there is an Hole about the fortieth Part of an Inch in Diameter; and if when the Sun ſhines through that Hole, an Hair be held in its Ray at ſome Diſtance from the Hole; the Progreſs of the Rays after they have paſſed by the Hair will be as expreſſed in the 72d Figure, where A is a Section of an Hair, BC an Hole in a Window-ſhutter; BL, UF, DE, &c. Rays paſſing through it, in the Middle of which is placed the Hair A; and let RP be a Paper held at ſome Diſtance behind the Hair. Things being thus diſpoſed, the Ray BL ſhall fall upon O, UF upon Q, DE upon R, GH upon P, &c. as repreſented in the Figure.

rival at the ſecond Surface, is ſuch, that the refracted Angle ought to be a right one; or when the Ray, were it not to return in Reflection, ought to paſs on parallel to the Surface, without going from it? For in this Caſe it is evident, that it ought to be returned by this very Power, and in ſuch Manner that the Angle of Reflection ſhall be equal to the Angle of Incidence: juſt as a Stone thrown obliquely from the Earth, after it is ſo far turned out of its Courſe by the Attraction of the Earth, as to begin to move horizontally, or parallel to the Surface of the Earth, is then by the ſame Power made to return in a Curve ſimilar to that which it deſcribed in its Departure from the Earth, and ſo falls with the ſame Degree of Obliquity that it was thrown with.

But ſecondly, as to the Reflection at the ſecond Surface, when it is *partial*; an attractive Force *uniformly* ſpread over it, as the Maintainers of this *Hypotheſis* conceive it to be, can never be the Cauſe thereof. Becauſe, it is inconceivable that the ſame Force, acting in the ſame Circumſtances in every Reſpect, can ſometimes reflect the *Violet* coloured Rays and tranſmit the *Red*, and at other times reflect the *Red* and tranſmit the *Violet*.

This Argument concludes equally againſt a repulſive Force *uniformly* diffuſed over the firſt Surface of a Body, and reflecting Light there; becauſe ſome Bodies reflect the *Violet* and tranſmit the *Red*, others reflect the *Red* and tranſmit the *Violet* at their firſt Surface; which cannot poſſibly be upon this Suppoſition, the Rays of whichever of theſe Colours we ſuppoſe to be the ſtrongeſt.

III. Some, being apprehenſive of the Inſufficiency of a repulſive and attractive Force diffuſed over the Surfaces of Bodies and acting uniformly, have ſuppoſed, that by the Action of Light upon the Surfaces of Bodies the Matter of theſe Forces is put into an *undulatory* Motion, and that where the Surface of it is ſubſiding, Light is tranſmitted, and in thoſe Places where it is riſing, Light is reflected[n]. But this ſeems not to advance us one Jot farther; for in thoſe Caſes, ſuppoſe where *Red* is reflected and *Violet* tranſmitted, how comes it to paſs that the *Red* impinges only on thoſe Parts when the Waves are riſing, and the *Violet* when they are ſubſiding?

IV. The next *Hypotheſis*, that I ſhall take notice of, is that remarkable one of Sir *Iſaac Newton*'s *Fits of eaſy Reflection and Tranſmiſſion*, which I ſhall now explain and examine.

That Author, as far as I can apprehend his Meaning in this Particular, is of Opinion, that

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[n] See *Muſchenbroek* Element.. Phyſic. Cap. 35.

Light, in its Paſſage from the luminous Body, is diſpoſed to be *alternately* reflected by and tranſmitted through any refracting Surface it may meet with; that theſe Diſpoſitions (which he calls *Fits of eaſy Reflection and eaſy Tranſmiſſion*) return ſucceſſively at equal Intervals: and that they are communicated to it at its firſt Emiſſion out of the luminous Body it proceeds from, probably by ſome very ſubtle and elaſtic Subſtance diffuſed through the Univerſe, and that in the following Manner. As Bodies falling into Water, or paſſing through the Air cauſe Undulations in each, ſo the Rays of Light may excite Vibrations in this elaſtic Subſtance. The Quickneſs of which Vibrations depending on the Elaſticity of the *Medium* (as the Quickneſs of the Vibrations in the Air, which propagate Sound, depend ſolely on the Elaſticity of the Air, and not upon the Quickneſs of thoſe in the ſounding Body) the Motion of the Particles of it may be quicker than that of the Rays; and therefore when a Ray, at the Inſtant it impinges upon any Surface, is in that Part of a Vibration of this elaſtic Subſtance which conſpires with its Motion, it may be eaſily tranſmitted, and when it is in that Part of a Vibration which is contrary to its Motion, it may be reflected. He farther ſuppoſes, that when Light falls upon the firſt Surface of a Body, none is reflected there, but all that happens to it there is, that every Ray that is not in a Fit of eaſy Tranſmiſſion is there put into one, ſo that when they come at the *other* Side (for this elaſtic Subſtance eaſily pervading the Pores of Bodies, is capable of the ſame Vibrations within the Body as without it) the Rays of one Colour ſhall be in a Fit of eaſy Tranſmiſſion, and thoſe of another in a Fit of eaſy Reflection, according to the Thickneſs of the Body; the Intervals of the Fits being different in Rays of a different Kind. This very well accounts for the different Colours of the Bubble and thin Plate of Air and Water (mention'd in Chap. XII.) as is obvious enough; and likewiſe for the Reflection of Light at the ſecond Surface of a thicker Body; for the Light reflected from thence is alſo obſerved to be coloured, and to form Rings according to the different Thickneſs of the Body, when not intermix'd and confounded with other Light, as will appear from the following Experiment. If a Piece of Glaſs be ground concave on one Side and convex on the other, both its Concavity and Convexity having one common Center, and if a Ray of Light be made to paſs through a ſmall Hole in a Piece of Paper held in that common Center, and be permitted to fall on the Glaſs; beſides thoſe Rays which are regularly reflected back to the Hole again, there will be others reflected to the Paper, and form coloured Rings ſurrounding the Hole, not unlike thoſe occaſioned by the Reflection of Light from thin Plates. The ſame will happen if the Rays be reflected from a metalline *Speculum*, but the Light will not be *coloured*; which ſhews that the Colours ariſe from that Light which is reflected from the Back-ſide, and that in the following Manner: Beſide that Light which is regularly reflected from the farther Surface of the Glaſs, there is ſome reflected irregularly, which paſſing from the back Surface under different Obliquities, does as it were paſs through Glaſſes of different Thickneſſes, and therefore is in part reflected back again when it comes to the firſt Surface, and is in part tranſmitted through it, the tranſmitted Light, when received upon the white Paper exhibiting the Rings of Colours abovementioned[o].

As to the Light which is ſuppoſed to be reflected at the firſt Surface, his Opinion ſeems to be, that it is not *there* reflected, as I obſerved above, but that it really enters the Surface, and is reflected from the Back-ſide of the firſt *Series* of Particles that lie therein; ſo that according as theſe Particles are larger or ſmaller, the Rays of Light which at their Entrance into them

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[o] This Experiment ſucceeds better, when the Back part of the Glaſs is Quick-ſilver'd over.

(for they are tranſparent, whether the Body they compoſe be ſo or not; *See Page* 154) are thereby put into Fits of eaſy Tranſmiſſion; at their Emerſion at the other Side are ſome in a Fit of eaſy Tranſmiſſion, others in a Fit of eaſy Reflection, according as the Interval of their Fits are large or ſmall. So that the Particles of a Body may be of ſuch a Size that they ſhall reflect the *Red* and tranſmit the *Violet*; or that they may reflect the *Violet* and tranſmit the *Red*; or, in general, that the ſtrongeſt and moſt forcible Rays may be tranſmitted, while the weaker are reflected; or the weaker may be tranſmitted, while the ſtronger are reflected.

Thus I have endeavour'd to clear up the Account Sir ISAAC NEWTON has left us of his own Sentiments concerning this Matter. But after all, I cannot ſay, that I think his Solution the true one. It is too much clogged with Suppoſitions; neither is it conſonant to that Simplicity, Uniformity, and Regularity, with which Nature is every where obſerved to act. The Time will come, when the Principles of *Attraction* and *Repulſion* will be found alone ſufficient to account for this perplexing *Phænomenon.* Would any one that has a *Genius* for a Work of this Kind, and *Opportunity* to make the neceſſary Experiments, aſſume thoſe Sir *Iſaac Newton* has left, and add others, as his Judgment ſhould direct him; he would ſoon be able to give us as eaſy and ſimple a Solution of the *Reflection of Light*, as we already have of any other *Phænomenon* whatever. [p]

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[p] Perhaps it may be of Service to one that ſhall undertake this, to acquaint him of a Fact relating to this Matter, which every Philoſopher is not appriſed of, *viz.* That a Piece of Iron, when heated, aſſumes all the Colours of the Rainbow before it becomes red hot.

DISSER-

DISSERTATION III.

Of Microſcopes *and* Teleſcopes.

THAT the Teleſcope is of modern Invention is moſt certain; neither does it appear that Microſcopes or *Optic* Glaſſes of any Kind were known to the Ancients, though there are two Paſſages brought to ſhew that ſuch Glaſſes were of Uſe among them. The one is quoted by *Pancirollus* from *Plautus*, *Cedo Vitrum, neceſſe eſt* Conſpicillo *uti*; the other is taken from *Pliny*, *C. Julius Medicus, dum inungit* Specillum, *&c.* The former of theſe Quotations is a Fiction, no ſuch Paſſage being to be found in the Writings of *Plautus* [q]; and the Word *Specillum* in *Pliny* is not to be underſtood of an Optic-glaſs of any Kind, but of a Probe or other Inſtrument made uſe of by the Surgeons of that Time [r].

It is contended, that *Alexander de Spina*, a Native of *Piſa*, was the firſt that made the Uſe of Glaſſes known to the World; but our Countryman, Frier *Bacon*, who died one and twenty Years before him [ſ], was, in all Probability, acquainted with them firſt, for he wrote a book of *Perſpective*, in which, he plainly ſhews that he did not only underſtand the Nature of convex and concave Glaſſes, but the Uſe of them when combined in Teleſcopes [t]; though he no where in that Treatiſe diſcovers the Manner in which they are to be put together.

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[q] Vid. Lettere Memorabili del Abbate Michele Giuſtiani. Parte terza, Let. 16.

[r] See *Molineux*'s Dioptrics, Part II. Ch. 6.

[ſ] In the Year 1292.

The Teleſcope with the concave Eye-glaſs was firſt invented by a Mechanic of *Middleburg* in *Zeland*, called *Z. Johannides*, about the Year 1590, though *J. Lipperhoy*, another *Dutchman*, is Candidate for the ſame Diſcovery [u]. From whence, this Sort of Teleſcope is called *Tubus Batavus* [x].

Franciſcus Fontana, a *Neapolitan*, contends, that he was the firſt Contriver of the Teleſcope compoſed of two convex Glaſſes, which is now the common aſtronomical Teleſcope [y]; and *Rheita* pretends to be the firſt that rendered that Teleſcope fit for terreſtrial Uſes, by adding two Eye-glaſſes to it [z]. This Kind of Teleſcope is called *dioptrical.*

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[t] See his *Perſpective*, Part III. and his *Epiſtola ad Pariſienſem*, Cap. 5.

[u] Vid. *Borellus* de vero Teleſcopii Inventore.

[x] This is by ſome called *Galileo's* Teleſcope, as invented by him; but *Galileo* acknowledges, that it was upon hearing that the *Dutchman* had contrived one, that he effected his.

[y] See his *Obſervationes cæleſtium terreſtriumq; Rerum.*

[z] Ocul *Enoch* & *Eliæ*. Lib. IV. Hiſt. Acad. Reg. Lib. V. Sect. 1, Cap. 7.

The *Catadioptrical* or reflecting Teleſcope was invented by Sir *Iſaac Newton*; of which we ſhall give a particular Deſcription when we have explained the former Sort, and ſhewn the Defects of them.

Microſcopes are of two Kinds, Simple and Compound. The firſt Sort conſiſts of one Glaſs; the other of two or more.

The Simple Microſcope is no other than a convex *Lens*, through which, as we have ſhewn (Chap. VII. Prop. 3.) Objects appear magnified.

An Object ſeen through this Microſcope appears magnified nearly in that Proportion which the Diſtance, at which an Object would be ſeen diſtinctly with the naked Eye, bears to the focal Diſtance of the Microſcope.

Thus, let AB (Fig. 73.) repreſent the Microſcope, CD an Object placed at the focal Diſtance of parallel Rays, or ſomething nearer, that the Rays of the ſame Pencil may be parallel to each other, or rather diverging in a ſmall Degree, when they enter the Eye (this Circumſtance being requiſite to diſtinct Viſion:) And let the Microſcope be ſo ſmall, that all the Rays that paſs through it from the Object may enter the Pupil of the Eye EF at the ſame time, when placed cloſe to it as in the Figure (for, unleſs it be ſo ſmall, it will ſcarce magnify ſufficiently to obtain the Name of a Microſcope.) Things being thus diſpoſed, the Angle under which this Object appears will be GIH, or CID; but this is nearly the ſame it would have appeared under, had there been no Microſcope interpoſed [a]. Notwithſtanding which, the Object is properly enough ſaid to appear magnified by this Microſcope, becauſe, without that, it could not have been ſeen diſtinctly at ſo ſmall a Diſtance from the Eye, but muſt have been ſituated eight or ten Inches from it; and therefore, ſince Objects appear under a larger Angle the nearer they are to the naked Eye, this Object appears larger, or is magnified by means of the Microſcope, in Proportion as it is ſeen diſtinctly at a leſs Diſtance with it than without it; that is, nearly as the focal Diſtance of the Microſcope is to that at which Objects are ſeen diſtinctly with the naked Eye [b].

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[a] For had there been no Microſcope interpoſed, the Angle CID would have been in the Middle of the Pupil (See the Note in Page 61.) and therefore ſomething leſs, as being farther from the Object; but this is an *accidental* Circumſtance depending on the Thickneſs of the *Lens*, and its Diſtance from the Center of the Pupil, and therefore not conſidered in the Theory.

[b] An Object will alſo appear diſtinct though it be ſituated at a very ſmall Diſtance from the Eye, by being viewed thro' a ſmall Hole in a piece of Paper, the Reaſon of which was explained in Note Page 64. But then this Hole muſt be made ſo very ſmall, that, unleſs the Object be ſtrongly illuminated, it will appear very obſcurely through it.

The Form of a Compound Microſcope is expreſſed in the 74th Figure, where AB repreſents a ſmall convex *Lens*, whoſe focal Diſtance is ſuch, that Rays flowing from the Point C may be collected in D; and EF is a larger *Lens* whoſe *Focus* of parallel Rays coincides with the Point D; and FG repreſents an Eye ſo ſituated that Rays proceeding from an Object at KL may enter the Pupil of it, after having paſſed through both Glaſſes. Things being diſpoſed in this Manner, the Object KL will appear *magnified* and alſo *diſtinct*.

For firſt, let RCS repreſent a Pencil of Rays flowing from the Point C, theſe will meet their *Axis* again in the Point D by Suppoſition, and croſſing there will enter the *Lens* EF diverging from its *Focus* of parallel Rays, and will therefore enter the Pupil of the Eye in Directions parallel to each other, and concur upon the *Retina* at Q; the Object will therefore appear *diſtinct*.

Secondly, A Pencil of Rays flowing from another Point of the Object, as L, will meet their *Axis* in M, and diverging from thence will, after being refracted by the Lens EF, become parallel with reſpect to each other; but with reſpect to the former they will converge, becauſe with regard to them, they diverged before they paſſed through the *Lens* EF from I, a Point more diſtant than its

Focus of parallel Rays. They will conſequently croſs them at ſome Diſtance from it, ſuppoſe at H, where the Pupil of an Eye being placed to receive them, the Point L will be repreſented at O. And for the like Reaſon, the Point K being repreſented at P, the Object will be ſeen under the Angle PHO or EHF, which, as will be demonſtrated in Note (*e*), is much larger than that under which it would have appeared to the naked Eye [c].

That Glaſs AB, which is ſituated next the Object, is called the *Object-glaſs*; that which is placed next the Eye, the *Eye-glaſs* [d].

The Proportion of magnifying, in a Microſcope of this Kind, is nearly in a *Ratio* compounded of the Proportion which the Diſtance of the Image from the Object-glaſs bears to its Diſtance from the Eye-glaſs; and of that which the Diſtance of the Object from the Eye bears to its Diſtance from the Object-glaſs [e].

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[c] It may be remarked here, that when we view an Object through an Inſtrument of this Kind, we are then in reality looking at the Image of that Object through a ſingle Microſcope. Thus, it is MN the Image of the Object KL, formed by the Concurrence of the Rays of each Pencil in their reſpective *Foci*, which we ſee through the *Lens* or ſingle Microſcope EF; ſo that the Addition of the Glaſs AB is only that we may have an Image of the Object to look at, larger than the Object itſelf.

[d] In ſome Microſcopes there is a third Glaſs placed between the Object-glaſs and the Image, and is called a *Midale-glaſs*. This is placed there only to bring the Rays to a *Focus* the ſooner, in order that the Image may fall nearer the Object-glaſs than it otherwiſe would do.

After

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[e] *Lemma*. Let EF (in Fig. 75.) be a *Lens*, whoſe *Focus* of parallel Rays is D; and let Rays flowing from the Point I be collected in H; I ſay, that if X be the middle of the *Lens*, IX will be to XH, as ID to DX.

Demonſtration of the Lemma. On D the *Focus* of parallel Rays erect the Perpendicular DM; and let IE be an Incident, and EH a refracted Ray: then whereas Rays flowing from D, and paſſing through the *Lens*, would after Refraction be parallel to each other and to DX, a Line drawn from thence through the Middle of the *Lens*; Rays flowing from M and paſſing through the *Lens*, will alſo be refracted into Lines parallel to themſelves, and conſequently to MX that which paſſes through the Middle of the *Lens*: conſequently EH is parallel to MX; the Triangles therefore IMX and IEH are ſimilar; and therefore IM is to ME as IX to XH: but IM is to ME alſo as ID to DX, the Triangles IMD and IEX being alſo ſimilar; IX is therefore to XH as ID to DX. Q. E. D.

Demonſtration of the Propoſition. To avoid the Confuſion which might ariſe from the Multiplicity of Lines in the 74th Figure, let only CXH and LEH, *viz.* the *Axes* of the Rays which proceed from the Points C and L, be repreſented as in Fig. 75, and draw the Line LH, then will CHL be the Angle under which half the Object would be ſeen by the naked Eye at H; but EHX is the Angle under which the ſame Half appears when viewed through the Miſcroſcope. Now this Angle is to the former in a *Ratio* compounded of the Angle EHX to EIX, and of the ſame EIX, or (which is equal to it becauſe vertical) CIL to CHL; *becauſe the* Ratio *that any two Quantities bear to each other, is compounded of the* Ratio *which the firſt bears to any other, and of the* Ratio *which that other bears to the ſecond.* But the firſt of theſe *Ratio's, viz.* EHX to EIX, is as IX to XH, or which, as demonſtrated in the *Lemma*, is the ſame thing as ID to DX; that is, as the Diſtance of the Image from the Object-glaſs to its Diſtance from the Eye-glaſs: and the other *Ratio, viz.* CIL to CHL, is as CH to CI, that is, as the Diſtance of the Object from the Eye to its Diſtance from the Object-glaſs. Therefore, &c. Q. E. D.

But

After what has been ſaid concerning the Structure of the Compound Microſcope, and the Manner in which the Rays paſs through it to the Eye, the Nature of the common *aſtronomical* Teleſcope will eaſily be underſtood, for it differs from the Microſcope only in that the Object is placed at ſo great a Diſtance from it, that the Rays of the ſame Pencil, flowing from thence, may be conſidered as falling parallel to one another upon the Object-glaſs; and therefore, the Image made by that Glaſs is looked upon as coincident with its *Focus* of parallel Rays.

The 76th Figure will render this very plain, in which ABC is the Object emitting the ſeveral Pencils of Rays ADF, BDF, &c. but ſuppoſed to be at ſo great a Diſtance from the Object-glaſs DF, that the Rays of the ſame Pencil may be conſidered as parallel to each other, they are therefore ſuppoſed to be

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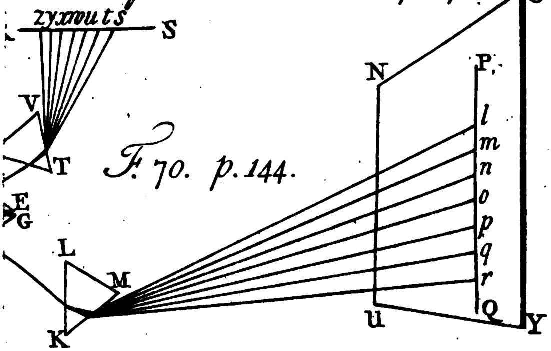
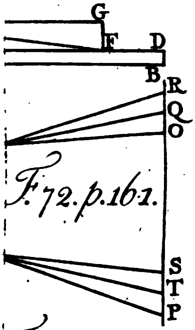
But it is proper to compare the Angle EHX with that under which the Object would appear to the naked Eye at a Diſtance proper for diſtinct Viſion; becauſe when a Perſon views an Object by the Help of a Microſcope, he is often obliged to place his Eye at a Diſtance from the Object, very different from that at which he would chooſe to place it, were he to look at it with the naked Eye; and then, inſtead of the Diſtance of the Object from the Eye in the foregoing Propoſition, we muſt ſubſtitute the Diſtance of diſtinct Viſion; in which Caſe it will ſtand thus: The Proportion of magnifying is nearly in a *Ratio* compounded of the Proportion which the Diſtance of the Image from the Object-glaſs bears to its Diſtance from the Eye-glaſs, and of that which the Diſtance of diſtinct Viſion bears to the Diſtance of the Object from the Object-glaſs.

Part III. Plate XI. p. 176.

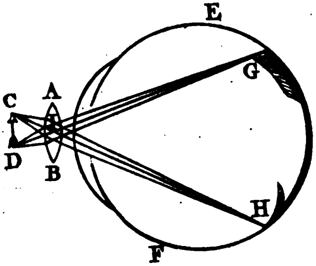


F. 70. p. 144.

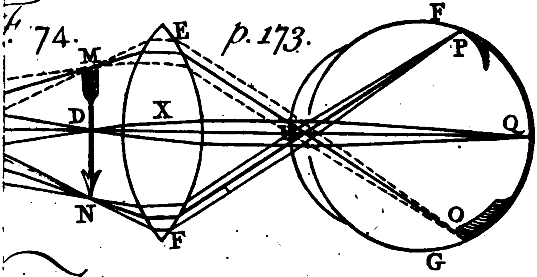
F. 73. p. 171.



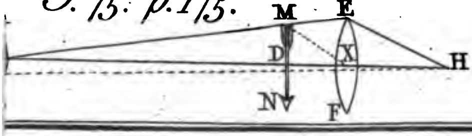
F. 72. p. 161.



F. 74. p. 173.



F. 75. p. 175.



collected into their reſpective *Foci* at the Points G, H, I, ſituated at the focal Diſtance of the Object-glaſs DF. Here they form an Image, and croſſing each other proceed diverging to the Eye-glaſs KM; which being placed at its own focal Diſtance from the Points G, H, I, the Rays of each Pencil, after paſſing through that Glaſs, will become parallel among themſelves, but the Pencils themſelves will converge conſiderably with reſpect to one another, even ſo as to croſs at P, very little farther from the Glaſs KM than its *Focus*; becauſe, when they entered the Glaſs, their *Axes* were almoſt parallel, as coming through the Object-glaſs at the Point E, to whoſe Diſtance the Breadth of the Eye-glaſs KM in a long Teleſcope bears very ſmall Proportion. So that the Place of the Eye will be nearly at the focal Diſtance of the Eye-glaſs, and the Rays of each reſpective Pencil being parallel among themſelves, and their Axes croſſing each other in a larger Angle than they would do if the Object were to be ſeen by the naked Eye, as we ſhall demonſtrate in the Notes, Viſion will be *diſtinct*, and the Object will appear *magnified*.

The Power of magnifying in this Teleſcope is as the focal Length of the Object-glaſs to the focal Length of the Eye-glaſs [f].

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[f] *Dem.* In order to prove this, we may conſider the Angle AEC as that under which the Object would be ſeen by the naked

It is evident from the Figure, that the viſible *Area*, or Space which can be ſeen at one View when we look through this Teleſcope, depends on the Breadth of the Eye-glaſs, and not of the Object-glaſs; for if the Eye-glaſs be too ſmall to receive the Rays GM, IK, the Extremities of the Object could not have been ſeen at all: a larger Breadth of the Object-glaſs conduces only to the rendering each Point of the Image more luminous by receiving a larger Pencil of Rays from each Point of the Object.

It is in this Teleſcope as was remarked of the compound Microſcope in the Notes, Page (174), that what we ſee, when we look through it, is not the Object itſelf, but only an Image of it at GI: Now that Image being inverted with reſpect to the Object, as it is, becauſe the *Axes* of the Pencils that flow from the Object croſs each other at E, Objects ſeen through a Teleſcope of this Kind neceſſarily appear *inverted*.

naked Eye; for in conſidering the Diſtance of the Object, the Length of the Teleſcope EP may be omitted, as bearing no Proportion to it. Now the Angle, under which the Object is ſeen by means of the Teleſcope, is KPM, which is to the other AEC, or its Equal KEM, as the Diſtance EL to LP, or which is the ſame Thing by *Lemma* to the foregoing Note, as EH to HL. The Angle therefore, under which an Object appears to an Eye aſſiſted by a Teleſcope of this Kind, is to that under which it would be ſeen without it, as the focal Length of the Object-glaſs to the focal Length of the Eye-glaſs.

This is a Circumſtance not at all regarded by Aſtronomers, but for viewing Objects upon the Earth, it is convenient that the Teleſcope ſhould repreſent them in their natural Poſture; to which Uſe the Teleſcope with three Eye-glaſſes, as repreſented Fig. 77. is peculiarly adapted, and the Progreſs of the Rays through it from the Object to the Eye is as follows:

AB is the Object ſending out the ſeveral Pencils ACD, BCD, *&c.* which, paſſing thro' the Object-glaſs CD, are collected into their reſpective *Foci* in EF, where they form an inverted Image, from hence they proceed to the firſt Eye-glaſs HI, whoſe *Focus* being at G, the Rays of each Pencil are rendered parallel among themſelves, and their *Axes*, which were nearly parallel before, are made to converge and croſs each other at K: the ſecond Eye-glaſs LM, being ſo placed that its *Focus* ſhall fall upon K, renders the *Axis* of the Pencils which diverge from thence parallel, and cauſes the Rays of each which were parallel among themſelves to meet again at its *Focus* NO on the other Side, where they form a ſecond Image inverted with reſpect to the former, but direct with reſpect to the Object. Now this Image, being ſeen by the Eye at XY through the Eye-glaſs QR, affords a direct Repreſentation of the Object, and under the ſame Angle that the firſt Image

EF would have appeared, had the Eye been placed at K, ſuppoſing the Eye-glaſſes to be of equal Convexity; and therefore the Object is ſeen equally magnified in this, as in the former Teleſcope, that is, as the focal Diſtance of the Object-glaſs to that of any one of the Eye-glaſſes, and appears *erect*.

If a Teleſcope exceeds 20 Feet, it is of no Uſe in viewing Objects upon the Surface of the Earth; for if it magnifies above 90 or 100 times, as thoſe of that Length uſually do, the Vapours, which continually float near the Earth in great Plenty, will be ſo magnified as to render Viſion obſcure.

The Teleſcope with the concave Eye-glaſs is conſtructed as follows:

AB (Fig. 78.) is an Object ſending forth the Pencils of Rays ADE, CDE, &c. which, after paſſing through the Object-glaſs, DE, tend towards FG (where we will ſuppoſe the *Focus* of it to be) in order to form an inverted Image there as before; but in their Way to it are made to paſs through the concave Glaſs HI, ſo placed that its *Focus* may fall upon S, and conſequently the Rays of the ſeveral Pencils which were converging towards thoſe reſpective focal Points F, S, G, will be rendered parallel among themſelves; but the *Axes* of thoſe Pencils croſſing each other at K, and diverging from thence, will be rendered more diverging, ſuppoſe in the Directions LM, NO. Now theſe Rays entering the Pupil of an Eye, will form a large and diſtinct Image P Q upon the *Retina*, which will be *inverted* with reſpect to the Object, becauſe the *Axes* of the Pencils croſs in K; and the Angle the Object will appear under will be equal to that which the Lines ML, ON, produced back through the Eye-glaſs, form at X.

'Tis evident, that the leſs the Pupil of the Eye is, the leſs is the viſible *Area* ſeen through a Teleſcope of this Kind; for a leſs Pupil would exclude ſuch Pencils as proceed from the Extremities of the Object AB, as is evident from the Figure. This is an Inconvenience that renders this Teleſcope unfit for many Uſes, and is only to be remedy'd by the Teleſcope with the convex Eye-glaſſes, where the Rays which form the extreme Parts of the Image are brought together in order to enter the Pupil of the Eye, as explained above.

It is apparent alſo, that the nearer the Eye is placed to the Eye-glaſs of this Teleſcope, the larger is the *Area* ſeen through it; for, being placed cloſe to the Glaſs, as in the Figure, it admits Rays that come from A and B, the Extremities of the Object, which it could not if it was placed farther off.

The Degree of magnifying in this Teleſcope is in the ſame Proportion with that in the other, *viz.* as the focal Diſtance of the Object-glaſs is to the focal Diſtance of the Eye-glaſs.

For there is no other Difference but this, *viz.* that as the extreme Pencils in that Teleſcope were made to converge and form the Angle KPM (Fig. 76.) theſe are now made to diverge and form the Angle MXO (Fig. 78.) which Angles, if the concave Glaſs in one has an equal refractive Power with the convex one in the other, will be equal, and therefore each Kind will exhibit the Object magnified in the ſame Degree.

There is a Defect in all theſe Kinds of Teleſcopes, not to be remedied by any Means whatever, which was thought only to ariſe from hence, *viz.* that *ſpherical* Glaſſes do not collect Rays to one and the ſame Point, as was obſerved (Chapter III. in the Notes) but it was happily diſcover'd by Sir *Iſaac Newton*, that the Imperfection of this Sort of Teleſcope, ſo far as it ariſes from the ſpherical Form of the Glaſſes, bears almoſt no Proportion to that which is owing to the different Refrangibility of Light. This Diverſity in the Refraction of Rays is about a twenty-eighth Part of the Whole, ſo that the Object-glaſs of a Teleſcope cannot collect the Rays which flow from any one Point in the Object into a leſs Room than the circular Space whoſe Diameter is about the fifty-ſixth Part of the Breadth of the Glaſs [g]. Therefore, ſince each Point of the Object will be repreſented in ſo large a Space, and the Centers of thoſe Spaces will be contiguous, becauſe the Points in the Object the Rays flow from are ſo, it is evident that the Image of an Object made by ſuch a Glaſs muſt be a moſt confuſed Repreſentation, though it does not appear ſo when viewed through an Eye-glaſs that magnifies in a moderate Degree; conſequently the Degree of magnifying in the Eye-glaſs muſt not be too great with reſpect to that of the Object-glaſs, leſt the Confuſion become ſenſible.

Notwithſtanding this Imperfection, a dioptrical Teleſcope may be made to magnify in any given Degree, provided it be of ſufficient Length; for the greater the focal Diſtance of the Object-glaſs is, the leſs may be the *Proportion* which the focal Diſtance of the Eye-glaſs may bear to that of the Object-glaſs, without rendering the Image obſcure. Thus, an Object-glaſs, whoſe focal Diſtance is about four Feet, will admit of an Eye-glaſs whoſe focal Diſtance ſhall be little more than one Inch, and conſequently will magnify almoſt forty-eight times: but an Object-glaſs of forty Foot *Focus* will admit of an Eye-glaſs of only four Inch *Focus*, and will therefore magnify 120 times; and an Object-glaſs of an hundred Foot *Focus* will admit of an Eye-glaſs of little more than ſix Inch *Focus*, and will therefore magnify almoſt 200 times[h].

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[g] To ſhew this, let AB, Fig. 79, repreſent a convex *Lens*, and let CDF be a Pencil of Rays flowing from the Point D, and let H be the Point at which the leaſt refrangible Rays are collected to a *Focus*, and I, that where the moſt refrangible concur: then, if IH be the twenty-eighth Part of EH, IK will be a proportionable Part of EC (the Triangle HIK and HEC being ſimilar;) Conſequently LK will be the twenty-eighth Part of FC. But MN will be the leaſt Space into which the Rays will be collected, as appears by their Progreſs repreſented in the Figure. Now MN is but about half of KL, and therefore it is but about the fifty-ſixth Part of CF, ſo that the Diameter of the Space, into which the Rays are collected, will be about the fifty-ſixth Part of the Breadth of that Part of the Glaſs through which the Rays paſs. Which was to be ſhewn.

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[h] The Reaſon of this Diſproportion in their ſeveral Degrees of magnifying is to be explained in the following Manner: Since the Diameters of the Spaces, into which Rays flowing from the ſeveral Points of an Object are collected, are as the Breadth of the Object-glaſs, it is evident that the Degree of Confuſedneſs in the Image is as the Breadth of that Glaſs (for the Degree of Confuſedneſs will only be as the Diameters or Breadths of thoſe Spaces, and not as the Spaces themſelves.) Now the focal Length of the Eye-glaſs, that is, its Power of magnifying, muſt be as that Degree; for, if it exceeds it, it will render the Confuſedneſs ſenſible; and therefore it muſt be as the Breadth or Diameter of the Object-glaſs. The Diameter of the Object-glaſs, which is as the Square Root of its Aperture or Magnitude, muſt be as the ſquare Root of the Power of magnifying in the Teleſcope, for unleſs the Aperture itſelf be as the Power of magnifying, the Image will want Light; the ſquare Root of the Power of magnifying will be as the ſquare Root of the focal Diſtance of the Object-glaſs; and therefore the focal Diſtance of the Eye-glaſs muſt be only as the ſquare Root of that of the Object-glaſs. So that in making Uſe of an Object-glaſs of a longer *Focus*, ſuppoſe than one that

But Teleſcopes of ſuch prodigious Length being very incommodious and unfit for Practice, the *catadioptrical* or reflecting Teleſcope as it is commonly called, invented by Sir *Iſaac Newton*, is infinitely preferable to them, for one of theſe, ſix Feet in Length, ſhall magnify as much as one of the other of an hundred. The Form of the Tube, and the Progreſs of the Rays through it, are as deſcribed in Figure the 80th, where ABCD is the Tube, BC a concave reflecting Metal, EF a plain reflecting Metal fixed to the Tube by Means of the Stem HI. MN repreſents a diſtant Object emitting Pencils of Rays from each Point, two only of which are here repreſented, and thoſe cut off before they reach the Metal, to prevent Confuſion in the Figure. Now it is evident from what has been explained above (Chap. X. Propoſit. 3. Caſe 4.) that theſe Rays, were they not intercepted in their Way, would return after Reflection at the concave Surface BC, and form an inverted Image at OP, ſuppoſing

is given, you are not obliged to apply an Eye-glaſs of a proportionably longer *Focus* than what would ſuit the given Object-glaſs, but ſuch a one only whoſe focal Diſtance ſhall be to the focal Diſtance of that which will ſuit the given Object-glaſs, as the ſquare Root of the focal Length of the Object-glaſs, you make Uſe of, is to the ſquare Root of the focal Length of the given one. And this is the Reaſon that longer Teleſcopes are capable of magnifying in a greater Degree than ſhorter ones, without rendering the Object confuſed or coloured.

that to be the Place of the *Focus* of reflected Rays. But in this Caſe the reflected Rays are intercepted in their Return to that Place by the plain Metal, and are thereby thrown ſide-ways, and, inſtead of forming the Image OP, are made to form the Image QR; which, becauſe the Rays have as yet ſuffered no Refraction, is not liable to the Imperfection which ariſes from the different Refrangibility of the Rays of Light, nor to any other except what may ariſe from an imperfect Poliſh, or the Want of the Form of one of the *conic* Sections in the Reflector BC; and therefore may be viewed by an Eye at T with a very ſmall *Lens* or Eye-glaſs KL, without appearing either coloured or confuſed.

It being inconvenient to find the Object with a Teleſcope of this Form, a ſmall dioptrical Teleſcope with two Hairs or Wires run through the Tube in the common *Focus* of the two Glaſſes, and croſſing each other at right Angles, is generally fixed upon it in ſuch a Manner that the *Axis* of one Teleſcope ſhall be parallel to that of the other, ſo that when the Object appears in one at the Interſection of the Hairs, the other may be duly poſited for viewing the ſame Object through its Side.

But this Method of finding an Object is very incommodious for viewing terreſtrial

Objects, and therefore the ſame Kind of Teleſcope has been contrived and effected in the follwing Manner:

ABCDEFGH (Fig. 81.) is the Tube, BG the concave reflecting Metal, with an Hole in it at IK. LM is another reflecting Concave fix'd to the Tube by means of the Stem NO, the common *Focus* of the two Metals being at P. Things being thus diſpoſed, let QR repreſent an Object emitting ſeveral Pencils of Rays, two of which are repreſented in the Figure. Theſe, after Reflection, will form the two Extremities of the inverted Image ST (as explained Chap. X. Prop. 3. Caſe 4.) where the ſeveral Rays of the ſame Pencil croſs each other, and, being afterwards reflected by the concave Surface LM, become parallel among themſelves, but the Pencils themſelves are made to converge, and, croſſing each other at V, paſs through the *Lens* CF, which having its focal Diſtance about V, makes the Pencils parallel, and at the ſame Time renders the Rays of each Pencil converging, ſo as to form an erect Image WX, which is ſeen by the Eye at Y through another *Lens* at DE.

This Kind of Teleſcope is called the *Gregorian*, as being attempted by *J. Gregory*, though in vain. (See his *Optica promota*, Propoſit. 59.) It is now grown common, and is excellently well adapted for the viewing terreſtrial Objects, becauſe a Tube of this Kind, of two or three Feet in Length, will magnify ſufficiently for that Purpoſe.

If the Reader would ſee a particular Hiſtory of the Invention of the ſeveral Sorts of reflecting Teleſcopes, he may conſult the *Appendix* to the laſt Edition of *Gregory*'s *Optics*, where he will find a full Account of it, together with the Letters that paſſed between the Inventors themſelves upon that Occaſion. And for ſuch Authors as have explained the Nature of Microſcopes and Teleſcopes in general, conſult Mr. *Johnſon's Quæſtiones Philoſoph.* Q. 34 & 35.

DIS-

DISSERTATION IV.

Of the Rainbow.

BEfore the different Refrangibility of Light was diſcovered, all Attempts to account for the Appearance of the Rainbow proved ineffectual; for it is no other than the Diverſity of Refrangibility to which that *Phænomenon* is to be aſcribed: as will appear from the following Explication of it; in which, becauſe it is a *Phænomenon* not eaſily apprehended by Beginners, I hope to be excuſed, if I am more than ordinarily particular. To begin then:

The Rainbow is never ſeen, but when the Sun ſhines upon Drops of Rain falling on that Side of the Spectator which is oppoſite to the Sun.

To illuſtrate this, let A (Fig. 82.) repreſent the Eye of a Spectator, and let B, C, D, E, be a Series of Drops of Rain falling from a Cloud, on which let the Sun be ſuppoſed to ſhine from the Parts about S, *&c.* then will there be exhibited the Appearance of a Rainbow in the Cloud; and it will be formed as follows. Let SB, SC, SD, *&c.* repreſent the Sun's Rays, which (becauſe of the Sun's great Diſtance) we will ſuppoſe parallel; and let the Ray SC fall upon the Drop C at the Point C: then will ſo much of it as enters the Drop be refracted towards the Perpendicular, and proceed on, ſuppoſe to F, where Part of it will be tranſmitted, and Part reflected, ſuppoſe to G[i]: Of that which is reflected to G, ſome will be there reflected and ſome tranſmitted; that which is tranſmitted will, on account of the Diverſity of Refrangibility to which Light is ſubject[k], be ſeparated by Refraction, and made to exhibit the ſeveral *priſmatic* Colours, *viz. Red*, *Orange*, *Yellow*, *&c.* And if the *Red* Light proceeds from the Drop in the Line GR, the *Orange*, ſuffering a greater Degree of Refraction, will proceed in one ſituated above this, ſuppoſe in GO, and the *Yellow* in GY, *&c.* and the *Violet* in GV; therefore, to an Eye placed any where in the Line GR, the Drop C will exhibit a *Red* Colour, that is, the Cloud will appear *Red* in that Place. To an Eye placed any where in the Line GO, the ſame Drop would exhibit the *Idea* of *Orange* Colour, and ſo on through all the Colours of the *Priſm*.

Now, let us conſider the Paſſage of a Ray of Light through another Drop at a certain Diſtance below this, *viz.* the Drop D, on which let SD be the incident Ray. This Ray, after having been refracted and reflected as the former was in the other Drop, will emerge ſeparated into the Rays HR, HO, HY, *&c.* of which, if HR exhibits the *Red*, HO will paint the *Orange*, HY the *Yellow*, *&c.* and HV the *Violet* Colour; and the Ray incident upon this Drop being parallel to that which was incident upon the former, the Rays, exhibiting the ſeveral Colours ſeparated by this Drop, will be reſpectively parallel to the Rays exhibiting the correſpondent Colours ſeparated by means of the other Drop; that is to ſay, the Ray HR, which exhibits *Red* in this Drop, will be parallel to the Ray GR, which exhibits the ſame Colour in the other Drop; and ſo of the other correſponding Colours. Conſequently the Rays HO, HY, *&c.* which exhibit *Orange*, *Yellow*, *&c.* in this Drop, will all converge towards GR, which exhibits *Red* in the other: and therefore each of theſe would croſs that, if produced far enough. Let then the Ray HV in the Figure before us, which exhibits *Violet* in this Drop, be produced till it croſſes that which exhibits *Red* in the other produced alſo, ſuppoſe at the Point A where the Eye of the Spectator is placed. To this Eye therefore, upon this Suppoſition, will be repreſented in the Cloud at the ſame Time two of the *priſmatic* Colours, *viz.* *Red* and *Violet*, the

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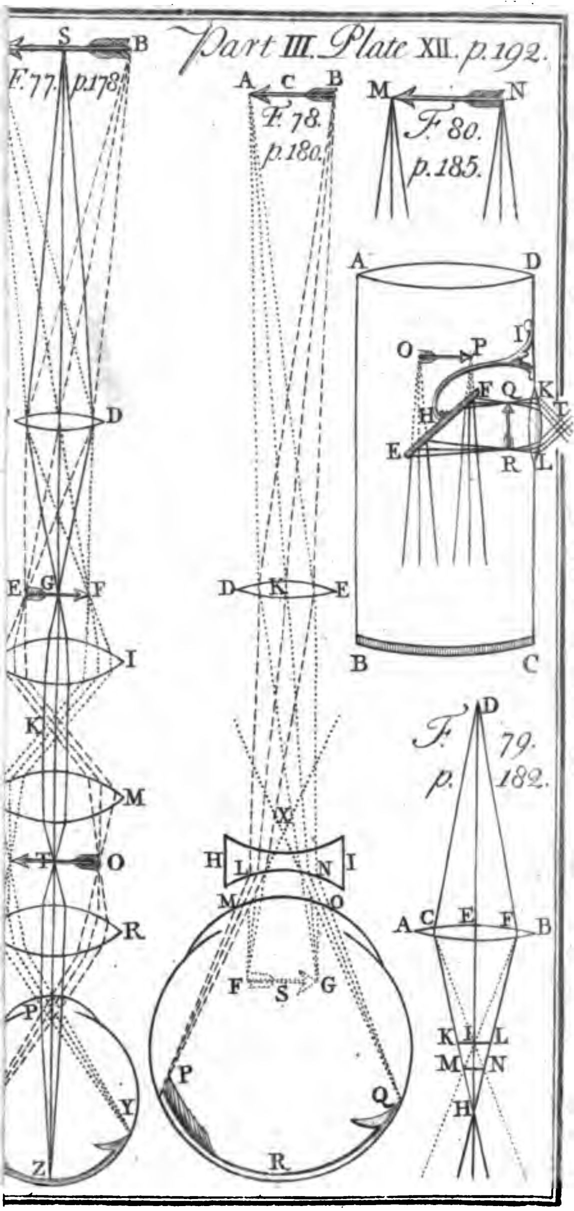
[i] See the Manner in which Light is reflected, Chap. VIII.

[k] See Chap. XI.

Red above at G, and the *Violet* below at H. But if we ſuppoſe the Eye placed farther back, where a Ray, that exhibits another Colour in the Drop D, would croſs the Ray GR, or which comes to the ſame Thing, if we ſuppoſe the Drop D ſo much nearer to the Drop C, that that Ray may enter the Eye along with the other at A; then would the Colour of that Ray be exhibited along with the *Red*. For Inſtance, if the Drop D be placed ſo much higher, that the Ray HO which exhibits *Orange*, may croſs the Ray GR at A, then to the Eye will be exhibited the Colours of *Red* and *Orange*; and if there be a third Drop below this, ſo placed that the *Yellow* proceeding from it ſhall enter the Eye alſo at the ſame Time, then will three of the *priſmatic* Colours appear to that Eye, and ſo on for the other Colours, till the Situation of the Drop be ſuppoſed as low as where the Drop D is, and then the *Violet* and moſt refrangible Light is tranſmitted to the Eye; but from Drops below this, no Colour is tranſmitted thither, all the Rays which iſſue from a lower Drop, as E, paſſing below the Eye. And as thoſe Rays, which paſs through the lower Drops, are too low for the Eye at A, ſo thoſe which come from the higher ones, as B, are too high, as appears by Inſpection of the Figure; ſo that there is nothing but total Darkneſs both

Part III. Plate XII. p. 192.



above and below. Now ſince the Colours of the Rainbow are the ſame with thoſe of the *Priſm*, it is evident, from what has been ſaid, that between the Points G and H, all the Colours of that beautiful *Phænomenon* will appear, provided a ſufficient Number of Drops be ſupplied from the Cloud, to prevent any *Hiatus* or Deficiency in the Light reflected by them.

But we have hitherto *tacitly* ſuppoſed, that the Rays SB, SC, SD, *&c.* were all incident with the ſame Degree of Obliquity upon the Surface of each Drop (that is, that they entered at the ſame Diſtance from that Point in the Surface of each Drop which is neareſt the Sun) and that that Obliquity was certain and determinate: For Rays which enter the Drops with other Obliquities conduce nothing towards exhibiting the Colours of the Rainbow, and are therefore to be looked upon as *ineffectual* and *inſignificant*. The Truth of which we ſhall now proceed to ſhew. After this, we ſhall be enabled to explain the remaining Particulars relating to the *Bow.*

Let then SA, SB, SC, *&c.* *(*Fig. 83.*)* repreſent the Sun's Rays falling upon the Drop XY, the firſt perpendicularly to the Surface of it, the other with different Degrees of Obliquity, according to their different Diſtances from the firſt; and let the Point at which the two firſt, *viz.* SA and SB, would meet by means of that Refraction which the oblique one SB ſuffers in paſſing through the firſt Surface of the Drop, be H. Now it was remarked (Chap. III. Obſervat. 2. in the Notes) that when parallel Rays fall upon a convex Surface, the nearer any one of the oblique ones is to that which enters the Surface perpendicularly, the greater ſhall be the Diſtance at which it will meet the perpendicular one after Refraction at that Surface; that is, in the preſent Caſe, that the oblique incident Ray SB ſhall after Refraction at B, (ſuppoſing it to paſs through the ſecond Surface of the Drop without Refraction) meet the perpendicular Ray SA produced at a greater Diſtance than SC ſhall; and SC ſhall meet it at a greater than SD ſhall; SD at a greater than SE, *&c.* H then being ſuppoſed to be the Point where SB meets SA, let I be that where SC, K that where SD, L that where SE, M that where SF would meet it, *&c.* From whence we may obſerve,

That the farther we take the Rays from SA, the *nearer* are the Points which the refracted Rays fall upon the ſecond Surface of the Drop ſituated at X, till we come to the Ray SD; after which, the farther we take them from SA, the *farther* the Points they fall upon are from X. For Inſtance, the Ray SB falls upon N; SC upon O; SD upon P; but SE does not fall beyond P, but upon O, and SF upon N, *&c.* So that upon every Point below P there are two Rays incident; and the one of them is ſuch as enters the Drop on one Side of the Ray SD, and the other on the other; and the farther from it the one is on the one Side, the farther the other is on the other; and alſo the farther they are from it, the farther the Point they meet at is from P. Thus SC and SE meet at O, the Rays SB and SF at N, *&c.* The Uſe of which Obſervation we ſhall ſee by and by. But let it be remembered, that I would be underſtood here and in what follows of the Rays of one particular Colour only.

Now it is remarkable, that when two Rays fall upon a Drop, and at their Entrance are ſo refracted, as to meet in a Point at the other Surface, and are from thence reflected to ſome other Part of the Surface, and there paſs out of it; they ſhall after ſuch Emergency have the ſame Inclination towards each other, that they had before they entered the Drop. To explain this, let AB, CD, (Fig. 84.) repreſent two Rays incident upon the Drop BEF, and let them, after Refraction at B and D, meet at the Point E, from whence being reflected, let them paſs out at F and G, and be refracted into the Lines FH and GI; then whatever Inclination the incident Rays AB, CD, have to each other, the emerging Rays FH and GI will have the ſame. Becauſe the Angles of Incidence and Reflection at E being equal, the Rays EF and EG will have the ſame Inclination to each other, and to the Surface at F and G, that the Rays EB and

ED have to each other, and to the Surface at B and D. For if we conceive all theſe Rays to flow from the Point E as a Radiant, and BA, DC, to be the refracted ones of the incident ones EB, ED, as FH and GI are of EF and EG, it is evident, that under theſe Circumſtances, the Rays BA and CA will have the ſame Inclination to each other, that FG and HI have; but the Degree of Refraction is the ſame, whether EB and ED, or AB and CD, be the incident Rays; becauſe the refractive Power of the Drop is the ſame, whether the Rays paſs one Way, or the other. The Propoſition therefore is true.

From hence it follows, that the parallel Rays, SB, SF, (Fig. 83.) which after Refraction meet at the ſame Point N, will if they are from thence reflected, ſuppoſe in the Lines N Q, NR, become parallel to each other, after their Emergency, ſuppoſe in the Lines QT, RV: But their intermediate ones SC, SD, and SE, which fall upon quite different Points at the ſecond Surface of the Drop, and are from thence reflected, will not do ſo, but will go out in Directions oblique to one onother and to them; and will therefore paſs on, not only a different Way from them, but from one another: So that the Rays QT and RV will be left to themſelves, being deprived of their *intermediate* ones, by which Means they are rendered, as to all Intents and Purpoſes of Viſion, entirely uſeleſs and *inſignificant*.

Again, the Rays SC and SE, which meet at the Point O, will alſo be parallel among themſelves after their Emergency from the Drop, but their *intermediate* ones will paſs off another Way, though not ſo obliquely, with reſpect to them and to one another, as thoſe in the foregoing Caſe; becauſe the ſeveral Points they fall upon at the ſecond Surface of the Drop, being ſituated between O and P, are nearer to each other, than the Points the *intermediate* Rays fell upon, in the former Caſe, were to N.

But ſuch as are incident very near SD on each Side of it, will with their intermediate ones all fall upon, or at an inſenſible Diſtance from the Point P; ſo that theſe, after their Emergency, will all paſs on parallel, or very nearly ſo, to each other; and therefore when they enter the Eye of a Spectator, though he be at a conſiderable Diſtance, will affect him ſenſibly enough to excite the *Idea* of their own Colour (for as was obſerved above, I ſpeak now only of Rays of one Colour) which the other Rays conſidered in the forementioned Caſes, for Want of their *intermediate* ones, were too weak to do, however near the Situation of the Eye might be. Theſe therefore are the only Rays that exhibit the Colours of the *Bow*, and are hence called in Contradiſtinction to ſuch as enter at other Points of the Drops, that is, with other Obliquities, *effectual* or *ſignificant*.

It was proper therefore, in the Explication above, to ſuppoſe none to enter the Drop, but theſe. As to the Degree of Obliquity with which Rays muſt fall upon the Drops to become effectual, the Method of finding that will be ſhewn in the next Note.

Since then the effectual Rays enter each Drop with the ſame Degree of Obliquity (I ſtill mean ſuch Drops as exhibit the ſame Colour) the emerging Rays muſt neceſſarily make the ſame Angle with the incident ones in every Drop. The Magnitude of which ſhall be determined in the Note below[a]. That is, the Angle which the Ray SC (Fig. 82.) makes with the emerging Ray GR which exhibits *Red*, will be the ſame in all the Drops that exhibit that Colour, and ſo of the reſt. Let then the Line

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[a] We are here to determine the Angle, which an incident *efficacious* Ray of any Colour makes with the emerging Ray of the ſame Colour. In order to this, let AB, CD, (Fig. 84.) be two Rays incident upon the Drop BG, and let them be refracted to E, and after Reflection there, and a ſecond Refraction at F and G, let them emerge in the Lines FH and GI, making with the incident ones the Angle AKI, which is the Angle to be determined.

Let us call the *Ratio*, which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, I to R. Then from the Center L to the Lines BK, BE, and DE, draw the Lines LM, LN, and LO, reſpectively perpendicular, and with the *Radius* LO deſcribe the Arch OP, and draw the Line LB, and produce it to Q. Then will ABQ, or its Equal LBM, be the Angle of Incidence of the Ray AB, and LM its Sine: LBN will be the Angle of Refraction, and LN its Sine: Likewiſe LR will be the Sine of the Angle of Incidence of the other Ray CD, and

Line AI be ſuppoſed to be drawn from the Sun through the Eye of the Spectator. This Line, becauſe of the Sun's immenſe Diſtance, will be parallel to the Sun's Rays SB, SC, SD; and therefore the Angle GAI which this Line makes with any one of the emerging Rays, for Inſtance GR, will be the ſame that the incident one makes with it. Let us then imagine the Line AI fixed, and the Line AG to revolve round it, always making the ſame Angle with it; then will the Line AG deſcribe the Surface of a Cone whoſe *Apex* will be at A, and its *Axis* AI, and the Surface of this Cone will in all Parts of it make the ſame Angle with the Sun's Rays, becauſe they are parallel to

one

LO the Sine of its Angle of Refraction. We ſhall therefore have for the firſt Step of the following Proceſs

this Proportion, *viz.*	1	LM : LN : : I : R
And for the ſecond	2	LR : LO : : I : R
But by Conſtruction.	3	LO=LP
Therefore from the 2d and 3d Steps	4	LR : LP : : I : R
Now if we ſubſtract the two firſt Members of the fourth Step from the two firſt of the firſt Step reſpectively, by which Means the Proportion between the Terms will not be deſtroyed, we ſhall have	5	LM—LR : LN—LP : : I : R
But by the Figure	6	LM—LR=MR
And ——— ——— ———	7	LN—LP=NP
Therefore from the fifth, ſixth, and ſeventh Steps, we have	8	MR : NP : : I : R

Parallel

one another, and to the Axis of the Cone; therefore Drops of Rain, whatever Part of this Surface they paſs through in falling, will, in the Inſtant of Time that they paſs through it, ſend a *red* Ray towards the Eye of the Spectator; for it is not neceſſary that the Drops ſhould be all at the ſame Diſtance from the Eye. In like manner Drops of Water, paſſing through the Surface of a leſſer Cone made by the Revolution of the Line HV about the Line AI, will exhibit *Violet*; and ſo for the intermediate Colours. So that the Rainbow, were we to ſee it entire, would be a compleat Circle having its Center in the Line AI, and conſequently directly oppoſite to the Sun with reſpect to the Spectator's Eye.

: Parallel to BE draw the Line DX, and on the Lines DR and DX let fall the Perpendiculars BT and BX, from the Point B. Then, becauſe we ſuppoſe the Rays AB and CD *efficacious* ones, and therefore infinitely near one another, the little Triangles BTD, BXD, and NOP may be conſidered as right-lined ones, and the latter, *viz.* NOP may be alſo looked upon as right-angled at P. Upon this Suppoſition, the Triangles BTD and BLM will be ſimilar, *for they are right-angled at T and M; and the Angles DBT and MBL want each the ſame Angle TBL to make either of them right, they are therefore alſo equal.* For the like Reaſon the Triangles DBX and BNL are ſimilar, *being right-angled at X and N, and wanting equally the Angle XBL to make their Angles at B right ones.* But to the Triangle BDX the Triangle NPO is alſo ſimilar, *for they are right-angled at P and X, they have their Sides NP and BX parallel, as being each perpendicular to the ſame Line BE; and their Sides NO and BD are alſo parallel, becauſe the Points N and O, where the Perpendiculars LN and LO fall, muſt be in the Middle of the Lines BE and DE.* Farther, ſince the Lines BE and DE are coincident at E, and the Points N and O are in the Middle of each, BD is double of NO; and conſequently, Eye. The Reaſon that it does not appear ſuch, is becauſe the Sun when the Rainbow is ſeen, is *in* or *above* the Horizon, and therefore the Center of the Bow being oppoſite to it, is *in* or *below* it, on the other Side the Spectator.

Since the Angle made by the Line SC with GR, or which is the ſame thing, GR with AI is 42 Degrees and two Minutes, as determined in the Note, it's evident that when the Sun is in the Horizon, the higheſt Point of the *Bow* is 42 Degrees and two Minutes above the Horizon,

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quently, the Triangles NOP and BDX being ſimilar, BX is double of NP. From all which we have the following Steps,

viz.	9	BL : BM :: BD : BT
And	10	BL : BN :: BD : BX
By comparing theſe two together	11	BM : BN :: BT : BX
But by Conſtruction	12	BT = MR
And, as was juſt now ſhewn,	13	BX = 2 NP
Conſequently from the three laſt Steps	14	BM : BN :: MR : 2 NP
But by the eighth Step	15	I : R :: MR : NP
Therefore from the laſt	16	I : 2 R :: MR : 2 NP
Conſequently from the fourteenth and ſixteenth Step	17	BM : BN :: I : 2 R
But from the firſt Step	18	LM : 2 LN :: I : 2 R
Therefore from the two laſt	19	BM : BN :: LM : 2 LN
And by changing the Places of the mean Terms in the laſt Step	20	BM : LM :: BN : 2 LN
And by ſquaring each Term	21	BMq : LMq :: BNq : 4 LNq
From whence by comparing the Antecedents and Conſequents with the Antecedents we have	22	BMq + LMq : BMq :: BNq + 4 LNq : BNq

rizon, for then the Line AI is parallel to it; and as the Sun riſes the Height of the *Bow* diminiſhes, and with it the Portion that is viſible, till it is 42 Degrees and two Minutes high; after which the *Bow* appears no more, becauſe then the Point I is above 42 Degrees below it.

The *Phænomenon*, we have been explaining, conſtitutes what is called the *primary* or *interior Bow*; there is alſo another exterior to this, whoſe Colours are much more dilute and faint,

which

But becauſe the Triangle BML is right angled at M	23	$BMq + LMq = BLq$
And for the like Reaſon BNq + LNq is equal to BLq, therefore	24	$BNq + 4\,LNq = BLq + 3\,LNq$
Therefore from the three laſt Steps	25	$BLq : BMq :: BLq + 3\,LNq : BNq$
But BLN being a right-angled Triangle	26	$BNq = BLq - LNq$
Therefore from the two laſt Steps	27	$BLq : BMq :: BLq + 3\,LNq : BLq - LNq$
Now becauſe BML is a right-angled Triangle	28	$BLq = BMq + LMq$
Therefore from the two laſt Steps	29	$BLq : BMq :: BLq + 3\,LNq \quad BMq + LMq - LNq$
And ſubſtracting the two firſt Terms, *viz.* BLq and BMq out of the two laſt Terms reſpectively, we have	30	$BLq : BMq :: 3\,LNq : LMq - LNq$
But by the firſt Step	31	$LM : LN :: I : R$
Therefore ſubſtituting I and R in the Room of LM and LN in the 30th Step, we have	32	$BLq : BMq :: 3\,Rq : Iq - Rq$

The

which for that Reaſon is called the *ſecondary* Bow. The Progreſs of the Rays of Light through the Drops of Water, in forming this, is as follows.

Let A (*Fig.* 85.) repreſent the Eye of a Spectator, SB and SC two of the Sun's Rays entering the Drops, as expreſſed in the Figure, and after being twice reflected in each Drop, *viz.* at D and D, let them paſs out, the one at E, the other at F, by which Means they will be ſeparated into their homogeneous Colours, the *Violet* and moſt refrangible Light being

The Proportion therefore which the Sine of the Angle of Incidence bears to that of the Angle of Refraction, when Rays of any particular Colour paſs out of Air into Water, being known, the Proportion, which the *Radius* BL bears to BM, will be thereby determined; and therefore the Angle BLM, of which BM is the Sine, will alſo be known, and therefore alſo the Angles LBM, which is equal to ABQ, the Degree of Obliquity wherewith the *efficacious* Rays enter the Drop. But the Line BM being known, the Line BN may be alſo had by the 17th Step, and therefore alſo the Angle BLN of which it is the Sine, and therefore the Angle LBN too, or its Equal LEN; and therefore alſo the Complement of this laſt to two right ones, *viz.* KEB. If now we ſubſtract the Angle LBN out of LBM, we gain the Angle EBK, and conſequently the third Angle in the Triangle EBK may be from hence known, *viz.* the Angle BKE. Now if we double this, we have the Angle AKI, which was the Angle ſought.

If a Computation be made after this Manner with the *Ratio* of 108 to 81 (or which is the ſame thing) that of 4 to 3, for the *Red* Rays, the Angle AKI (that is, the Angle GAI in Fig. 82.) will be found 42 Degrees and 2 Minutes; and if we uſe the Proportion of 109 to 81, which is the Proportion of Refraction in the *Violet* coloured Light, the Angle AKI (or HAI in Fig. 82.) will be 40 Degrees 17 Minutes. And the Difference between theſe two Angles (that is, the Angle GAH in Fig. 82.) will be the Breadth of the Bow.

conveyed from the uppermoſt Drop to the Eye at A in the Line EV; while the *Red*, and leaſt refrangible, is carried from the lower one in the Line FR, and the intermediate Colours from the intermediate Drops: So that in this Bow the Colours will be in an inverted Order, with reſpect to thoſe of the other, the *Red* being the innermoſt in this, which was the outermoſt in that. The Colours in this will be more dilute than in that, becauſe the Rays in this ſuffer two Reflections, in that but one; and the Angles, which the incident Rays in this make with the emerging ones, are larger than the like Angles in the other, as ſhall be determined in the next Note, *viz.* 50 Degrees 57 Minutes for the *Red*, and 54 Degrees 7 Minutes for the *Violet*; this Bow therefore is exterior to, and encompaſſes the former.

As to the Means by which Rays of Light become *efficacious* in the Formation of this Bow, it is exhibited in Figure 86. where AB, CD repreſent two parallel Rays incident with ſuch Obliquity upon the Drop, that they ſhall croſs each other before they reach the other Side; which that it is poſſible appears from what was ſaid, with regard to the Progreſs of the Rays through the Drop XY in Figure 83. Let them then croſs in the Point E ſuch, that after Reflection at F and G they may become parallel, ſuppoſe in the Lines FH and GI, then from the Nature of the Circle it is plain, that after

Reflection at H and I they will croſs again, ſuppoſe at K, and after Refraction at V and W, will become parallel as at firſt. And ſuch of theſe as alſo enter ſo very near one another, that their intermediate ones may ſuffer the like Refractions and Reflections with themſelves, will be the *efficacious* ones, and exhibit the *Idea* of their own Colour, at a conſiderable Diſtance from the Drop. What the Obliquity is with which theſe muſt enter the Drops, and the Angle the emerging ones of each particular Colour will make with their incident ones, ſhall be determined in the Note below [b].

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[b] The Progreſs of the *efficacious* Rays through the Drop BH (Fig. 86.) being as explained above, it is evident, becauſe the Angles of Incidence are every where equal to the Angles of Reflection, that the Lines BG, GI and IW are all equal, and therefore the Arches BG, GI, and IW are ſo too; and likewiſe, that the Lines DF, FH, and HV are equal, and therefore alſo the Arches DF, FH, and HV: It is alſo apparent, that the Arch FG is equal to HI, therefore FG is half the Difference between the Arches FH and GI, and conſequently it is half the Difference alſo between the Arches FD and GB which are reſpectively equal to theſe. Now, the whole Difference between theſe Arches is what remains when FG is taken from BD, therefore the Remainder when FG is taken from BD is double of FG, conſequently FG itſelf is but one third Part of BD; for, if when one Quantity is taken from another, the Remainder be double to the Quantity taken away, it is plain that other muſt contain the Quantity taken away three times.

Now the Rays AB and CD being ſuppoſed infinitely near one another, the curvilineal Spaces BED, and FEG may be conſider'd as ſimilar Triangles, and therefore EG is equal to a third Part of EB, conſequently N (the Point where the Perpendicular LN falls upon BG) being the middle Point of the Line BG, EN is equal to EG, and therefore alſo a third Part of EB.

If now as in the 84th Figure, the Triangles BTD and BXD be formed, as alſo the Triangle NOP, NO will be a third Part of

Now if we imagine the Lines EA and FA (Fig. 85.) to revolve about the Line AI which paſſes through the Eye of the Spectator and the Center of the Sun as before, and always to make the ſame Angles with it at A, they will deſcribe the Surface of two Cones, in the larger of which will be ſituated the Drops that exhibit *Violet*, and in the leſſer thoſe which exhibit *Red*. So that this Bow alſo, were it to appear intire, would be a compleat Circle, and

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BD, and NP a third Part of BX. Thereſore reſuming the former Proceſs at the 11th Step, we may proceed as follows,

Viz.	11	$BM : BN :: BT : BX$
By Conſtruction	12	$BT = MR$
And by what was juſt obſerv'd	13	$BX = 3\ NP$
Therefore from the three laſt Steps	14	$BM : BN :: MR : 3\ NP$
But by the eighth Step	15	$I : R :: MR : NP$
Therefore from the laſt	16	$I : 3\ R :: MR : 3\ NP$
Conſequently from the 14th and 16th Steps	17	$BM : BN :: I : 3\ R$
But by the firſt Step	18	$LM : LN :: I : R$
Therefore from the laſt	19	$LM : 3\ LN :: I : 3\ R$
Therefore from the 17th and 19th	20	$BM : BN :: LM : 3\ LN$
Or by changing the Places of the mean Terms in the laſt Step	21	$BM : LM :: BN : 3\ LN$
And ſquaring the Terms	22	$BMq : LMq :: BNq : 9\ LNq$
Therefore by comparing the Antecedents and Conſequents with the Antecedents, we have	23	$BMq + LMq : BMq :: BNq + 9\ LNq : BNq$
But by the Figure	24	$BMq + LMq = BLq$
And	25	$BNq + 9\ LNq = BLq + 8\ LNq$

Therefore

and the ſeveral Cones, through whoſe Surfaces

Therefore from the three laſt	26	$BLq : BMq :: BLq + 8\,LNq$ BNq
But by the Figure	27	$BNq = BLq - LNq$
Therefore from the two laſt	28	$BLq : BMq :: BLq + 8\,LNq : BLq - LNq$
But by the Figure	29	$BLq = BMq + LMq$
Therefore from the two laſt	30	$BLq : BMq :: BLq + 8\,LNq : BMq + LMq - LNq$
And ſubſtracting the two firſt Terms out of the two laſt Terms, we have	31	$BLq : BM :: 8\,LNq : LMq - LNq$
But by the firſt Step	32	$LM : LN : I : R$
Therefore	33	$BLq : BMq :: 8\,Rq : Iq - Rq$

Now the Proportion of I to R being known, the Proportion which the *Radius* BL bears to BM is had by the laſt Step. But to avoid the Confuſion which a Multiplicity of Lines may occaſion, let the 86th Figure be transferred to the 87th with as many Lines as ſhall be neceſſary, in which let AB be the incident Ray, BG the refracted one as before. Then, becauſe the Proportion between BL and BM is known, the Angle LBM may be had, which is equal to ABQ, the Meaſure of the Obliquity with which the efficacious Rays enter the Drop; and therefore alſo its Complement to two right ones SBL. And the Line BM being known, the Line BN may be had, becauſe by the 17th Step BM is to BN as I to 3 R, and therefore alſo the Angle LBN, or its Equal LGB, and conſequently BLG the remaining Angle of the Triangle BGL; but to this is equal the Angle GLH or HLV; and if theſe three be added together, and their Sum taken from four right ones, it will give the remaining Angle about the Center, *viz.* VLB, which being halved, gives the Angle SLB; but the Method of determining the Angle SBL was ſhewn before, and therefore LSB the remaining Angle of the Triangle LBS, may be had, which Angle doubled gives the Angle VSB or its Equal ASY, which is the Angle ſought.

If a Computation be made after this manner with the *Ratio* of 108 to 81 for the *Red* Rays, this Angle will be found to be 50 Degrees 57 Minutes; if with 109 to 81, for the *Violet*, it will be 54 Degrees 7 Minutes; and the Difference, *viz.* 3 Degrees 10 Minutes, will be the Breadth of the *Bow*.

the Drops as they form the Colours of it paſs, having one common *Axis* AI with thoſe in whoſe Surfaces the Drops forming the Colours of the other *Bow* were placed, this will be exterior to, and concentric with it, and will therefore ſurround it, as obſerved above.

As Rays of Light when they arrive at the Surface of a Drop of Water never all paſs out, but are in Part reflected and in part refracted, it is evident that ſome Rays will paſs out of each Drop after having ſuffered three Reflections, ſome after four, &c. theſe alſo will conſtitute *Rainbows*; but becauſe the greateſt Part of the Rays will be loſt in ſuffering ſo many Reflections, that *Rainbow*, which is made by three Reflections, is ſcarce ever ſeen, much leſs ſuch as are made by more, &c.

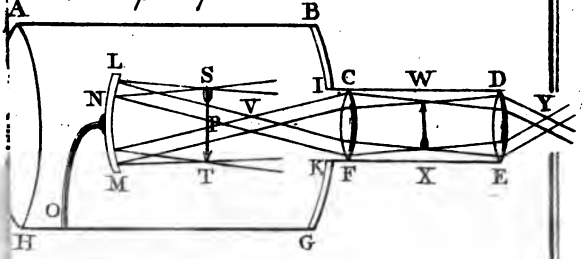
It is evident, that ſince the Line AI, *viz.* the common *Axis* of the Cones, on whoſe Surfaces the Colours of the *Bow* are formed, paſſes through the Eye of the Spectator, no two Perſons can ſee the ſame *Bow* at the ſame Time, or rather, that the Rainbows, ſeen by two Perſons at the ſame Time, are formed in different Drops of Rain and in different Parts of the Heaven.

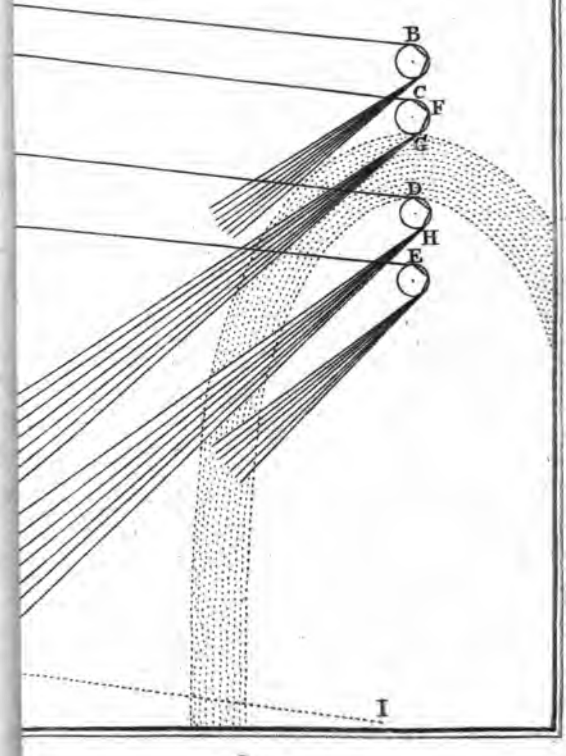
Accordingly, if a Perſon obſerves the Dew as it hangs upon the Graſs when the Sun ſhines, he ſhall ſee the Colours of the *Bow* in the Drops of Dew; but as he walks along, the Colours ſhall remove from Drop to Drop.

Some-

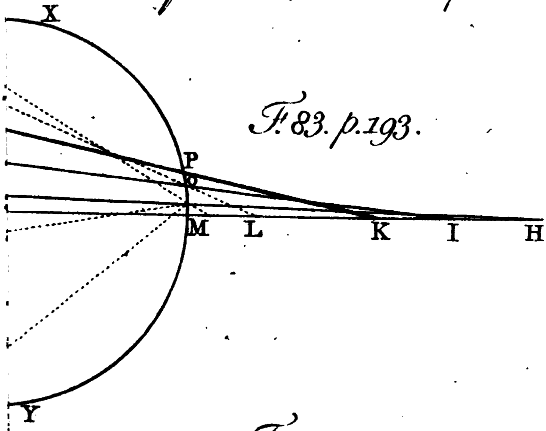
Part III. Plate XIII. p. 208.

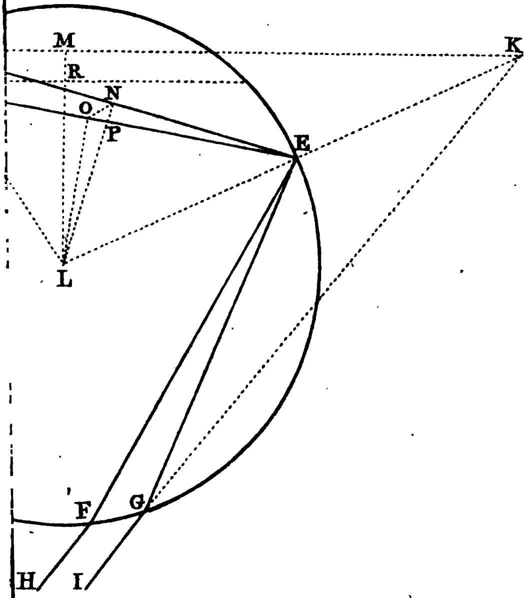
F. 81. p. 187.





Part III. Plate XIV. p. 208.


F. 83. p. 193.


F. 84. p. 195.

Sometimes the lower Part of the *Bow* ſhall appear upon the Ground, and the upper Part of it not at all, and then it looks like a Rainbow lying along the Ground with the Extremities of its Legs turned upward into the Air: This is when the Sky is clear towards the Sun, but foggy on the oppoſite Parts, and only to a ſmall Height from the Ground.

The Moon ſometimes occaſions the Appearance of a *Rainbow* after the ſame Manner that the Sun does, but the Colours are much more faint and dilute.

And laſtly, if Water be continually thrown up into the Air oppoſite to the Sun, as from a Fountain, and there breaks into ſmall Drops, the Appearance of the Rainbow will be exhibited in them.

See more on this Subject in *Antonius de Dominis de Radiis Viſus & Lucis*; and the Authors referred to by Mr. *Johnſon*, in his *Philoſoph. Quæſt.* Chap. VII. Q. 45 & 46.

Of the Obſcura Camera *and the* Magic Lanthorn.

THE *Obſcura Camera* is of two Sorts; the one is no other than a convex *Lens* fixed in an Hole in a Window-ſhutter, which *Lens*, when no other Light is permitted to enter the Room except what paſſes through it, will repreſent all the external Objects that are viſible through that Hole upon a white Paper held at the focal Diſtance of it, painted in their proper Colours. To illuſtrate this, let AB (Fig. 88.) repreſent a Window-ſhutter, CD a convex *Lens* fixed in an Hole therein, and let EF be an external Object; then will this Object emit Rays of Light of its own Colour from each Part, which paſſing through the *Lens*, as the Figure repreſents them, will be collected into Points at GH, and being there received upon a white Paper or other Surface, will repreſent the Object painted in its proper Colours, which Colours will be the ſtrongeſt of all when the Sun ſhines upon that Side of the Object that is next the Glaſs. But the Repreſentation will be *inverted*, becauſe the Pencils of Rays that flow from the Object croſs in the Middle of the Glaſs.

The other Sort of *Obſcura Camera* is that which is called the *Portable one*, and is of Uſe in drawing Pictures, taking Landſkips, *&c.* It is contrived after the following Manner, AIKB (Fig. 88.) is a Box, in an Hole in whoſe Side the *Lens* CD is fixed (or rather at the Extremity of a ſhort Tube fixed in that Hole) and in the Situation LM is fixed a Piece of Looking-glaſs making an Angle with the Side of the Box of 45 Degrees; this, receiving the Rays in their Paſſage to GH, throws them upwards, and cauſes the Repreſentation to be made in NO, which is there received upon the under Side of ſome thin Subſtance PQ, that is in a ſmall Degree tranſparent, (as thin Paper or Glaſs about half poliſhed) and ſo upon opening the Box appears ready to be drawn or copied out. But that the Colours may appear ſtrong, the Light, when the Box is opened, muſt be kept from falling upon the Paper or Glaſs, as much as may be.

The *Magic Lanthorn* is an Inſtrument invented by *Kircher*, in order to repreſent Objects much larger and more luminous than they are. It is no other than a dark Lanthorn, in the Side of which there is fixed a ſhort Tube, and in the Tube two convex *Lenſes*, and between them a tranſparent painted Image of the Object we would repreſent. The Paſſage of the Rays through the

Lenſes and the Image is thus. Let A (Fig 89.) repreſent a burning Lamp placed as in a common Lanthorn, and let CD, EF, be the two *Lenſes* placed in the Tube abovementioned, and the Picture at GH. And let the Situation of the *Lens* CD be ſuch, that the Light which falls upon it from the Lamp may be all thrown upon the Picture GH, by which Means it will be ſtrongly illuminated, and being tranſparent, will throw out Rays in Plenty the other Way: Which Rays, in paſſing through the other *Lens* EF, let us ſuppoſe to be collected into their reſpective *Foci* on an oppoſite Wall at IK, and to form an Image there. Which Image will be larger than the Picture in Proportion as the Diſtance IL is greater than LH; becauſe the Angles ILK and GLH are equal; and the Room being dark every where elſe, it will appear very bright, if the Picture be ſtrongly illuminated by the Lamp. And beſides the abovementioned *Apparatus*, there is ſometimes a concave Reflecter placed within the Lanthorn behind the Lamp, as at MN, to give a ſtronger Illumination to the painted Image at GH.

A
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OF
Natural Philoſophy.
With NOTES
Containing the
MATHEMATICAL DEMONSTRATIONS,
AND
Some Occaſional REMARKS.

PART IV.
ASTRONOMY.
Containing the *real* and *apparent* Motions of
the *Heavenly BODIES.*
With an Attempt to account for the *Phænomena*
of the *COMETS TAILS,*
After a new Manner.

By J. ROWNING, M. A.
Rector of ANDERBY in LINCOLNSHIRE, and late
Fellow of MAGDALEN COLLEGE in CAMBRIDGE.

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A COMPENDIOUS SYSTEM OF Natural Philoſophy.

PART IV.

ASTRONOMY.

The INTRODUCTION.

THE Science of *Aſtronomy* may be diſtinguiſhed into two Parts, the one relating to the Motion of the heavenly Bodies, as they really are in themſelves; the other as they appear to a Spectator upon the Surface of the Earth: and theſe two are frequently the very Reverſe of each other; that Body appearing to move *Weſterly*, which in reality at the ſame time moves *Eaſterly*; and that ſeeming to tend towards the *North*, which is ſtanding ſtill, or haſtening towards the *South*: and the contrary. And becauſe the apparent Motions of theſe Bodies depend upon their real ones, the Order I ſhall proceed in, ſhall be firſt to lay down their real Motions; and then to ſhow how the apparent ones ariſe therefrom. But ſince the generality of People are apt to form their Judgment by their Senſes only, and are loath to imagine Things different from what they appear to be, to prepare ſuch Readers for a more eaſy Reception of what follows, it may not be amiſs to premiſe the following Particulars by way of Introduction.

I. Let us imagine only one great Body in the Univerſe, plac'd in the midſt of infinite Space, like a Ball in the Air; and then enquire whether of the two following Suppoſitions has the greater Probability on its Side; *viz.* That this Body will continue in its Place; or that it will not.

In perſuance of this Inquiry, perhaps it may be argued thus: Whereas no Bodies on which we can make tryal are obſerved to continue in their Places without a Support, neither will this Body remain in the Place it is put, but will immediately deſcend, as it were, towards the lower Regions of Space.

This Argument is founded upon the Obſervation of Bodies falling to the Ground for want of ſupport, which is not a parallel Caſe. For to ſay the leaſt that may be, we cannot be ſure but that the Earth itſelf may be the Cauſe why Bodies deſcend towards it; as the Loadſtone is the Cauſe why Steel approaches it when placed within a certain Diſtance from it. And if ſo, then for any thing that appears from hence to the contrary, this Body, being the only one in the Univerſe, may remain in its Place without a Support. In ſhort, the Argument is juſt as if one ſhould reaſon thus: It has never yet been obſerved, but that Steel when placed at a ſmall Diſtance below a Loadſtone will aſcend upwards; therefore by Parity of Reaſon, if all the Bodies in the Univerſe were transformed into a Maſs of Steel, and placed alone in the midſt of infinite Space, that Maſs would aſcend. The Fallacy of which way of arguing is ſufficiently apparent.

There is a much greater Probability of Truth in the oppoſite Way, and it is more rational to argue thus: By all the Experiments and Obſervations made upon Bodies, it does not appear, that any ever moves out of its Place, unleſs impelled by ſome other; or when it moves towards another, as in the Caſe of Bodies falling towards the Earth, or of Steel tending towards the Loadſtone. Now the Loadſtone is certainly ſome way or other the cauſe why the Steel tends towards it, and therefore (by Parity of Reaſon) in all Probability the Earth is ſome way or other the Cauſe, why Bodies are diſpoſed to move towards it: and if ſo, then ſuppoſing all the Bodies in the Univerſe to be collected into one, it would moſt likely continue where it was put, without requiring any thing to ſupport it, as having nothing which might impell it any way, or towards which it might move. This, I ſay, being the moſt natural and eaſy Deduction we can make in this Caſe, ſeems to have the greater Probability on its Side. And farther,

II. Since there are no Bodies, we can make Tryal upon, but what are diſpoſed to move towards ſome other Body, as Bodies towards the Earth, or Steel toward the Loadſtone, it is moſt probable that the ſeveral Bodies whereof the Univerſe conſiſts, would all move towards one another, if not obſtructed.

III. Let us ſuppoſe all the Bodies in the Univerſe to be united in one, and that Body to be put into Motion. Now whereas upon that Suppoſition there is nothing which may ſtop or deſtroy its Motion, it does not appear that it would ever ſtand ſtill, or loſe the leaſt Part of it. For by all the Experiments we can make upon the Bodies about us, there are none which, when put into Motion, can be ſtopped without ſome Force applied to them for that Purpoſe. They will *ſeem* indeed to ſtop of themſelves, which is always owing to ſome Impediment. Accordingly it is found, that the leſs Reſiſtance or Rubs Bodies meet with in their Way, the longer they continue their Motion*(a)*. Whence we may infer, that this Body once put into Motion, would ever continue in that State*(b)*.

IV. Let us now ſuppoſe that all the Bodies in the Univerſe are compriſed in two, the one of which let us imagine to be immenſely greater than the other. Theſe Bodies, if not obſtructed, will (agreeably to the Concluſion made in Section II.) approach towards each other; but becauſe a little one requires proportionably leſs Force to put it into Motion than a larger one, the leſs will be the only one in which the Motion will be ſenſible, becauſe we have ſuppoſed the other to be immenſely greater, which will therefore be ſcarcely moved out of its Place by the leſs; juſt as it is with a Loadſtone and a Piece of Steel hung up by two Strings near one another; if the Steel be much leſs than the Loadſtone, it ſhall move almoſt all the Way towards the Loadſtone, while that ſhall ſcarce ſtir out of its Place; but if the Steel much exceeds the Loadſtone in Bulk and Weight, the Loadſtone ſhall then move to the Steel, and not the Steel to the Loadſtone, as in the other Caſe: This being granted,

Let us conceive the leſs Body to have a ſwift Motion given it in a Direction parallel to the Surface of the larger Body. This Motion

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(a) See Part I. Chap. IV.

(b) See more in Confirmation of this, Part I. Chap. IV.

Motion will in ſome Meaſure prevent its approaching to that Body, and the greater the Velocity is that is thus given to it, the longer will the Time of its Approach be protracted thereby, and the greater Space it will move over before it comes to the Body. The Velocity therefore which is thus given to it, may be ſo adjuſted to the Degree of Tendency it has towards the other Body, that it ſhall move quite round it without ever touching it, returning to the ſame Place where it was when the Motion was at firſt given it. In this Caſe if it meets with no Oppoſition, it will move round it in the ſame Path over and over again, not unlike to a Stone whirl'd round the Hand in a Sling*(c)*. And as a ſmaller Body may continually move round a larger in this Manner, ſo may ſeveral leſs ones move round the ſame larger one at the ſame Time, provided the Diſtances of the leſs ones from each other be ſo great, that their mutual Tendencies towards each other do not diſturb their Motions*(d)*.

This is the Caſe of the Planets and Comets with reſpect to the Sun, which is a Body immenſely greater than any of them, placed as it were in the midſt of an infinite Space, and

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(c) See this explained more at large and illuſtrated, Part I. Chap. VIII.

(d) For the ſame Reaſon that the ſeveral ſmaller ones tend towards the larger, they will alſo tend one towards another, but with leſs Force in Proportion to their Smallneſs.

round which they perform their reſpective Revolutions(*e*).

As to what we call the *Sky*, in which the Heavenly Bodies ſeem as it were fixed; it is no real Subſtance, but mere empty Space. The Reaſon why it appears to be a Subſtance, is as follows. Was there no Atmoſphere ſurrounding the Earth(*f*), whoſe Particles might reflect other Rays of Light to our Eyes, than thoſe which come directly from the Sun, all Parts of the Heavens, even in ſpite of Sunſhine, would be quite dark; and the Stars would be viſible at Noon-day (*g*). But ſince

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(*e*) This Doctrine being allowed, we may ſee the Poſſibility of the Earth being habitable quite round: for if Bodies are not diſpoſed to move any otherwiſe than as they tend towards ſome other, there can be no more Danger of our *Antipodes* falling from the Earth, than there is of our riſing up into the Air. And as to their walking with their Head downwards with reſpect to us, there is neither Inconvenience, nor ſeeming Odneſs to them in that; for firſt, their Feet are turned towards the Surface of the Earth on which they walk, as well as ours are; and ſecondly, ſince both the Heavens and the Earth, which are the only Things they can judge of their Situation from, have the ſame Poſition with reſpect to their Bodies, when they ſtand or walk, that they have to us in the like Caſe, their Poſition cannot ſeem otherwiſe to them than ours does to us; for a Man cannot tell which way his Feet are turned, any otherwiſe than as his Judgment is directed by the Things about him; juſt as one cannot diſtinguiſh North from South in a ſtrange Place, without a Compaſs, or ſeeing either the Sun or Stars.

(*f*) See the Nature and Conſtitution of the Atmoſphere explained, Part II. Chap. III.

(*g*) See Chap. XIII.

the Atmoſphere of the Earth abounds with Particles capable of reflecting Light every way, ſome of it will fall upon our Eyes, to whatever Part of the Heavens they are directed. This Light, as all other, gives us the *Idea* of ſome Colour: having an Impreſſion of Colour upon our Minds, our Imagination preſents us with a Subſtance for it to inhere in. Juſt as when a Man views an Object in a Glaſs, the Light regularly reflected, gives him an Idea of the Colours of that Object duly ranged; but it is his Imagination that forms the Image he thinks he ſees.

Thus much by way of Introduction; which muſt not be underſtood as containing all the Reaſons for theſe Things; but only as tending to ſhew the Poſſibility, and in ſome Degree the Probability of them.

That which will be delivered afterwards, when we treat of the Phyſical Cauſes of the Motions of the Heavenly Bodies will ſet this Doctrine in a clearer Light, and abundantly evince the Reaſonableneſs of it. Proceed we in the mean time to conſider the Order theſe Bodies move in, and their reſpective Diſtances from the Sun, which makes the Subject of the firſt Chapter.

CHAP. I

Of the Bodies which compoſe the Solar Syſtem, *and their* real *Motions.*

THE Sun is a Body of prodigious Magnitude, ſituated in the midſt of an unbounded Space, and the Fountain of Light and Heat to a certain Number of Planets and Comets, which continually move round it *(a)*: all theſe taken together make up what is called the *Solar Syſtem.*

The Planets are in number ſixteen, ſix of which are called *Primary* ones; the other ten are diſtinguiſhed by the Names of *Secondary* ones, *Moons*, or *Satellites.*

The *Primary* Planets are *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter* and *Saturn.* They revolve about the Sun at different Diſtances from it, moving from Weſt to Eaſt *(b)*, in Orbits *(c)* nearly

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(a) The Sun itſelf is not abſolutely at Reſt, but is ſubject to a very ſmall Degree of Motion, which ſhall be conſider'd when we treat of the Phyſical Cauſes of the Motions of the Heavenly Bodies.

(b) When any of the Heavenly Bodies are ſaid by Aſtronomical Writers to move from Weſt to Eaſt, or from Eaſt to Weſt, it is always to be underſtood by way of the South. So that by the latter Expreſſion is meant the ſame Way that the Sun appears to move in its daily Courſe, by the former is intended the contrary.

(c) By the Orbit of a Planet is meant the Way or Path it deſcribes in moving round the Sun.

nearly circular, having the Center of the Sun directly within each of them, and nearly in the Middle; and they are not coincident one with another, but every one has about one Half a little above, and the other Half a little below each of the reſt. Or, to expreſs it in the common Way, the Planes *(d)* of their Orbits paſs through the Center of the Sun, and are ſo ſituated, that they make but ſmall Angles with each other.

Figure the firſt repreſents the *Solar Syſtem*, wherein the Point S is deſigned to denote the Center of the Sun; the Circle A B the Orbit or Path, which the neareſt Planet *Mercury* deſcribes in moving round it; CD, that in which *Venus* moves; FG, the Orbit of the Earth; H K, that of *Mars*; LN, that of *Jupiter*; and OP, that of *Saturn (e)*. Theſe Bodies, as they move round the Sun at very different

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(d) The Plane of an Orbit is the Space included therein. Thus, if the Curve Line ADME, in Fig. II. repreſents an Orbit, the Surface of the Paper within that Curve repreſents the Plane of that Orbit. Or in other Words, the Plane of the Orbit of a Planet is a flat broad Space, out of which the Planet never ſtrays.

(*e*) That the Sun is at Reſt, and that the Planets move round it as deſcribed above, is an Opinion received of old by *Philolaus*, *Ariſtarchus* of *Samos*, and the whole Sect of the *Pythagoreans*. The *Egyptians* were early Obſervers of the Heavens, and from them probably this Notion was received in *Greece*; tho' the Notion of the Earth ſtanding ſtill in the Center, and the whole Heavens revolving round it, was generally received and defended till the Time in which *Copernicus* flouriſhed, *viz.* about the Year 1500, who reſtored the ancient Aſtronomy, and ſhewed in ſo clear a Manner how the Appearances of the Heavens might be ferent Diſtances, the true Proportion of which is nearly repreſented in the Scheme, ſo they perform their reſpective Periods in very different Times; *viz.* *Mercury* in about three Months; *Venus* in about ſeven and a Half; the Earth in a Year; *Mars* in about two Years; *Jupiter* in twelve; and *Saturn* in not much leſs than thirty. And as they differ in their Periods, ſo they do ſurpriſingly in their Mag-

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be accounted for by it, that it became generally received, and from him was called the *Copernican* Syſtem.

There are two other *Hypotheſes* of Note, *viz.* the *Ptolemaic* and the *Tichonic.* The *Ptolemaic* is ſo called from *Ptolemæus* a Mathematician of *Peluſium* in *Egypt*, a great Defender of this *Hypotheſis.* He ſuppoſes the fix'd Stars to be ſtuck in the Firmament as they appear to the Eye; the Earth in the Center, round which he ſuppoſes the Moon to revolve; and at greater Diſtance, *Mercury*; at a greater ſtill, *Venus*; at a greater than this, the Sun; at a ſtill greater, *Mars*; then *Jupiter*; then *Saturn*; and beyond all, the Firmament with the fixed Stars; all in the ſame Time they appear to move round us. This *Hypotheſis* was excellently well adapted to amuſe the Vulgar, the Motions of the Sun and Planets being repreſented by it much after the ſame Manner as they appear to Senſe. But ſince by the Help of Teleſcopes it has been diſcover'd, that *Venus* puts on different *Phaſes*, like the Moon, the Reaſon of which will be ſhewn in Chap. VI. this *Hypotheſis* is entirely laid aſide, as inconſiſtent therewith.

The *Tychonic Hypotheſis* has its Name from *Tycho Brahe*, a *Daniſh* Nobleman. This Philoſopher ſuppoſes that the Earth ſtands ſtill, and that as the Sun moves round the Earth, the Planets move round that, juſt as the Secondaries move round their Primaries in the *Solar Syſtem* laid down above. There is another *Hypotheſis* called the *Semitychonic*, and differs from the former only in this; that whereas that makes the Earth ſtand ſtill, and the Sun with the Planets to move round it every Day; this gives the Earth a diurnal Motion round its *Axis*, and only ſuppoſes the Sun with the Planets revolving about it, to move round the Earth once a Year.

nitudes. For ſuppoſing the Circumference of the Sun to be repreſented by a Circle of ten Inches in Diameter, the ſeveral Circles Q, V, W, X, Y, Z, would nearly expreſs thoſe of the Planets *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn* reſpectively *(f)*.

Theſe Planets have no Light of their own; for when viewed through a Teleſcope, the Side only next the Sun is obſerved to be enlighten'd. They are alſo of the Form of a Globe; for one Edge of the ſhining Part ſometimes appears hollow, ſometimes ſtrait, and ſometimes convex, according as the Planet is ſituated with regard to the Spectator and the Sun; which it could not do, unleſs they were of that Form *(g)*.

Farther, the Planets do not only move about the Sun, as we have obſerved, but turn alſo about their own *Axes* *(h)* at the ſame Time,

and

(f) Their Magnitudes, Diſtances, Times of Revolution, with their Rotation about their *Axes*, are more accurately expreſs'd in' the following Table.

	Diameters in Engliſh Miles.	Diſt. from the Sun in Engliſh Miles.	Periodical Times.				Rotation about their *Axes*.			
			Days	H.	M.	S.	D.	H.	M.	S.
The Sun	763,460						25	6	0	0
Mercury	4,240	32,000,000	87	23	15	53	not diſcovered			
Venus	79,06	59,000,000	224	16	49	24	00	23	16	0
The Earth	7,970	81,000,000	365	6	9	14	00	23	56	4
Mars	4,444	123,000,000	686	23	27	30	00	24	40	0
Jupiter	81,155	424,000,000	4,332	12	20	25	00	9	55	54
Saturn	67,870	777,000,000	10,759	6	36	26	not known.			

(g) See the Reaſon why one Edge of the ſhining Part of a round Body appears ſometimes ſtrait, ſometimes hollow, ſometimes convex, largely explained Chap. VI.

(h) By the turning of a Planet about its *Axis* is meant, its turning about any two Points in its Surface that are oppoſite to one

another;

and the ſame Way they do about the Sun, *viz.* from Weſt to Eaſt. As the former of theſe is generally compared to the revolving of a Stone in a Sling about the Hand, this latter may be compared to the ſpinning of a Top or the turning of a Wheel upon its Axle-tree. The Sun itſelf is not exempt from a Motion of this latter Kind.

This Motion of the Sun and ſome of the Planets about their *Axes* is diſcoverable by certain Spots, or diſtinguiſhable Parts on their Surfaces; which appearing firſt on one Extremity of their Diſks(*i*), do by Degrees come forwards towards the Middle, and ſo paſs on till they reach the oppoſite Edge thereof, where they diſappear; and after they have lain hid about the ſame ſpace of Time that they continued viſible, they appear again as at firſt. By the Motion of theſe Spots paſſing in a right Line over the Diſk of a Planet, or riſing upwards one Half of the Way, and deſcending the other

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another; and an imaginary Line ſuppoſed to be drawn through the Planet from one of thoſe Points to the other, is, while the Planet moves about thoſe two Points, called its *Axis*. Thus if the Reader holds a Ball between his Finger and Thumb, and turns it round, a Line ſuppoſed to paſs through it from Finger to Thumb, will in Aſtronomical Language be the *Axis* of that Ball. And if he takes it up by two other Points, and turns it, then will a Line paſſing through thoſe other Points become its *Axis*.

(*i*) This is a Term uſed by Aſtronomers for the Face of the Sun, Moon, or other Heavenly Body, when it is conſidered as a flat round Surface, as it appears to be: and the Breadth of it is conceived to be divided into twelve equal Parts, which they call *Digits*.

ther Half, the Situation of the Planet's *Axis*, about which it turns with reſpect to us, is evidently diſcovered. Theſe Spots are very viſible on the Surface of the Sun, *Venus*, *Mars* and *Jupiter*; but by reaſon of the Nearneſs of *Mercury* to the Sun, and the great Diſtance of *Saturn* from us, no Obſervations have been made that could diſcover any Spots in them, ſo that it is uncertain whether theſe Bodies revolve about their *Axes* or not. As to the reſt, the Times in which they thus revolve is expreſſed above in Note *(f)* in page 14. and the Inclination of their *Axes* in the Note below *(k)*.

In *Jupiter*, beſides his Spots, there are ſeveral broad Spaces running parallel to each other, called his *Belts*. Theſe, as alſo his Spots, are obſerved to undergo ſeveral Changes, neither keeping the ſame Magnitude nor Diſtance from one another. And ſome of his Spots have appeared only for a Time.

As to the Spots of the Sun, they are ſubject to much greater Variety of Changes. New ones appear, and old ones vaniſh, others ſucceeding in their Room; ſeveral ſmaller ones run together, and form one larger; and larger

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(k) The Inclination of the Axis of the Sun to the Plane in which the Earth moves, is ſeven Degrees, and a half: that of *Venus* to the ſame Plane fifteen Degrees, according to an Obſervation of Signior *Blanchini*'s at *Rome* in the Year 1726. the Earth's 23 Degrees and 29 Minutes: and *Mars* and *Jupiter*'s are nearly at right Angles with their own Orbits.

er ones are ſometimes divided into ſmaller *(l)*. However, ſeveral have remained on the ſame Part of its Body long enough to determine the Time of its Motion about its Axis. See Note *(f)* in page 14.

In the Beginning of this Chapter it was obſerved, that the Planets move round the Sun in Orbits nearly circular, and in Planes croſſing each other in ſuch Manner, as to make very ſmall Angles with each other. As to the Form of their Orbits, they are *Ellipſes* (*m*), having one of their *Foci (n)* in the Center of the Sun. They differ very little from Circles, or to ſpeak properly, their Excentricities (*o*) bear a very

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(l) We learn from Hiſtory, that the Sun has wanted its uſual Brightneſs, ſhining with a dim and obſcure Light for a Year together. This was probably owing to its being in a great Meaſure covered with Spots; for now there are ſometimes Spots ſeen upon it, that are larger than the Surface of the whole Earth.

(m) An *Ellipſe* is a Figure like an Oval; but ſtrictly ſpeaking, a Geometrical Curve, made by the tranverſe Section of a Cone or a Cylinder. But the Nature of it may be underſtood from the following Deſcription. Stick up two Pins at F and G (Fig. 2.) over which put a Thread, both Ends being tied together; then with a third Pin at P, within the Thread, keeping it upon the Stretch, deſcribe the Curve ADBE, which is an Ellipſe. And if we make Uſe of the ſame Thread, but increaſe or diminiſh the Diſtance between the Pins, the Figure deſcribed will ſtill be an Ellipſe, though of a different Kind. And when the Pins are brought cloſe together, the Figure deſcribed degenerates into a Circle.

(n) The Points F and G where the Pins were fixed, are called the *Foci* of the Ellipſe, and the middle Point between them the Center.

(o) The Diſtance between the Center and either of the *Foci* is the *Excentricity*.

a very ſmall Proportion to their longer *Axes* *(p)*. And as to the ſituation in which their Planes lie in, it is in all nearly the ſame with that of the Earth's Orbit. See the Excentricities of their Orbits, and their Inclinations to that of the Earth's, in the Notes *(q)* and *(r)*.

The Primary Planets are again diſtinguiſhed into two Kinds, *viz. Inferior* and *Superior*; thoſe which are nearer the Sun than the Earth is, as *Mercury* and *Venus*, are called the *Inferior* Planets; the others the *Superior* ones.

The Secondaries are conſtant Attendants of the reſpective Primary ones to which they belong, revolving round them, while they themſelves move round the Sun. Of the Primary Planets there are only the three largeſt, as far as the Obſervations of Aſtronomers have been able to diſcover, that have their Secondaries, *viz.* the *Earth*, *Jupiter*, and *Saturn*.

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(p) The Line AB, which paſſes through the *Foci*, and is terminated at each End in the Ellipſe, is called the *longer Axis*. The Line DE, which paſſes through the Center of the Ellipſe in a Direction perpendicular to the former, and is terminated by the Sides of the Ellipſe, is the *ſhorter Axis*.

(q) The Excentricities of the ſeveral Orbits of the Planets are as follow: Suppoſing the Diſtance of the Earth from the Sun 1000 equal Parts, the Excentricity of *Mercury*'s Orbit is about 80 ſuch; of that of *Venus* 5; of that of the *Earth* 17; of *Mars* 141; of *Jupiter*'s 250; and the Excentricity of *Saturn*'s Orbit 247.

(r) Their Inclinations are as follows: *viz.* The Orbit of the Planet *Mercury* is inclined to that of the Earth 6 Degrees, 54 Minutes; that of *Venus* is inclined to it 3 Degrees, 24 Minutes; that of *Mars*, 1 Degree, 51 Minutes; that of *Jupiter*, but 1 Degree, 19 Minutes; and that of *Saturn*, 2 Degrees and 33 Minutes.

The *Earth* is attended but by one; which is the Moon. This Secondary revolves round the Earth from Weſt to Eaſt in 27 Days, 7 Hours and 43 Minutes, at the Diſtance of about 60 Semidiameters and an half of the Earth from its Center *(s)*; and performs its Rotation about its Axis, which is nearly perpendicular to the Plane of the Earth's Motion, in the ſame Time and the ſame Way that it revolves about the Earth.

The Planet *Jupiter* has four Moons, which revolve about it at different Diſtances and in different Times. The innermoſt performs its Revolution in about one Day and eighteen Hours, nearly at the Diſtance of ſix Semidiameters of *Jupiter* from its Center; the ſecond revolves about it in about three Days and an Half, at the Diſtance of about nine ſuch Semidiameters; the third in about ſeven Days, at the Diſtance of 14 Semidiameters; the fourth and outermoſt Satellite requires almoſt 17 Days to perform its Courſe in, and is diſtant from its Center 25 Semidiameters nearly *(t)*.

(s) The Excentricity of the Moon's Orbit is about 3,3 of the Semidiameter of the Earth; and its Inclination to that of the Earth is 5 Degrees and 18 Minutes. The Reaſon that the Moon appears ſo large, and to outſhine all the other Planets, both Primary and Secondary, is becauſe of the Smallneſs of its Diſtance from us in reſpect of theirs; for had it been as far removed from us as ſome of them, it had never been ſeen at all by us.

(t) More accurately as in the following Table.

Saturn has no leſs than five Satellites: The firſt and innermoſt revolves about it in one Day and 21 Hours, at the diſtance of about 5 Semidiameters and an half of *Saturn* from its Center; the Second performs its Courſe in two Days aud 17 Hours, at the Diſtance of 6 Semidiameters and a Quarter; the Third in about 4 Days and an half, and is diſtant from it about 8 Semidiameters and 3 Quarters; the fourth compleats its Period in about 16 Days, at the Diſtance of 20 Semidiameters; the fifth and outermoſt requires above 79 Days to perform a Revolution in, and is 59 Semidiameters of *Saturn* diſtant from its Center(*u*).

Theſe Satellites, as alſo thoſe of *Jupiter*, perform their Revolutions about their Primaries from Weſt to Eaſt, as their Primaries do about the Sun.

The Orbits of the ſecondary Planets are expreſſed in Fig. I. by the pricked Circles; *viz.*

Satell.	Periodical Times.				Diſt. in Semid. of *Jupiter*.
	D.	H.	M.	S.	
1	1	18	27	34	5,667
2	3	13	13	42	9,017
3	7	3	42	36	14,384
4	16	16	32	09	25,299

(*u*) More accurately as in the following Table.

Satell.	Periodical Times.				Diſt. in Sem. of *Saturn*'s Ring.
	D.	H.	M.	S.	
1	1	21	18	27	2,10
2	2	17	41	22	2,69
3	4	12	25	12	3,75
4	15	22	41	14	8,70
5	79	07	48	0	25,35

viz. that of the Moon at E, thoſe of the Satellites of *Jupiter* at I; and thoſe of *Saturn* at T; the Centers of the *Earth*, *Jupiter* and *Saturn* being ſuppoſed to be in the Points E, I, and T reſpectively.

As to the Orbits of the Satellites of *Jupiter* and *Saturn*, Aſtronomers have not obſerved that they differ from Circles; and whether theſe Secondaries revolve round their *Axes* or not, has alſo not been diſcovered (*x*).

The Secondaries are alſo Bodies having no Light of their own, being obſerved to be eclipſed as oft as they enter the Shadows of their Primaries.

Beſides its Satellites, *Saturn* is alſo encompaſſed with a flat broad Ring, not unlike the Verge of an Hat, but it no where touches the Body of *Saturn*, the Space between it and *Saturn* being equal to the Breadth of the Rim. The Diameter of the Ring, meaſuring acroſs from one Outſide to the other, is to the Diameter of *Saturn*'s Body as 7 to 3 (*y*).

The Form of the Ring encompaſſing the Body of *Saturn*, is expreſs'd in Fig. I. at RR. The

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(*x*) The Orbits of *Jupiter*'s Satellites lie almoſt in the ſame Plane in which *Jupiter* himſelf moves round the Sun; but thoſe of *Saturn* deſcribe their Orbits nearly in the Plane of the Ring produced, except the fifth, the Situation of whoſe Orbit deviates a little therefrom. See the Inclination of the Ring in the next Note.

(*y*) The Ring, according to Meſſrs. *Roëmer*, *Picard* and is inclined to the Orbit of the Earth 31 Degrees: *Halley* affirms it to be nearly parallel to the Plan Earth's Equator.

The Comets revolve alſo round the Sun in Orbits, whoſe Planes paſs through its Center; but they are not confined to any particular Direction, as the Planets are, ſome moving one Way, and ſome another: their Orbits are of a very elliptical or oval Form, ſo that their Lengths vaſtly exceed their Breadths; and they lie not nearly in the ſame Plane, like thoſe of the Planets, but admit of any Inclination to one another. But more of theſe in another Place(z). A Portion of the Orbit of a Comet is expreſſed in Fig. I. at *abcd* (a).

(z) Chap. XI.

(a) The Claim which the Syſtem here laid down has to a Preference to all others, will appear from the following Conſiderations.

I. The Planet *Venus* moves round the Sun, and not round the Earth. For through a Teleſcope it ſometimes appears with a bright and round Face like the full Moon, and ſometimes horned like the new Moon; (ſee Chap. VI.) and when it appears full, it is ſeen near the Body of the Sun, as well as when it appears horned: which things could not be, did it move either round the Earth alone, or round neither the Sun nor the Earth. For did it move round the Earth alone, or round both the Sun and the Earth, it muſt be ſeen ſometimes in that Part of the Heavens which is oppoſite to the Sun. Did it move round neither the Sun nor the Earth, it could not appear both horned and full when it is ſeen near the Body of the Sun: it remains therefore, that it revolves round the Sun, and not round the Earth. Its Orbit therefore, agreeably to the abovementioned Syſtem, includes the Sun, and is within the Orbit of the Earth.

II. In like Manner we are aſſured, that *Mercury* turns round the Sun in a leſſer Orbit than that of *Venus*; becauſe it puts on the ſame Phaſes with *Venus*, and is never ſeen ſo far from the Sun as that Planet is.

III.

Thoſe bright and ſhining Bodies, which beſides the Planets and Comets are diſperſed every

III. The Orbit of *Mars* includes both the Earth and the Sun; and the Earth is not in the Center thereof. For this Planet is capable of appearing oppoſite to, or in any other Situation in reſpect of the Sun, which it could not be, unleſs it moved round the Earth; and it always appears full, or nearly ſo; which it could not do if it ever came between the Sun and the Earth: it moves therefore alſo round the Sun. And farther, when *Mars* is in the oppoſite Part of the Heavens to the Sun, it appears about five times larger than when it is near the Sun; which ſhews that it is ſo many times nearer the Earth in one Situation than in the other: the Earth therefore is not in the Center of its Motion. All which is agreeable to the Syſtem above laid down

IV. Since the like is obſervable both of *Jupiter* and *Saturn*, (though, by Reaſon of their greater Diſtance from the Sun and us, the Diverſity in their apparent Magnitudes in different Parts of their Orbits is not ſo great as in *Mars*) it is reaſonable to conclude, that theſe Planets alſo have both the Earth and the Sun within their Orbits, and that rather the Sun than the Earth is in the Center of the ſame.

V. Since then the Earth is placed within the Orbits of the Planets *Mars*, *Jupiter*, and *Saturn*, theſe Planets cannot appear to us to ſtand ſtill or to go backward, as Obſervation ſhews they ſometimes do appear, unleſs the Earth moves: and ſince, as we have ſhewn, the Planets *Venus* and *Mercury* revolve about the Sun, and not about the Earth; ſince likewiſe the Earth is placed between the Orbits of *Mars* and *Venus*, and the Periodical Time of the Earth, if it does move, is, in point of Magnitude, between the Periodical Times of thoſe two, it has been thought reaſonable to ſuppoſe that the Earth revolves in like Manner about the Sun, as the Planets do, and is therefore reckoned in the Number of them.

VI. But what wonderfully confirms this Concluſion, is, that Harmony which upon this Suppoſition runs through the whole *Solar Syſtem*; *viz.* that the Motions of all the Planets, both Primary and Secondary, are governed and regulated by one and the ſame Law; which is, that *the Squares of the Periodical Times of the Primary Planets are* to each other, *as the Cubes of their Diſtances* from the Sun; and likewiſe *the Squares of*

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every where throughout the Heavens, are the *fixed Stars.* They are termed *fixed*, becauſe they are obſerved to keep the ſame Diſtances from each other in all Ages, having no Motion like that of the Planets or Comets. They are far removed out of the *Solar Syſtem*, and ſhall therefore be conſidered in a Chapter by themſelves.

the periodical Times of the Secondaries of any Primary are to each other, *as the Cubes of their Diſtances* from that Primary. Now the Moon, which in the Syſtem above laid down is a Secondary of the Earth, in the other Hypotheſes is a primary One; and ſo the Rule cannot take Place; becauſe her Periodical Time, conſidered as that of a Primary one, does not agree therewith. Whereas according to the other Suppoſition, the Motions of the Heavenly Bodies, as we ſhall ſee in its proper Place, are to be accounted for with the greateſt Simplicity; and a regular and beautiful Fabrick is thereby exhibited, worthy its divine Architect.

By the *Square* of the periodical Time of a Planet is meant the Time it takes up in revolving about the Sun, or about its Primary, multiplied by itſelf *Once*; and by the *Cube* of its Diſtance, is meant its Diſtance multiplied by itſelf *Twice*. Thus if the Time a Planet moves once round its Orbit in, be 30 Years, then 30 times 30 is the *Square* of its periodical Time: and if the Diſtance of a primary Planet from the Sun, or of a Secondary from its Primary, be 81 Millions of Miles, then 81 Million multiplied by 81 Million, and that again by 81 Million, will expreſs the *Cube* of the Diſtance of that Planet.

CHAP. II.

Of the fixed Stars.

THE *fixed Stars* are thoſe bright ſhining Bodies, which, beſides the Planets and Comets, are diſperſed throughout the Heavens: they are termed fixed, as appearing always at the ſame Diſtance from each other.

It is obſervable of the *fixed Stars*, that, when viewed through a Teleſcope, they appear only as mere Points deſtitute of all ſenſible Magnitude. This is ſuppoſed to be owing to their very great Diſtance from us. That they appear larger to the naked Eye than with a Teleſcope, is aſcribed to that Irradiation or Brightneſs which is obſerved to ſurround ſhining Bodies when ſeen in the Dark, and which the Teleſcope takes off *(a)*.

From their ſhining with ſo ſtrong and ſparkling a Light, when at the ſame Time they are ſo far off that through a Teleſcope they appear as mere Points, is inferr'd, that they ſhine with

(*a*) That the Stars would appear of no ſenſible Magnitude to the naked Eye, were it not for that Irradiation or Brightneſs mentioned above, is confirmed from their being obſerved to *twinkle*; which the Planets and other Bodies of ſenſible Magnitude do not. The Reaſon of their *twinkling* is, that the leaſt Particle of Duſt or Vapour paſſing between the Eye and them is capable of intercepting their Appearance for a Time, which in larger Bodies it is not.

with their own proper and unborrowed Light: for if they borrowed their Light, they would be inviſible to the naked Eye, ſince the Satellites of *Jupiter* and *Saturn*, which appear of very diſtinguiſhable Magnitudes through a Teleſcope, are ſo.

There is another remarkable Circumſtance relating to the ſtarry Heavens; that the better Teleſcopes we make uſe of, the more Stars appear: from whence it is thought, that all theſe *fixed Stars* are not placed at equal Diſtances from us, but that they are every where interſperſed at great Diſtances beyond one another throughout the Univerſe; and that probably the different Appearances which they make in point of Splendor and Magnitude may be rather owing to their various Diſtances from us, than to any Difference in their Magnitudes.

The Diſtance of the neareſt *fixed Star* is computed to be at leaſt 500 thouſand Millions of Miles from us; and their Magnitude is judged to be ſuch, that if they were as near us as the Sun is, they would not appear unlike it (*b*).

Their appearing under various Magnitudes has given Occaſion to a Diſtribution of them into ſeveral Orders or Claſſes: thoſe which appear largeſt, are called *Stars of the firſt Magnitude*,

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(*b*) The Method of computing the Diſtance of the *fixed Stars* is ſhewn in Chap. V.

Magnitude; thoſe which are neareſt to them in Luſtre are reckoned of the *ſecond Magnitude*; and ſo on, till we come to the leaſt Stars that are viſible to the naked Eye, which conſtitute the laſt Claſs, and are ſaid to be of the *ſixth Magnitude*. For ſuch as are to be ſeen only by the help of a Teleſcope, are not included in thoſe ſix Claſſes, but are diſtinguiſhed by the Name of *Teleſcopic* Stars; the Diſtribution of the Stars into Claſſes being made long before the *Teleſcopic* ones were diſcovered. But we are not to underſtand from hence, that all the Stars which are viſible to the naked Eye, are reducible to ſome one of theſe Claſſes preciſely; for there are ſcarcely two Stars to be found exactly of the ſame Size: they are only to be ranked in that Claſs to which their Magnitude and Luſtre gives them the juſteſt Claim.

There is alſo another Diſtribution of the Stars into *Conſtellations*. The Ancients, that they might the better diſtinguiſh the Stars with regard to their Situation in the Heavens, divided them into ſeveral *Aſteriſms* or *Conſtellations*, that is, Syſtems of Stars, each Syſtem conſiſting of ſuch as are near each other. And to diſtinguiſh theſe Syſtems from one another, they gave them the Names of ſuch Men or Things as they fancied the Space they took up in the Heavens repreſented, or of thoſe whoſe Memories, in Conſideration of ſome notable Exploit, they had a Mind to tranſmit to future Ages.

Beſides the Stars contained within theſe Conſtellations, there are ſome others, which for that Reaſon are called *Informes.* Of theſe the later Aſtronomers have made ſome new Conſtellations.

There is alſo a Diviſion of the Heavens into three Parts; *viz.* 1. The *Zodiac*, or that Portion of the Heavens in which the Planets would appear to move to an Eye placed in the Sun. The Breadth of this Space depends on the Inclination of the Orbits in which the Planets move to one another, and includes twelve Conſtellations, commonly called the Signs of the *Zodiac*; *viz. Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*, *Sagittarius*, *Capricornus*, *Aquarius*, and *Piſces:* and becauſe moſt of theſe are Animals, that Space is called the *Zodiac.* 2. All that Region of the Heavens which lies on the North Side the *Zodiac*; which contains twenty-one Conſtellations: and 3. That on the Southern Side, which contains fifteen;

There is a remarkable Tract that goes quite round the Heavens, called the *Milky Way.* This Space has a peculiar Whiteneſs, occaſion'd by an infinite Number of ſmall Stars therein, none of which are to be ſeen diſtinctly without a Teleſcope, yet conſpire to render that Part of the Heavens, where they are, much more luminous than the reſt. This Tract is in ſome Parts ſingle, in others double (*a*).

Notwithſtanding that ſeeming Impoſſibility of *numbring* the Stars, their relative Situations one to another have been ſo carefully obſerved by Aſtronomers, that they have not only been able to number them, but even to diſtinguiſh the Places of each in the Heavens, and that with greater Accuracy, than any Geographer could ever point out the Situations of the ſeveral Cities or Towns upon the Surface of the Earth. And not only the Places of thoſe few, if I may call them ſo, which are to be ſeen with

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(*a*) *Hugenius*, in the Year 1656, looking by chance through a large Teleſcope at three ſmall Stars very cloſe to one another in the Middle of *Orion*'s Sword, ſaw ſeveral more as uſual: But three little Stars very near one another, together with four larger ones, ſhone out as it were through a whitiſh Cloud much brighter than the ambient Sky; which being very black and ſerene, cauſed that lucid Part to appear like an Aperture that gave a Proſpect into a brighter Region. He viewed it many Times, and found it continued in the very ſame Place and of the ſame Shape. But in the *Philoſophical Tranſactions*, Nº· 347. there is an Account of a later Diſcovery of five more ſuch lucid Spots, though leſs conſiderable than this of *Hugens*; the Middle of which, we are there told, is at preſent in ♊. nineteen Degrees, with South Latitude twenty-eight Degrees, forty-five Minutes; and that it ſends forth a radiant Beam into the South-Eaſt; as another in the Girdle of *Andromeda* ſeems to do in the North-Eaſt. It is alſo there remarked, that tho' theſe Spots are in Appearance but ſmall, and moſt of them but a few Minutes in Diameter, yet ſince they are among the *fixed Stars*, as it appears they are by their having no Parallax, they cannot fail to occupy Spaces immenſely great, and perhaps not leſs than our whole *Solar Syſtem*: in all which Places, it ſhould ſeem, there is a perpetual uninterrupted Day.

with the naked Eye have been pointed out and regiſtered by them, but even of ſuch as are diſcoverable only by the Teleſcope. The firſt among the *Greeks*, who numbered thoſe which are viſible to the naked Eye, and regiſter'd their Places in a Catalogue, was *Hipparchus*. He flouriſh'd about 120 Years before *Chriſt*, and number'd 1022 Stars. To this Catalogue the Places of ſeveral more have been added by ſucceeding Aſtronomers, as *Ptolemy*, *Copernicus*, *Tycho Brahe*, *Ricciolus*, and Dr. *Halley*; which laſt went to the Iſland of St. *Helena*, where he reckoned up and ſet down the Places of ſuch Stars in the Southern Hemiſphere as are not viſible to us in this.

The laſt Catalogue is that of *Flamſtead*, who reckons 3000, of which a great Part are *Teleſcopical*. The Number of Stars which may be ſeen at one Time with the naked Eye in one Hemiſphere ſeldom exceed a Thouſand; which perhaps may appear ſtrange, ſince at firſt Sight they ſeem to be innumerable. But this Appearance is only a Deception of Sight, ariſing from a confuſed and tranſient View. Let a Perſon ſingle out a ſmall Portion of the Heavens, and after ſome Attention to the Situation of the more remarkable Stars therein, begin to count, and he will ſoon be ſurpriſed to find how few there are therein.

Yet nevertheleſs, if we conſider the Teleſcopic Stars along with thoſe which are viſible to the naked Eye, we may venture to pronounce them infinite: ſo prodigious are the Numbers that may be ſeen in a good Teleſcope.

Thoſe in Mr. *Flamſtead*'s Catalogue mention'd above, that are Teleſcopical, are only the more remarkable ones, ſuch whoſe Longitudes and Latitudes (that is, their true Situation in the Heavens) it was thought worth while to regiſter and put down. Dr. *Hook*, with a Teleſcope of 12 Feet, ſaw 78 Stars among the *Pleiades*, that is, thoſe which are commonly called the ſeven Stars; and with a longer Teleſcope ſaw more *(b)*. And in the ſingle Conſtellation of *Orion*, which in *Flamſtead*'s Catalogue has but 80 Stars, there have been ſeen 2000 *(c)*.

There is one Thing more which is very remarkable in the fixed Stars, *viz.* that ſome of thoſe which were taken Notice of, and had their Places regiſter'd, have ſince diſappeared. Some have diſappeared for a Time: Some new ones have appeared, and afterwards diſappeared; particularly a remarkable one in the Conſtellation *Caſſiopeia*, in the Year 1572, which ſome time after its firſt Appearance outſhone the biggeſt of the fixed Stars, and in ſixteen Months Time by degrees vaniſhed quite away, and was never ſeen ſince. In the *Pleiades*, which uſed to be reckoned 7, there are but 6 to be ſeen with the naked Eye.

From

(b) Hook's Micrograph. pag. 241

(c) Antonius Maria de Rheita, Rad. Sydereomiſt. pag. 197.

From the Similitude there appears to be between the *fixed Stars* and the Sun, it is generally ſuppoſed by Philoſophers, that they are not placed in the Heavens by way of Ornament only, or to ſupply us with a faint Light in the Abſence of the Moon; but that each of them is placed in the midſt of a Syſtem of planetary Worlds, and that it directs their Motions, and ſupplies them with Light and Heat, in the ſame Manner that the Sun does the ſeveral Bodies of which the Solar Syſtem is compoſed *(d)*.

As to thoſe Alterations in the Appearances of ſome of the fixed Stars mentioned above, Sir *Iſaac Newton* conjectures, that, as it is poſſible the Sun may ſometimes receive an Addition of Fewel by the falling of a Comet into it (the only Uſe *(e)* they ſeem to be of) ſo the ſudden Appearance of ſome of the Stars, which formerly were not viſible to us, may be owing to the falling of a Comet upon them, and occaſioning an uncommon Blaze and Splendor for ſome Time. But that ſuch as appear and diſappear periodically, and increaſe by very ſlow

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(d) How the Sun directs the Motions of the Planets will be ſhewn afterwards.

(e) Sir *Iſaac Newton* computes, that the Comet which appeared in 1680 approached towards the Sun within leſs than a ſixth Part of the Sun's Diameter; and from thence concludes, that it muſt have been retarded by the Reſiſtance of the Sun's Atmoſphere, which is very large; the Conſequence of which is, that it muſt come nearer and nearer the Sun every Revolution, till at laſt it falls into its Body.

flow Degrees, ſeldom exceeding thoſe of the third Magnitude, may be ſuch as having large Portions of their Surfaces obſcured by Spots, may by revolving about their *Axes*, like the Sun, expoſe their lighter and darker Parts to us ſucceſſively.

CHAP. III.

Of ſuch of the apparent *Motions of the Heavenly Bodies, as ariſe from the Motion of the* Earth *about its* Axis.

HAving now laid down the *true* and proper Motion of the Planets, and ſhewn, as far as the Obſervations of Aſtronomers have been able to inform us, the Nature of the Sun, the Planets, and the fixed Stars; we are in the next Place to enquire particularly into their *apparent* Motions, and ſhew how it comes to paſs that theſe ſeem to us ſo different from what they really are. And theſe are principally of two Sorts, *viz.* thoſe which ariſe from the Motion of the Earth round its *Axis*, and thoſe which are owing to the Motion of the Earth and Planets about the Sun. The former of theſe ſhall be conſider'd in this Chapter; the latter in thoſe which follow.

Since then, as we obſerved above, the Earth turns round its own *Axis* from Weſt to Eaſt, every Spectator upon its Surface muſt neceſſarily be carried round it the ſame Way; and conſequently thoſe Parts of the Heavens which lie hid towards the Eaſt, will by and by come into his Sight, and thoſe which are viſible to him will depart out of it towards the Weſt. From hence it is that the Spectator, not being ſenſible of his own Motion (the Reaſon of which is, becauſe all things about him move along with him) imagines the whole Heavens to turn round the contrary Way, *viz.* from Eaſt to Weſt, every 24 Hours, which is nearly *(f)* the Time the Earth performs one Revolution about its *Axis* in. This may be illuſtrated in the following Manner.

Let the Circle STV (Fig. 2.) repreſent the Earth, S the Place of a Spectator, ABC ſo much of the Heavens as is viſible to him in that Situation, and let A be the Place of the Sun or any other of the heavenly Bodies: When S, the Place of the Spectator, is carried by the Rotation of the Earth about its *Axis* to T, the viſible Part of the Heavens will become B A D; and the Point A, which before was juſt at one Edge of the viſible Portion of the Heavens, is now in the midſt of it, or directly over the Spectator's Head. Again, when

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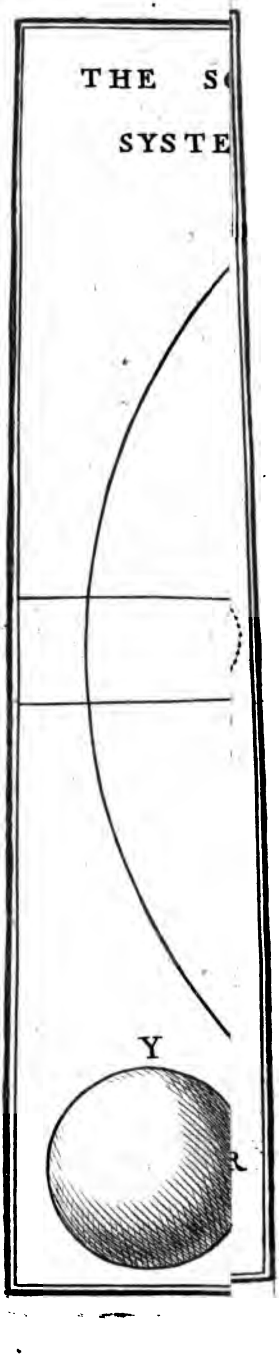
(f) The Reaſon why this is not the *exact* Time, is, becauſe of the Motion of the Earth in its Orbit performed in the mean while; as ſhall be explained in Chapter the XV.



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when the Spectator is got to V, the viſible Part of the Heavens is become ADX, and the Point A is got to the other Extremity of it, and juſt ready to diſappear; after which it is ſeen no more till the Spectator arrives at the Point S again.

From this Motion of the Earth ariſes that apparent Revolution of the Planets and fixed Stars once in 24 Hours; as alſo that of the Sun, and therewith the Succeſſion of Day and Night.

That the Days vary from each other both in Point of Length and Heat, is owing to the Motion of the Earth round the Sun, and will be accounted for in the next Chapter.

CHAP. IV.

Of the apparent *Motion of the* Sun, *ariſing from the* Earth's *revolving about it.*

AS when a Perſon ſails along the Sea-Coaſt, the Shore, the Villages, and other remarkable Places upon Land appear to change their Situation, and to paſs by him; and this apparent Change of Place is of two Sorts, (the one, is that of Bodies at reſt, the

Change of whoſe Places depend ſolely on that of the Spectator; the other is, that of Bodies in Motion, whoſe apparent Change of Place depends as well on their own Motion as on that of the Spectator;) ſo it is in the Heavens: To a Spectator upon the Earth, as it moves along its Orbit, the Sun, the Planets, and the fixed Stars appear to change their Places; which Change in thoſe which are at reſt is owing wholly to the Motion of the Earth; in thoſe which move, it is to be aſcribed to their own conjointly with that of the Earth: The former Kind ſhall be conſidered in this Chapter and the next, the latter in thoſe which follow. To begin with that of the Sun.

As a Spectator in the Sun would ſee the Earth deſcribing a Circle in the Heavens, which is that where the Plane of the Earth's Orbit (were we to ſuppoſe it extended far enough) would cut them; ſo a Spectator on the Earth, being carried round the Sun therewith, would imagine *that* to move through the ſame Circle, and in the ſame Direction, paſſing from Star to Star till it has appeared to move quite round the Heavens. This Circle is called the *Ecliptic*, and from what was obſerved above of the Zodiac, neceſſarily paſſes through twelve Signs thereof. From hence it is, that the Sun is ſaid to paſs through the Signs of the Zodiac once a Year.

To illuſtrate this, let S (Fig. 4) repreſent the Sun, ABC the Orbit of the Earth, and DEF the ſtarry Firmament. Then if we ſuppoſe the Spectator to be in the Sun, as the Earth moves from A to B, it will ſeem to move from D to E, deſcribing the Arch DE in the Heavens, and ſo on; but if the Spectator be upon the Earth, then whilſt it moves from A to B, the Sun will ſeem to deſcribe the Arch GH in the oppoſite Part of the Heavens: and whilſt the Earth is paſſing from B to C, the apparent Place of the Sun will paſs from H to I; and ſo on through the whole Circle. So that the Sun appears to deſcribe the ſame Circle among the fixed Stars in a Year, that the Earth would ſeem to deſcribe in the ſame Time to a Spectator in the Sun.

Having thus ſhewn, that the annual Motion of the Earth gives the Sun an apparent one in the *Ecliptic*, we ſhall now proceed to enquire how the Variety of Days and Nights, as to their Lengths and the different Seaſons of the Year, are to be accounted for. In order to which we will premiſe the three following Obſervations.

I. As the Earth turns round its *Axis*, there are two Points in its Surface which have no Motion from thence, *viz.* the two Extremities of its *Axis*. Theſe Points are called the *Poles* of the Earth; that which reſpects the

North, is called the *North Pole*, the other the *South Pole*. But every other Point of the Earth's Surface deſcribes a Circle, which will be greater or leſs, as the Point that deſcribes it is farther from or nearer to either Pole; and conſequently that Circle, whoſe deſcribing Point is equally removed from either, will be the largeſt of all. This Circle is called the *Equator*, by Mariners the *Line*; and its correſponding Circle in the Heavens, or that which the Plane of this Circle, were we to ſuppoſe it extended thither, would mark out, the *Equinoctial* Circle; and thoſe leſſer Circles deſcribed by other Points of the Earth's Surface are called *Parallels* of this.

II. The *Axis* of the Earth, as was obſerved above, making an Angle with the Plane of its Orbit of 23 Degrees and an half, does the ſame with that of the *Ecliptic*, thoſe two Planes being coincident one with another. Thus ſuppoſing the Line AB (Fig. 5.) to repreſent the Plane of the *Ecliptic* ſeen edgeways, and the Circle DFE the Earth; the *Axis* of it is not coincident with the Perpendicular DE, but ſtands inclined in the Situation FG, ſuppoſing the Arch DF to contain 23 Degrees and an half. And,

III. In whatever part of its Orbit the Earth is, its *Axis* has the ſame Inclination to the

Plane thereof, and is every where parallel to itſelf: that is, if a Line be conceived as drawn parallel to the *Axis* while the Earth is in any one Point of its Orbit, the *Axis* will in every other Situation of the Earth be parallel to the ſaid Line. Thus ſuppoſing the Line FG to repreſent the Situation of the *Axis* of the Earth when at DFG, and to be parallel to the Line HI; then when the Earth is at *dfg*, or any other Part of its Orbit, its *Axis* *f g* will ſtill be parallel to the ſame Line HI *(a)*.

Theſe things being obſerved, let us imagine the Plane of the *Ecliptic* to be repreſented by the Line AB (Fig. 6.) ſeen edgeways as before. Let the Point S repreſent the Place of the Sun, and PEpQ the Earth, whoſe *Axis* let be Pp, and its Poles P and p, the former the North, the latter the South; and let EQ be the Equator and TC, RN, *&c.* Parallels thereof. And let us in the firſt Place conſider the Earth in the Situation KL, or directly beyond the Sun with reſpect to the Eye. It is evident from the Figure, that in this Situation of the Earth, the Sun's Place S is in the Plane of the Equator EQ produced; and conſequently, while the Earth

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(a) N. B. The Axis of the Earth deviates a little from its Paralleliſm every Revolution of the Earth about the Sun, but in ſo ſmall a Degree that the Conſideration of it is better omitted in this Place; it ſhall be taken Notice of in the next Chapter.

Earth revolves upon its *Axis* Pp, the Sun will appear to deſcribe that Circle, and will enlighten one Side of the Earth from Pole to Pole. Whence half of the Equator, and half of each of its Parallels RN, TC, *&c.* will be enlightened thereby, while the other half being on the oppoſite Side the Earth, will be in the Dark. And conſequently every Place on the Surface of the Earth, ſince it muſt deſcribe ſome one of thoſe Parallels as it is carried round by the diurnal Motion, will be as long in the Light as in the Dark; that is, the Days and Nights will then be of equal Length all over the Earth *(b)*.

But becauſe the *Axis* of the Earth is every where parallel to itſelf, as was juſt now obſerved; when the Earth is carried by its annual Motion into the Situation MO (which we will ſuppoſe to be a quarter of a Circle diſtant from its former Situation, the Poles of the Earth being ſtill turned the ſame Way as before) it is manifeſt that the Sun's place S is not now in the Plane of the Equator EQ produced, (for that would paſs below the Sun towards L) but directly over C, a Point in the Parallel TC; and therefore as the Earth revolves about its *Axis*, all places of the Earth that

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(b) When the Sun appears to enter the firſt Degree of the Sign *Aries*, the Earth is in that Part of its Orbit where theſe Phænomena happen.

that lie in this Parallel will come directly under the Sun once in 24 Hours, as thoſe that lay under the Equator did in the former Situation of the Earth: and ſince the Rays of the Sun always enlighten one half of the Globe of the Earth at a Time, they will reach beyond the North Pole as far as F; and on the other Side no farther than I. From whence it follows, that the Tract of the Earth which lies within the Circle FG enjoys continual Day-light, while the Earth is in this Part of its Orbit; and on the contrary, that it is continual Night to that Part of the World that lies within the oppoſite Circle HI: that is, the Sun will not appear to ſet in the former, nor riſe in the latter. And farther, greater Portions of thoſe Parallels which lie between the Equator and the Circle FG will be in the illuminated Hemiſphere than in the dark one, as appears by Inſpection of the Figure, in which the latter Hemiſphere is diſtinguiſhed from the other by being ſhaded; and on the contrary, greater Portions of ſuch as lie between the Equator and the Circle HI will be in the Dark, than in the Light. And hence it is that the Earth being in this Part of the Orbit, the Days are longer than the Nights on the Northern Side the Equator; whereas on the Southern Side, the Nights are longer than the Days; and the Difference between the Lengths of the Days and Nights is ſo much the greater, as the Place is more remote from the Equator on either ſide, becauſe the Difference between the Portions of the Parallels which are in the illuminated Hemiſphere, and thoſe which are in the contrary one, are greater the farther they are from the Equator: But to ſuch as live under the Equator, Day and Night are in this Situation of the Earth alſo equal to one another, one half of the Equator EQ being here alſo in the illuminated Hemiſphere (c).

Let us now conceive the Earth to have moved through another quarter Part of its Orbit, and to be in the Situation KL again, but with this Difference, that it is now on the contrary ſide the Sun to what we ſuppoſed it at firſt. In this Situation alſo it is evident, that the Plane of the Equator EQ being produced, will paſs through the Sun, and that the illuminated Hemiſphere will paſs from Pole to Pole as before; for we are ſtill to conceive the *Axis* of the Earth parallel to its firſt Situation; ſo that the Sun will ſeem to deſcribe the ſame Circle in the Heavens, and the Days and Nights will be equal every where, as when the Earth was in the oppoſite Point of its Orbit (*d*).

(c) In this Situation of the Earth the Sun appears to be in the firſt Degree of *Cancer*, and to the Inhabitants of the Northern Hemiſphere of the Earth introduces the Summer; to thoſe of the other, the Winter Quarter.

But when the Earth has paſſed through another quarter of its Orbit (which we will ſuppoſe it to have done, when it is in the Situation X Y) its *Axis* Pp ſtill remaining parallel to its former Direction, the Place of the Sun will now be vertical to R, a Point under the Parallel RN; ſo that to all the Inhabitants that live under this Parallel, the Sun will be vertical once in 24 Hours, as it was to thoſe who live under the Parallel TC, when the Earth was at MO; and that Portion of the Earth's Surface which is included within the Circle FG, will be wholly in the Dark, and that within the oppoſite one HI, will be wholly in the Light; and leſſer Portions of the Northern Parallels will be in the Light than in the Dark, and greater Portions of the Southern. So that the Inhabitants of the Northern Hemiſphere will then have their Days at the ſhorteſt, and thoſe of the Southern at the longeſt. Thoſe who live within the Circle FG will have no Day, and thoſe within HI no Night. Tho' to thoſe who live under the Equator, the Days and Nights are in this, as in all other Situations of the Earth, of an equal Length, the Equator

being

(*d*) In this Situation of the Earth, the Sun enters the firſt Degree of *Libra*, and makes *Autumn* to us, and *Spring* to thoſe on the other ſide the Equator.

being every where cut into two equal Parts by the Boundary of Light and Darkneſs *(e)*. From

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(e) When the Earth is in that Part of its Orbit where theſe Phænomena happen, the Sun is then entering the firſt Degree of *Capricorn*, and brings on Winter to us, and Summer to the Inhabitants of the other Hemiſphere.

There is another Way in which theſe *Phænomena* may be illuſtrated; and that is as follows. Let S (Fig 7.) repreſent the Sun, and the Circle ABDD the Earth's annual Courſe round it; EFGH the Earth, P the North Pole, RT the Arctic Circle, IKLE the Tropic of *Cancer*, FNH the Northern half of the Equator; the ſhaded Part the obſcure Hemiſphere, the other the illuminated one: And let X be the Place of the Sign *Aries* in the Heavens, Y that of *Cancer*, M that of *Libra*, and Q that of *Capricorn*.

When the Earth is in that Part of its Orbit that reſpects M or *Libra*, the Sun appears in the oppoſite one, *viz.* at X, or in *Aries*; and the Circle GPE, which ſeparates the light Hemiſphere from the dark one, paſſes through the Poles of the Earth, and therefore cuts each Parallel of the Equator into two equal Parts, and thereby renders the Days and Nights every where equal.

But while the Earth in its annual Courſe moves from M towards Q, which we will ſuppoſe to be a quarter of a Circle diſtant from it, its *Axis* being directed as before, the Pole P will gradually advance into the illuminated Hemiſphere, and that Portion of the Parallels that are therein will continually encreaſe until the Earth is arrived there: in which Poſition the Boundary of Light and Darkneſs paſſes through FRH, the Pole P, and all the Space within the Arctic Circle falling wholly within the illuminated Hemiſphere; and thoſe Portions of the ſeveral Parallels that are in the Light, will be longer than thoſe which are in the Dark, all which is evident from the Inſpection of the Figure. So that in the Northern Hemiſphere of the Earth, the Days will be at longeſt, and the Nights at the ſhorteſt.

Again, as the Earth is moving from Q towards X, the Pole P begins to incline towards the Line that diſtinguiſhes Light from Darkneſs, for the like Reaſon that before it receded from it; and therefore thoſe Portions of the Parallels that are in

From this apparent Shifting of the Sun from one Side to the other, thoſe Parallels of the Equator, mentioned in the foregoing Explication, have been diſtinguiſhed from the reſt by particular Names, as the two *Tropics*, and the two *Polar* Circles. The *Tropics* are TC and RN; the former is called the *Tropic of Cancer*; becauſe when the Sun ſeems to deſcribe that Circle, it then appears to be in the firſt Degree of that Part of the Ecliptic which is diſtinguiſhed by the Sign *Cancer*: the latter is called the *Tropic of Capricorn*; becauſe at that time of the Year when it appears to deſcribe that Circle, it is entering the firſt Degree of the Sign *Capricorn* The two *Polar* Circles are FG and HI; the former the *Arctic*, the latter the *Antarctic*. Theſe are juſt as far diſtant from their reſpective Poles, as the Tropics are from the Equator; *viz.* 23 Degrees and an half; this being a neceſſary Conſequence of the Sun's illuminating but one half of the Globe of the Earth at a Time.

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in the Light gradually leſſen till the Earth arrives there; at which Time that Line will paſs through it again, and ſo make the Days and Nights every where equal.

But afterwards the Pole will begin to fall into the obſcure Hemiſphere, and ſo recede gradually from the Light, until the Earth comes to Y; in which Situation not only the Pole, but the whole Arctic Circle will be involved in Darkneſs; and thoſe Portions of the ſeveral Parallels that are in the Light, will be ſhorter than thoſe which are in the Dark; ſo that the Days will be at the ſhorteſt, and the Nights at the longeſt.

The Days and Nights being of an equal Length, when the Sun appears to be in thoſe Points where the Equator cuts the *Ecliptic*, (*viz.* the firſt Degree of *Aries* and *Libra*) they are from thence called the *Equinoctial Points*, and the Times of the Year anſwering thereto the *Equinoxes*; one the *Vernal*, the other the *Autumnal*; and of the *Equinoctial Points*, that where the *Ecliptic* croſſes the Equator in its Aſcent towards the North Pole, is called the *vernal Equinoctial Point*; that where it croſſes it in its Deſcent towards the South Pole, the *Autumnal*; and a Circle imagined to paſs through them and the Poles of the Earth, is called the *Equinoctial Colure*.

And becauſe when the Sun ſeems to deſcribe either of the *Tropic* Circles (that is, when it enters the firſt Degree of *Cancer* or *Capricorn)* it being about to revert to the Equator, does as it were ſeem to ſtand ſtill, neither receding from, nor approaching towards it, theſe Points are called the *Solſtitial Points*; and the Times of the Year anſwering thereto are termed the two *Solſtices*; the one, the *Summer*, the other the *Winter*. And a Circle conceived to paſs through the Poles of the Earth and theſe Points, is called the *Solſtitial Colure*. And laſtly, theſe four Points of the Heavens being thoſe in which the Sun appears to be when the four Seaſons of the Year (*viz.* the *Spring*, *Summer*, *Autumn*, and *Winter*)

ter) begin, are diſtinguiſhed by the Names of *Cardinal Points.*

If a right Line perpendicular to the Plane of the *Ecliptic* be ſuppoſed to paſs through the Center of the Sun, and to be extended both ways to the Heavens, this Line is called the *Axis of the Ecliptic*; and the two Extremities thereof, the *Poles of the Ecliptic.* But this Line is oft conceived to paſs through the Center of the Earth inſtead of the Sun; for they are ſo near to each other with reſpect to the Diſtance of the Heavens, that the Difference is not material.

The Sun being not placed in the Center of thoſe *Ellipſes* which the Planets deſcribe in moving round it, but in one of their *Foci*, as was obſerved above, it follows, that they muſt be farther from the Sun in one Part of their Orbits than in another. And not only ſo, but they muſt alſo move faſter in one Part of their Orbits than in another; becauſe, as has been already demonſtrated in Part I. Chap. VIII. page 46. whatever Orbit a Body deſcribes, if it be retained therein by any centripetal Force whatever, it will deſcribe equal *Areas* in equal Times; that is, it will move faſter in Proportion as it approaches nearer to the Body it revolves about, and ſlower the farther it is from it. And accordingly it is obſerved by Aſtronomers, that the nearer a

Planet comes to the Sun in its Orbit, the ſwifter it moves.

The Conſequence of this is, that the Sun does not appear to move uniformly along the *Ecliptic*, but in ſome Days to deſcribe a greater Portion of it than in others. In Winter it moves the faſteſt, in Summer the ſloweſt; inſomuch that it takes up eight Days more in paſſing from the *Vernal* to the *Autumnal Equinox*, than in its paſſage from the *Autumnal* to the *Vernal*; altho' in either of theſe Intervals of Time it moves through juſt half of the *Ecliptic (f)*.

When an *Ellipſe* is conſidered as the Orbit of a Planet, the longer *Axis* thereof is called the *Linea Apſidum*; and if the *Focus* F be the Place of the Sun *(g)*, the Point A is called the *Summa Apſis (h)*, or the *Aphelion*; and the oppoſite

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(f) Hence it is, that in one Part of the Year the Sun is farther towards the Eaſt, and in another farther towards the Weſt, than it would ſeem to be, was its apparent Motion in the *Ecliptic* uniform; ſo that it paſſes the Meridian ſometimes ſooner, ſometimes later, than the Time at which a Clock that moves uniformly would indicate the Hour of *Twelve*. This is one Reaſon that Sun-dials are obſerved to go ſometimes ſlower, and ſometimes faſter. When the Sun appears too far toward the Eaſt, they are too ſlow; when it appears too much to the Weſtward, they are too quick. But ſee this farther explained, Chap. XIV.

(g) See Fig. 2.

(h) This is the general Name to diſtinguiſh that Point of an Orbit, in which a revolving Planet is at the greateſt Diſtance from the Body it revolves about. With reſpect to a Body revolving about the Sun, it is called the *Aphelion*; but with reſpect to the Moon conſidered as revolving about the Earth, it is

oppoſite one B, the *Ima Apſis*, or the *Perihelion*. The Diſtance of a Planet from its *Aphelion* is called its *Anomaly*; and this is of two Kinds, *mean* and *real*; the latter is its real Diſtance from its *Aphelion*; the former is that Diſtance at which it would be from it, ſuppoſing its Motion in its Orbit to be uniform; and the Difference between theſe two, Aſtronomers call the *Equation*, or *Proſthaphæreſis* of the Planet.

The *Ellipſes* the Planets deſcribe do themſelves turn round the Sun, though with a very ſlow Pace, their *Aphelia* reſpecting ſometimes one Part of the Heavens, and ſometimes another. In this Age the *Aphelion* of the Earth's Orbit points towards the former Part of the Sign *Capricorn*. This is the reaſon that the Sun is farther from us in Summer than in Winter; for the Sun appearing in *Cancer* in the former Seaſon, the Earth is then in *Capricorn*, and therefore in its *Aphelion*, that is, its greateſt Diſtance from the Sun.

The Sun being farther removed from us in Summer than in Winter, it may be aſked how it comes to paſs, that ſince the Sun is the Fountain of Heat, as well as Light, our Winters are ſo much colder than our Summers? In Anſwer to this it is to be conſidered, that the Rays

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is called the *Apogeon*. And the *Ima Apſis* is the neareſt Point, which with reſpect to a Body moving about the Sun is called the *Perihelion*, but with reſpect to the Moon the *Perigeon*.

Rays of the Sun fall with much leſs Obliquity upon the Surface of the Earth on our ſide the Equator in the Summer, than in the Winter; and therefore they not only act more forcibly upon it, but a greater Quantity of them fall upon a given Place. Again; in the Winter the Sun being much lower in the Heavens when at its Meridian Height, than in Summer, its Rays paſs through a longer Portion of the Earth's Atmoſphere, by which great Part are intercepted, and ſome by various Refractions and Reflections turned another Way *(i)*. And laſtly, in Summer the Sun continues with us ſixteen Hours, and is abſent but eight; whereas in Winter it is with us but eight Hours, and is abſent ſixteen. All which things conſpire to make a conſiderable Alteration with reſpect to Heat and Cold.

If this be ſo, why is not the Weather hotter when the Sun is in the Tropic of *Cancer*, its Rays then falling with the leaſt Obliquity, and paſſing the ſhorteſt Way through the Atmoſphere, and the Days being then at the longeſt, than it is about a Month afterwards, when the Sun is in the beginning of the next Sign? In anſwer to this it is to be remembred, that Bodies are not always the hotteſt at that very Inſtant the greateſt Degree of Heat is applied to them: they require Time

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(i) See the State of the Atmoſphere, Part II. Chap. III.

H
A
D
A

to heat as well as to cool; it is the Length of Time therefore that the Heat is applied to them, as well as the Degree of it, that determines the Quantity of Heat communicated to them.

For the like Reaſon we commonly find it warmer about one or two of the Clock in the Afternoon, than at twelve, when the Sun is in its Meridian Altitude, and its Rays fall thickeſt and moſt forcibly upon the Earth.

The Sun appearing by means of the annual Motion of the Earth (as was illuſtrated in the beginning of this Chapter) to move from Weſt to Eaſt in the Heavens, it happens, that if any Star riſes or ſets along with the Sun at any time, that Star will after a few Days riſe or ſet before it; becauſe the Sun's apparent Place in the Heavens will be removed to the Eaſtward of the Star. So that thoſe Stars which at one time of the Year ſet with the Sun, and therefore never appear at all, ſhall at another time of the Year riſe when the Sun ſets, and ſhine all Night. And as any one Star ſhifts its Place with reſpect to the Sun, and in Conſequence of that with reſpect to the Hour of the Night, ſo all the reſt do; and from hence it is, that all thoſe Stars which at one time of the Year appear on any one ſide of the Pole-Star in the Evening, ſhall half a Year after, at the ſame Hour, appear on the contrary Side thereof.

CHAP. V.

Of the Earth's annual Parallax, *the* Nutation *of the* Poles, *and the* Preceſſion *of the* Equinoctial Points.

SInce the *Axis* of the Earth is always parallel to itſelf, it is in different Parts of the Year directed towards different Parts of the Heavens; and therefore to a Spectator on the Surface of the Earth, who cannot be ſenſible of his Change of Place, the apparent Places of the fixed Stars are different, when viewed from oppoſite Points of the Earth's Orbit. Thus let S (Fig. 8.) repreſent the Place of the Sun, AB the Orbit of the Earth, and when the Earth is at A, let its *Axis* be directed towards the Point C in the Heavens; then when the Earth is arrived at B, the oppoſite Point of its Orbit, its *Axis* being parallel to its former Poſition, will be directed to a different Point in the Heavens, *viz.* to D, as far diſtant from the former as the Point A is from B, that is, a Space equal to the Diameter of the Earth's Orbit: And therefore to a Spectator on the Surface of the Earth, the Places of the fixed Stars appear removed as far as from C to D, *viz.* by the Quantity of the Angle, CBD; or, becauſe the Lines AC and BD are parallel, by the Angle BCA, which is that under which the Orbit of the

Earth would appear from a fixed Star at C. This Angle is by Aſtronomers called the *Parallax of the Orbis magnus*, or the *Earth's annual Parallax*.

If this Angle could be accurately taken, the Diſtance of a fixed Star at C, with reſpect to that of the Sun at S, might be found by the Rules of *Trigonometry (a)*. But by reaſon of the great Diſtance of the fixed Stars, that Angle is ſo exceeding ſmall that it cannot be taken with any tolerable Accuracy, even with the beſt Inſtruments. It has appeared to ſome to contain about 47 Seconds of a Minute, according to which the Diſtance of the Stars muſt be five hundred thouſand Millions of Miles from us; but to others, particularly to *Flamſtead*, who has taken great Pains to obſerve it accurately, it has appeared to contain but 42 Seconds, according to which the Diſtance of the Stars muſt be much greater.

Altho' we hitherto have looked upon the *Axis* of the Earth as being always parallel to itſelf, yet it is not ſo, but is ſubject to a twofold Motion; to the one is owing the Nu-

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(a) For the Angle DBC being taken by Obſervation, its Equal ACB is had; and the Angle BAC, or SAC, ſubtends an Arch of the Heavens that meaſures the Diſtance between the Place of the Sun and the ſaid Point, and may therefore eaſily be obtained: and conſequently, by the Rules of *Trigonometry*, the Proportion which the Line AC or the Diſtance of the Star from the Earth, bears to the Line AB, which is double the Sun's Diſtance, may be found.

tation of the Poles, to the other the Preceſſion of the Equinoctial Points, which are now to be explained. The firſt is a Deviation of the Earth's *Axis* from its Paralleliſm with itſelf twice in the Year, and a Returning to it as oft. For inſtance, the Inclination of its *Axis* to the Plane of the *Ecliptic* increaſes while the Earth is moving from the Solſtitial to the Equinoctial, and diminiſhes as much in its Paſſage from the Equinoctial to the Solſtitial Points. So that the Inclination of the *Axis* of the Earth to the Ecliptic is, notwithſtanding this, at oppoſite Points of its Orbit always the ſame. This Nutation therefore does not in the leaſt interfere with what was ſaid with regard to the Earth's annual Parallax, in which though we ſuppoſed that *Axis* always parallel to itſelf, yet we conſider'd it only in oppoſite Points of its Orbit. This is called the Nutation of the Poles *(b)*. The other indeed claſhes a little with our former Suppoſition; but taking a ſingle Revolution of the Earth by itſelf, as we have hitherto done, it is ſo ſmall as to be quite inſenſible, tho' not ſo when we take a many Revolutions together. Accordingly, by comparing ſeveral Years together, Aſtronomers have obſerved that the *Axis* of the Earth deviates

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(b) As the Inclination of the *Axis* of the Earth is thus twice augmented and twice diminiſhed, while the Earth revolves about the Sun, in like manner it is twice augmented, and twice diminiſhed in each Revolution of the Moon about that.

viates a little from the Direction it formerly had; notwithſtanding which Deviation, its Inclination to the *Ecliptic* remains the ſame. To illuſtrate the manner in which this Alteration in its Direction is made, let S (Fig. 9.) repreſent the Sun, the Circle XEY the Earth, AB the Plane of the *Ecliptic*, EL the *Axis* of it, which we will now ſuppoſe to paſs thro' the Center of the Earth, and let PQ denote the *Axis* of the Earth. On the Surface of the Earth, and round the Points E and L, let us conceive the two ſmall Circles PTX and QVY to be deſcribed. Then if at any Time the Poſition of the Earth's Axis is in the Line PQ, as we have ſuppoſed, after ſeveral Revolutions of the Earth about the Sun it ſhall be found in the Situation CD; and after ſeveral more Revolutions it ſhall be removed into the Situation TV, *&c.* the one Pole of the Earth deſcribing the Circle PCTX, and the other QDVY: ſo that notwithſtanding this Change in the Direction of the *Axis* of the Earth, its Inclination to the Plane of the *Ecliptic* remains the ſame, the Poles of the Earth being every where at the ſame Diſtance from E and L, the Poles of the *Ecliptic*. But this Motion of the Poles is ſo very ſlow, that they do not perform their Revolution in leſs than 25,920 Years, which is at the Rate of one Degree in 72 Years. It is performed in a Direction contrary to that of the Order of the Signs, or from Eaſt to Weſt. Which is termed by Aſtronomers, a Moving in *Antecedentia*; whereas when a Body moves according to the Order of the Signs, its Motion is ſaid to be in *Conſequentia*.

From hence it follows, that the Solſtitial Colure, which paſſes both through the Poles of the *Ecliptic* and of the Earth, muſt neceſſarily turn round the *Axis* of the *Ecliptic* in the ſame Direction, or contrary to the Order of the Signs; and with that the Equinoctial Colure alſo, becauſe theſe croſs one another at right Angles in the Poles of the Earth. Whatever Star therefore the Equinoctial Points are directed towards at any particular Time, after 72 Years they will not be directed to the ſame, but to ſome other Star or Point of the Heavens, ſituated one Degree towards the Weſt. This Motion of the Equinoctial Points is called their *Preceſſion*. And from hence the Stars ſeem to move towards the Eaſt, and thereby to have their Longitude (which is always reckon'd upon the *Ecliptic* from the vernal Equinoctial Point) increaſed. The Conſtellations therefore ſeem all to have deſerted the Places allotted to them by the firſt Aſtronomers. For Inſtance, the beginning of the Sign *Aries*, which was near the vernal Equinoctial Point, and gave Name to that Point of the *Ecliptic*, is now advanced about an whole Degree forwards: So that *Aries* is now where

Taurus uſed to be ; *Taurus* where *Gemini*, &c. But to avoid Confuſion, Aſtronomers have thought fit to let the ſeveral Portions of the *Ecliptic*, where theſe Conſtellations were at firſt obſerved to be, retain their old Names *(c)*; ſo that the vernal Equinoctial Point is ſtill reckoned the firſt *Degree of Aries (d)*.

(c) Thoſe Portions of the Ecliptic, where the Conſtellations were at firſt, are called *Anaſtra*, thoſe where they now are, are diſtinguiſhed by the Titles of *Stellata*.

(d) The ancient Aſtronomers obſerving this Motion of the fixed Stars, and not knowing how to account for it, concluded that they really had ſuch a Motion, and called the Time of an entire Revolution of them, *Annus Magnus*, or the great Year. And imagined that when that great Year was ended, and the Stars reinſtated in their proper Places, all Things would begin to be done over again in the World, not only in the ſame Order and Manner, but by the ſame Perſons they were done before.

The Cauſes of this Preceſſion, as alſo of the Nutations above mentioned, will be accounted for when we treat of the Cauſes of the Motions of the heavenly Bodies.

CHAP. VI.

Of the Phænomena *which ariſe from the* Motion *of the* Earth *and of the Inferior Planets,* Mercury *and* Venus *conjointly.*

SInce the Orbits of *Mercury* and *Venus* are included within that of the Earth, it is evident neither of them can come to an Oppoſition to the Sun; that is, appear in the oppoſite Part of the Heavens with reſpect to it, but muſt always accompany it in its apparent Motion through the Ecliptic, juſt as the Satellites of the primary Planets do in reality accompany them. But whereas they perform their reſpective Revolutions about the Sun in Periods different from that of the Earth, it is obvious they muſt be ſometimes on one ſide of the Sun, and ſometimes on the other with reſpect to us; ſometimes be between the Sun and us, and ſometimes directly beyond it. When they are between the Sun and us, they are ſaid to be in their *inferior Conjunction*; when beyond it, in their *ſuperior Conjunction*: and when they appear to be removed the fartheſt from the Sun, on either Side, they are then ſaid to be in their greateſt *Elongation*.

To illuſtrate this, let S (Fig. 10.) repreſent the Sun, ABCD the Orbit of one of the inferior Planets, ſuppoſe *Venus*; E the Place of the Earth in its Orbit; FG and HI a Portion of the Heavens. Then will A be the Place of its *inferior*, C that of its *ſuperior Conjunction*; in both which Caſes it will appear in the Heavens to be at K, which is the apparent Place of the Sun. When it is at B or D, where a Line joining its Center and that of the Earth touches its Orbit, it then appears at I or H, and is ſaid to be in its greateſt Elongation, which is about 48 Degrees (*a*). But this is liable to ſome Alteration; becauſe as its Orbit is an *Ellipſe*, it is ſometimes nearer the Sun, and ſometimes farther from it, in its greateſt Elongation.

From hence we ſee the Reaſon why the Planet *Venus* is ſometimes ſaid to be our Morning Star, and ſometimes our Evening Star: For after it has paſſed its *inferior Conjunction* with the Sun, and is aſcending towards its *ſuperior* one, it is all that Time on the *Weſtern* ſide of the Sun, and therefore neceſſarily riſes

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(*a*) Hence the Diſtance of *Venus* from the Sun, with reſpect to that of the Earth from the ſame, is found. For if in the Triangle EBS, we take the Angle at E, when that at B is a right one, the Proportion between the oppoſite Sides is had from the known Rules of *Trigonometry*. The Reaſon why the Angle at B is not always a right one, is, becauſe the Orbit of the Planet is not a Circle, but an *Ellipſe*.

every Morning before it, and is then our *Morning Star.* On the other hand, while it deſcends from its *ſuperior* to its *inferior Conjunction*, it is to the *Eaſtward* of the Sun, and therefore riſes and ſets after it; and appears to us only in the Evening after the Sun is down.

Farther; altho' the Motion of the inferior Planets in their Orbits is direct, or according to the Order of the Signs, yet they frequently appear to be *retrograde*, or to move the contrary Way; and ſometimes to ſtand ſtill. When they are near their *inferior Conjunction* with the Sun, they ſeem to move one Way, and when near their *ſuperior Conjunction*, they appear to go another. And at that Time when their direct or progreſſive Motion is changing into a regreſſive one, or the contrary, they ſeem to ſtand ſtill. All which may be illuſtrated in the following Manner.

Let ABCD (Fig. 10.) repreſent the Orbit of the Planet *Venus*, S the Place of the Sun, E the Earth, FG a Portion of its Orbit, and HI the Sphere of the fixed Stars, as before; and becauſe *Venus* moves faſter than the Earth, let us ſuppoſe the latter to ſtand ſtill, and the former to move with the Differences of their Velocities: Then while *Venus* deſcribes that Portion of its Orbit that is repreſented by DAB, it will ſeem from the Earth to move from H to I in the Heavens, which is contrary to its real Motion, and to the Order of the Signs;

and in moving over the remaining Part of its Orbit, it will ſeem to paſs from I to H. And was the Earth really to ſtand ſtill, as we have ſuppoſed, it would be ſtationary, or ſeem to have no Motion either Way, at the Points B and D.

But as the Earth does not ſtand ſtill, but continues to move on in its Orbit, *Venus* will not ſeem ſtationary exactly at the Points B and D; but during its Motion through a certain Part of its Orbit, as LM, where ſuppoſing the Lines EL and OM parallel to each other, their Orbits are ſo inclined to each other, that the Space LM ſhall be to the Space EO as the Velocity with which *Venus* moves is to that with which the Earth moves; for it is evident, that in that caſe the Earth will paſs through EO while *Venus* paſſes through LM, and therefore it will appear all the Time in the ſame Part of the Heavens, becauſe the Diſtance between thoſe parallel Lines is nothing, when compared to that of the Heavens.

There is alſo another Particular to be taken Notice of with regard to the apparent Motion of the Inferior Planets, *viz.* that they do not ſeem to deſcribe the *Ecliptic* in the Heavens as the Sun does, but are obſerved to be ſometimes above and ſometimes below it. The Reaſon of this is, that their Orbits are inclined to the Plane of the Earth's, having one Half above it and the other below it, on which account they interſect the Plane of the *Ecliptic* in a Line that paſſes through the Center of the Sun (the Center of the Sun, as was obſerved above, being in the Planes of each Orbit.) This Line is called the *Line of the Nodes*, and the two Points of the *Ecliptic*, through which it paſſes when produced to the Heavens, are called the *Nodes*. Theſe Planets therefore never appear in the *Ecliptic* except when they are in their *Nodes*, and in all other Parts of their Orbits ſeem to be more or leſs diſtant from it, according as they are ſituated with reſpect to them and the Earth. But if we ſuppoſe them to be ſeen from the Sun, their greateſt Diſtance from the *Ecliptic* will be when they are fartheſt removed from their *Nodes*.

Their Diſtances from the *Ecliptic*, as it would appear if ſeen from the Center of the Earth, is called their *Geocentric Latitude*; as it would appear from the Center of the Sun, is called their *Heliocentric Latitude*.

There is a remarkable *Phænomenon* relating to the inferior Planets, which was not known to Aſtrónomers, till diſcovered by the Teleſcope; and that is, that in their different Situations with reſpect to the Earth, they aſſume different *Phaſes*, or Appearances, like thoſe of the Moon. Which may be explained in the following Manner.

Let A, B, C, D, (Fig. 11.) repreſent the Situation of one of the inferior Planets (*Venus* ſuppoſe) in ſeveral Parts of its Orbit, and let S be the Place of the Sun, and I that of the Earth in its Orbit KL : and if from the Center of the Sun to that of *Venus* in its ſeveral Stations we draw the Lines SA, SB, *&c.* reſpectively; and at right Angles to theſe the Lines MN, MN, *&c.* theſe laſt Lines will repreſent Circles upon the Surface of the Planet, ſeparating the enlightened from the obſcure *Hemiſphere.* And if from the Earth to the Planet we conceive the Lines IA, IB, *&c.* to be drawn, and at right Angles to theſe the Lines OP, OP, *&c.* reſpectively, theſe latter will repreſent Circles ſeparating that Part of the Planet which is turned towards us from that which is not. Theſe we may call *Circles of Viſion.* Now it is evident, that when *Venus* is in its inferior Conjunction at A, the Circles MN and OP coincide, and no Part of the Planet beſides its unilluminated Hemiſphere is turned towards us; in which Caſe it is inviſible, unleſs it happens at that Time to be in or near the Plane of the *Ecliptic*, which if it be, it appears as a Spot upon the Surface of the Sun. When it has advanced in its Orbit as far as B, there is a ſmall Part of the illuminated Hemiſphere included between the Lines OB and BM, on that ſide the Circle of Viſion which is next the Earth; in which caſe, ſince by reaſon of its great Diſtance from us we cannot diſtinguiſh the *Convexity* of its Surface, it appears as a flat round Surface, a ſmall Portion of which is illuminated, the reſt being dark; and ſo appears *horned*, like the new Moon *(b)*. When it has arrived at C, then is half its illuminated Hemiſphere turned towards us, and it appears as a round Surface, half of which is enlightened, and half not. When it has got to D, then is the greateſt Part of its illuminated Hemiſphere turned towards us, and it appears as a round plain Surface almoſt wholly enlighten'd, or what the Aſtronomers call *gibbous*. When it is in its ſuperior Conjunction

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(b) The Reaſon why *Venus* or any other Planet (the Moon ſuppoſe) appears *horned*, when a ſmall Portion only of her illuminated Hemiſphere is turned towards the Eye of the Spectator, will perhaps in ſome Meaſure appear from the Conſideration of the 12th Figure. In which let ABCD repreſent the Planet *Venus*, or any other Body of a globular Form; which that it might the better do, I have drawn ſome of thoſe Circles which are uſually found on Globes deſigned to repreſent the Earth. Now if half this Globe be illuminated, and half be in the dark, and if only ſo much of the illuminated Part as is included within the Space ABCE be turned towards the Eye, the remaining Part AECD being obſcure, it is plain, that the illuminated Part ABCE, when the whole Surface ABCD, by reaſon of the great Diſtance of the Globe, appears *flat*, muſt ſeem to be *horned*. And again, if the Space ABCE was a Portion of the dark Hemiſphere, and AECD of the illuminated, 'tis certain that when both are conſidered as making up one flat Surface, the illuminated Part muſt appear convex, or *gibbous*, on the Side AEC, which is contiguous to the unilluminated Hemiſphere ABCE.

junction at E, the Circles MN and OP coincide again, and the whole enlightened Hemiſphere is turned towards the Earth; in which Situation the whole illuminated Face of the Planet is turned towards the Earth, and, if not intercepted by the Body of the Sun, appears like the Moon at the *Full.* After which it puts on the ſame Appearances again, but in an inverted Order, till it arrives at A, where it is *inviſible*, as before *(c)*.

What has been here illuſtrated with regard to *Venus*, is to be underſtood in Reference to the like *Phænomena* of *Mercury*; only with this Difference, that the Orbit of *Mercury* being leſs than that of *Venus*, its greateſt *Elongation* is not ſo great as that of *Venus*, not exceeding about 33 Degrees. Upon which Account this Planet is rarely to be ſeen with the naked Eye. And as *Mercury* revolves round the Sun in leſs Time than *Venus* does, its *direct Motions*, *Stations* and *Retrogradations*, as alſo its Conjunctions with the Sun, ſucceed one another quicker than thoſe of that other Planet.

(c) Before the Invention of the Teleſcope, when *Copernicus* revived the old *Pythagorean Hypotheſis*, it was objected to him, that, according to that Scheme, *Venus* ought to aſſume the like Phaſes with the Moon. To which he replied, that poſſibly ſome Means would be found out, whereby ſucceeding Aſtronomers would diſcover that it did ſo. The firſt that obſerved it was *Gallilæo*, who thereby greatly confirmed the Truth of the *Copernican* Syſtem.

CHAP. VII.

Of the Phænomena *which are owing to the* Motion *of the* Earth, *and that of the ſuperior Planets* Mars, Jupiter, *and* Saturn *conjointly*

THE ſuperior Planets do not always ſeem to accompany the Sun, as we have ſhewn in the foregoing Chapter that the inferior ones do; but are ſometimes in Conjunction with it, ſometimes in Oppoſition: and are liable, as was obſerved before of the Planet *Mars*, to be ſeen at any Diſtance from it.

To illuſtrate this, let S (Fig. 13.) repreſent the Place of the Sun, ACE the Orbit of the Earth, NRM that of one of the ſuperior Planets (ſuppoſe *Mars*) and let OP be a Portion of the *Zodiac*. Now ſince theſe Planets perform their Revolutions about the Sun in different Times from that wherein the Earth goes round its Orbit, it is poſſible, that when the Earth is at E, *Mars* may be at M, or it may be at N; in the former Situation, it is in Conjunction with the Sun, in the latter it is in Oppoſition: or it may be at R, or at T, or in

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in any Situation with reſpect to the Sun and the Earth whatever.

The ſuperior Planets have this in common with the inferior ones, *viz.* that according as the Earth is ſituated with reſpect to them, they ſometimes ſeem to move forwards according to the Order of the Signs, ſometimes contrary to that Order; and between the Times of their being thus *direct* and *retrograde*, they ſeem to ſtand ſtill, as ſhall be made evident by the following Illuſtration.

Things remaining as before in the thirteenth Figure, ſince the Motion of the Earth is quicker than that of *Mars*, let it be ſuppoſed that *Mars* ſtands ſtill at N, and that the Earth is moving with the Difference of their Motions from B to C: It is obvious, that in this Caſe *Mars* will ſeem to paſs from O to P, which is contrary to the Order of the Signs; but while the Earth is paſſing over the other Part of its Orbit, *viz.* CAB, the Planet *Mars* would ſeem to move back again from P to O, even though it had no Motion of its own. Since then it is moving that Way all the Time, it neceſſarily ſeems to move according to the Order of the Signs; tho' with this Difference, that it ſeems to move faſter than it would do were it not for the Motion of the Earth in the mean time. Analogous to what was obſerved of the inferior Planets (ſince *Mars* does not ſtand ſtill, as we have ſuppoſed) it will not appear to be *Stationary* when the Earth is at B and C, but during the Motion of the Earth through a certain Part of its Orbit, as LH, which, ſuppoſing the Lines LN and HI parallel, is to the Portion of *Mars*'s Orbit NI (through which that Planet is moving at the ſame Time) as the Celerity with which the Earth moves is to that of *Mars*.

What was ſaid in reſpect of the inferior Planets, that they do not always appear in the *Ecliptic*, holds alſo with regard to the ſuperior ones. For their Orbits are alſo inclined to that of the Earth, one Side of them being above, and the other below it; ſo that thoſe have alſo their *Heliocentric* and *Geocentric* Latitudes as well as the other, and are never to be ſeen in the *Ecliptic*, but when they are in their *Nodes*.

What has been illuſtrated with regard to *Mars*, is applicable to the other two ſuperior Planets, only with this Difference; that the Motion of *Saturn* is more frequently changed from progreſſive to regreſſive, than that of *Jupiter*; as alſo its Oppoſitions and Conjunctions with the Sun are more frequent. The Reaſon of this is, becauſe as *Saturn* moves much ſlower than *Jupiter*, the former is more frequently overtaken and paſs'd-by by the Earth than the latter. And as theſe *Phænomena* happen ofter in *Saturn* than in *Jupiter*, ſo they do more frequently in *Jupiter* than in *Mars* for the ſame Reaſon.

So ſmall is the Diſtance of the Earth, with reſpect to that of *Jupiter* and *Saturn*, from the Sun, that in whatever Part of their Orbits they be ſituated with reſpect to the Earth, that Side of them, as to Senſe, which is turned towards the Sun, is always towards the Earth; on which Account they always appear to us as ſhining with a full Orb. But it is not ſo with reſpect to *Mars*; for ſince that Planet is not much farther removed from the Sun than the Earth, it is obvious that that Side of it which is turned towards the Sun will not always be towards the Earth. For inſtance; though when *Mars* is at N, and the Earth at A or E, its illuminated Hemiſphere is directly towards the Earth, yet it is turned away from it conſiderably when the Earth is at B or C; at which time therefore it appears to want ſomething of its uſual Rotundity, or to be *Gibbous* like the Moon, a little before or ſoon after the *Full.*

As the inferior Planets *Mercury* and *Venus* have their reſpective Elongations from the Sun when ſeen from the Earth; ſo the Earth, if we imagine it to be viewed from the ſuperior Planets, hath alſo its Elongation from the Sun. But this being a Phænomenon, which has not happened to our Aſtronomers to ſee, they give it a different Name, and call it when at the greateſt, from a Phænomenon in the ſuperior Planets depending upon it, *the Parallax of the Semidiameter of the Earth's annual Orbit in thoſe Planets:* by which is meant, the Difference between the Place in the Heavens the Planet would appear in if ſeen from the Center of the Sun, and that in which it would appear if ſeen from the Center of the Earth, when at its greateſt Elongation from the Sun with regard to that Planet. To explain this, let N be the Place of the Planet, and B that of the Earth in its greateſt Elongation from the Sun, as ſeen from that Planet. Then will the aparent Place of the Planet as ſeen from the Sun (that is, its true or *Heliocentric* Place) be Q, but its *Geocentric*, or apparent Place as ſeen from the Earth, will be O. Now becauſe the Arch QO ſubtends an Angle (*viz.* QNO) at the Planet which is equal to SNB, *viz.* that which SB, the Semidiameter of the Earth's annual Orbit ſubtends, the abovementioned Difference in the apparent Situation of the Planet is called *the Parallax of the Semidiameter of the Earth's annual Orbit in that Planet (a).*

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(*a*) Hence, as was obſerved of the Inferior Planets, the Diſtances of the ſuperior ones from the Sun with reſpect to that of the Earth from thence, may be found. For in the Triangle SBN, all the Angles being known, the Relation between the two Side BS and SN is had by the known Rules of *Trigonometry*. The Angle SBN is had from immediate Obſervation, This Angle in *Mars* is but 42 Degrees, in *Jupiter* not above 11: In *Saturn* it is but the fourth part of the greateſt Elongation of *Mercury* ſeen from the Earth, *viz.* about 6 Degrees. So that ſince *Mercury* by reaſon of its Nearneſs to the Sun is rarely ſeen by us, in all Probability the Aſtronomers in *Saturn* (if there be any) do not know that there is ſuch a Body in the Univerſe as our Earth.

ſervation, as being ſubtended by an Arch in the Heavens between the apparent Place of the Planet and that of the Sun. The Angle at N is equal to the Difference between the real and apparent Place of the Planet; the firſt of which is found by Computation, the latter by Calculation. In *Jupiter*, the beſt Way of finding that Angle is from his *Satellites*, the Method of doing which ſhall be ſhewn when we have conſidered the Phænomena of thoſe Planets.

CHAP. VIII.

Of the Phænomena *of the Moon.*

HAving now finiſhed what relates to the *Phænomena* of the *Primary* Planets, we proceed to conſider thoſe which are owing to the Motion of the *Secondary* ones; and firſt of the Moon, which is the conſtant and only Attendant of the Earth, and is always ſo near it, that if the Earth and the Moon were viewed together from the Sun, the

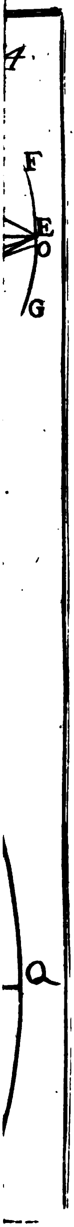
Moon at ſuch Time as it would appear the fartheſt from the Earth would not ſeem to be at a greater Diſtance from it than what is equal to one third Part of the Breadth of the Sun's Body, as it appears to us.

Since the Moon is an opake ſpherical Body, and revolves about us in ſuch Manner as to paſs between the Sun and the Earth, it is abſolutely neceſſary it ſhould put on different *Appearances*, according to its various Poſition with reſpect to the Sun and the Earth; its illuminated Hemiſphere being conſtantly turned towards the Sun the Fountain of Light, and for that reaſon ſometimes a greater and ſometimes a leſs Portion of it turned towards us. To ſhew this, little need be ſaid, after ſo full an Explication of the ſeveral Phaſes of *Venus*, as was delivered in Chap. VI.

When the Moon is between the Earth and the Sun, that is, in Conjunction with the Sun, its whole illuminated Hemiſphere is turned from us, in which Station it is ſaid to be *New*; afterwards when by its Motion in its Orbit it emerges out of the Sun's Rays, a ſmall Portion of that Hemiſphere which is turned towards the Sun is alſo turned towards the Earth, and then (as was obſerved of *Venus*) the Moon, by reaſon of its great Diſtance from us, appearing as a flat Surface, one Edge of that Surface appears brighter than the reſt of it, and ſo the Moon in this Situation appears *horned*; and becauſe it moves from Eaſt to Weſt, as was obſerved above, it at this Time appears to the Eaſtward of the Sun; upon which account it ſeems to follow the Sun in its daily Courſe, riſing and ſetting after it. When the Moon is got to a *quartile Aſpect*, as Aſtronomers call it, (that is, to the Diſtance of a quarter of a Circle in the Heavens from the Place of the Sun) then is half of its illuminated Hemiſphere turned towards us, and half from us, in which Caſe it appears to us as a plain round Surface *half illuminated*, and half not. When it has advanced a little farther it its Orbit, and got farther from the Sun, more of its illuminated Hemiſphere becomes viſible to the Inhabitants of the Earth; whence it appears like a plain round Surface, ſomewhat defective of Light on that ſide which is from the Sun, and is ſaid to be *Gibbous*. When it comes to an Oppoſition with reſpect to the Sun, it then turns the ſame Side directly towards the Earth that is towards the Sun, at which time it appears to us wholly enlightened, and is therefore called the *full Moon*. And as it performs the other Part of its Revolution, it aſſumes firſt a *gibbous*, then an *halved*, afterwards an *horned* Face; and when it arrives at its Conjunction with the Sun, it diſappears for a while, the obſcure part only being turned towards us, as before.

When the Moon is horned, the darker Hemiſphere, or that which is turned from the Sun, has a faint Light diffuſed over it, ſufficient to render it viſible. This Light is owing to a Reflection of the Sun's Rays from the Surface of the Earth to that part of the Moon. For as the Moon when at the *Full*, by means of its enlightene Hemiſphere which is then turned towards the dark Side of the Earth, ſhines upon it, and affords a certain Degree of Light; ſo the Earth when the Moon is at the *New*, has its enlightened Hemiſphere, (or that ſide where it is *Day*) turned towards the obſcure Hemiſphere of the Moon, and enlightens that, but in a much greater Degree than the unilluminated Hemiſphere of the Earth (or that ſide where it is Night) is enlightened by the *full* Moon; the Earth being conſiderably larger than the Moon. So that as that Body is a Moon to us, our Earth is a much larger and more luminous one to that.

The Time in which the Moon performs one entire Revolution about the Earth from any Point of the Zodiac to the ſame again, is called a *Periodical* Month. This Month conſiſts of 27 Days, 7 Hours, and 43 Minutes. But the *Synodical* Month, which is the Time the Moon takes up in paſſing from one Conjunction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junction with the Sun to another, is larger than this by about two Days and 5 Hours. The Reaſon of this is, that while the Moon is paſſing from one Conjunction to another, the Sun by means of the Earth's Motion in its Orbit, is in appearance advanced towards the Eaſt; ſo that when the Moon comes round to that Part of the Heavens where the Sun was at the foregoing Conjunction, it is not then in Conjunction with it again, but has ſuch a Portion of the Heavens to deſcribe, as takes up ſo much more time, as, being added to the *Periodical* Month, makes it up 29 Days, 12 Hours, 44 Minutes, and 3 Seconds; which is the *Synodical.* But perhaps the 14th Figure may make this clearer: in which let S repreſent the Sun, E the Earth moving in its Orbit AB from A towards B, and let MNO denote the Orbit of the Moon, and M its Place therein when in Conjunction with the Sun. Now while the Moon deſcribes its Orbit MNO, let it be ſuppoſed that the Earth advances in its Orbit from E to *e*, in which Caſe the Moon's Orbit will be found at *mno*, and the Point M will be at *m* in the Line *ed*, if we ſuppoſe it parallel to ES. And therefore when the Moon comes to *m*, it will have performed an entire Revolution about its Orbit; but will not be in Conjunction with the Sun, but has ſtill the Arch *mp* to deſcribe before it reaches that Point of the Heavens in which the Sun will then appear to be.

The Plane of the Moon's Orbit is not coincident with that of the Earth, but makes with it an Angle of about 5 Degrees, cutting it in two Points oppoſite to each other, after the ſame Manner as the Equator and the Ecliptic were obſerved to cut each other; ſo that the Moon does not ſeem to deſcribe the Ecliptic in moving round its Orbit, but one half of the Way it declines towards the *North*, the other half towards the *South*. The two Points where the Moon's Orbit paſſes through the Plane of the Ecliptic, are called the *Nodes*; and a right Line drawn from one of theſe Points to the other, will paſs through the Center of the Earth (that being in the Plane of the Moon's Orbit as well as in that of its own) and is called the *Line of the Nodes*. That Node through which the Moon paſſes from the *South Side* the Ecliptic to the Northern is called the *Aſcending Node*; the other thro' which it paſſes from the *North* to the *South* Side, is called the *Deſcending Node (a)*.

The Extremities of the Line of *Nodes* are not always directed towards the ſame Points of the Ecliptic, but continually ſhift their Places from Eaſt to Weſt, or contrary to the Order

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(*a*) Aſtronomers often expreſs the former of theſe Nodes by this Mark ☊, which they call the *Dragon's Head*; the lat- by ☋, which they call the *Dragon's Tail*.

Order of the Signs; performing an entire Revolution about the Earth in the Space of ſomething leſs than 19 Years.

As was obſerved of the primary Planets, the Moon alſo moves not in a Circle, but in an *Ellipſe*, one of whoſe *Foci* is in the Center of the Earth. And the *Linea Apſidum* of it is not carried along with the Earth in a Direction parallel to itſelf, but continually turns round the Center of the Earth thro' which it paſſes from Weſt to Eaſt, or according to the Order of the Signs, in the Space of about 9 Years.

The Moon being ſubject to the Influence of the Sun as well as the Earth, it is not only liable to various Irregularities in its Motion, but its Orbit is wont to put on different Forms and Motions, all which depend on its Situation with reſpect to thoſe two Bodies. They are theſe that follow.

I. The nearer the Moon is to its *Syzigies*, (that is, its Conjunction or Oppoſition with the Sun) the greater is its Velocity; and the nearer it is to the *Quadratures* (that is, the firſt and laſt Quarter) the ſlower it moves *(b)*.

II. It does not deſcribe *Areas* proportionable to the Times, except in the Quadratures and Conjunctions.

(b) This Irregularity was obſerved by *Tycho Brahe*, who gave it the Name of the Moon's *Variation*.

III. Its Orbit is ſomewhat more curved in the Quadratures, than it would be if the Moon were only influenced by the Earth; and leſs curved in the Syzigies, the Moon running farther off from the Earth in the Quadratures than in the Syzigies.

IV. When the Earth is in its Perihelion, (that is, in that Part of its Orbit which is neareſt the Sun) the Periodical Time of the Moon is greater than when it is in its Aphelion, or fartheſt from the Sun: ſo that the Lunar Months of one Part of the Year exceed thoſe of another.

V. The *Linea Apſidum* of the Moon's Orbit goes forwards when the Moon is in the Syzigies, and backwards when in the Quadratures; but it goes farther forwards than backwards every Time, ſo that at length it revolves quite round, as obſerved above.

VI. The Excentricity of the Moon's Orbit is greater and leſs in every Revolution of the Moon; it is greateſt when the Moon is in the Syzigies, and leaſt when it is in the Quadratures: and conſequently while it paſſes from the Quadratures to the Syzigies, it is encreaſing; and while it paſſes from the Syzigies to the Quadratures it is diminiſhing. And if we compare ſeveral Revolutions of the Moon together, its Orbit is the moſt Excentrical when the Situation of it is ſuch, that the *Linea Apſidum* is in the Syzigies, and leaſt when in the Quadratures. So that when both the Moon and the *Linea Apſidum* of its Orbit are in the Syzigies, its Excentricity ſhall be half as great again, as when they both happen to be in the Quadratures.

VII. The Line of the Nodes does not move uniformly backwards, that is, contrary to the Order of the Signs; but when it is in the Syzigies it ſtands ſtill, and moves the faſteſt of all when in the Quadratures.

VIII. The Inclination of the Lunar Orbit to the Ecliptic (*c*) is alſo liable to Change; it is greateſt when the Nodes are in the Syzigies, and leaſt when they are in the Quadratures.

All theſe Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion; and alſo greater when the Moon is in Conjunction with the Sun than in Oppoſition.

IX. There is another Particular in the Moon's Motion, which is by ſome reckoned among its Irregularities; which is, that it moves faſter in its Perigeon, than in its Apogeon; but this is not owing to any Perturbation in its Motion ariſing from the Influence of

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(*c*) Some Aſtronomers meaſure the Inclination of the Moon's Orbit to the Ecliptic, not from the Ecliptic, but from a Perpendicular to that, and ſo call its Inclination 85 Degrees inſtead of 5. And ſo of the Inclination of the Equator to the Ecliptic; reckoning that 66 Degrees and an half, and not 23 and an half, as is the moſt uſual Way.

of the Sun; but is common to all the Planets, and is occaſion'd only by its deſcribing equal Areas in equal Times in an elliptical Orbit, as the other Planets do.

All theſe Irregularities obſervable in the Motion of the Moon, naturally flow from the Suppoſition, that the Earth and the Moon are retained in their Orbits by Gravity, in the Manner laid down and explained in the Introduction to this Part, as will appear when we have conſidered the Phyſical Cauſes of the Motion of the heavenly Bodies.

The only uniform Motion the Moon has, is its Rotation about its *Axis*, which it performs the ſame Way, and in the ſame Time, that it does its Revolution about the Earth. The Conſequence of this would be, that if its Revolution in its Orbit were performed with an uniform Celerity, and that Orbit were a Circle having the Earth in its Center, and alſo if the *Axis* of the Moon were perpendicular to the Plane of that Orbit, the Moon would in all Poſitions turn exactly the ſame Side towards the Earth. This is ſo obvious as to need no Illuſtration. But ſince its Motion in its Orbit is not uniform, nor its Orbit a Circle, neither its *Axis* perpendicular to the Plane thereof, the Moon does not keep the ſame Face exactly towards us, but turns it a little, firſt one Way, then another; appearing thereby ſubject to a twofold *Libration*, *viz.* from Eaſt to Weſt, and from North to South. To the one of which the two firſt Cauſes abovementioned concur; the other is owing to the third. And firſt to illuſtrate that *Libration* which is from Eaſt to Weſt. Let the Elliptical Orbit BBB (Fig. 15.) repreſent that of the Moon, let E be the Place of the Earth, and let the Circle ABCD repreſent the Moon in its Apogeon at A; and for the preſent, let us ſuppoſe the *Axis* of the Moon perpendicular to its Orbit. Then becauſe its Motion about its *Axis* is uniform, and its Motion from its Apogeon the ſloweſt of all; it is evident, that at the Time when it has revolved a quarter of the Way round its *Axis* and obtained, ſuppoſe the Situation FBCD, it will not have advanced a quarter part of the Way through its Orbit; and therefore the Face BCD, which was turned towards the Earth at firſt, cannot now be turned to I the Center of the Ellipſe, much leſs to E the Place of the Earth, but to ſome other Place, as K, on the other Side the Center: But when it has advanced to G, the ſame Face BCD will be turned towards E, that was when the Moon was at A; becauſe as it has advanced half through its Orbit, it has advanced half round its own *Axis*. Again, as it moves quicker in that Part of its Orbit, that's near its Apogeon, it will have proceeded above a quarter part of the Way through its Orbit, and ſo will have got into the Situation HBCD ſuppoſe, when it has turned a quarter round its *Axis*; and that Part of the Moon which was viſible when the Moon was at A, will be turned towards K, as it was when the Moon was at F. And it is obſervable from the Figure, that the Face BCD, which was turned directly towards the Earth at E, when the Moon was either at A or G, is, when the Moon is at F, turned to the *Right*, or towards the Weſt, with reſpect to a Spectator viewing it from E; but to the *Left*, or towards the Eaſt, when the Moon is at H. And this is the firſt kind of *Libration* which was to be explained. As to that which is from North to South, it may be thus illuſtrated. Let HI (Fig. 16.) repreſent the Plane of the Moon's Orbit, being ſuppoſed to be ſeen edgeways; let E be the Place of the Earth, and let the Circle AMB*m* repreſent the Moon, the Line AB being its *Axis*, and M*m* one of the Parallels of its Equator; and when the Moon is at H, let the Point M be that which is turned directly towards the Earth. Now when the Moon has revolved half round its *Axis*, which will be when it has got to I the oppoſite Point of its Orbit, the Point M will be found at *m*; but becauſe of the Paralleliſm of its *Axis*, not this, but ſome other Point, as N, is that which is turned directly towards the Earth at E; ſo that to a Spectator at E, the viſible Part of the Moon librates from North to South, and returns to its place once in each Revolution, one of the Poles as A being only viſible, when the Moon is in one Part of its Orbit, ſuppoſe at I, and the Pole B only when it is at H; becauſe the Circle repreſented by the pricked Line CD terminates the View of a Spectator at E, in either Situation of the Moon *(a.)*

As to the Surface of the Moon, whoever views it with a good Teleſcope, will perceive ſome Parts of it ſhining very bright, while others are as dark and obſcure. The bright-

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(a) It may be worth conſidering here, what kind of Motion the Earth ſeems to have to the Inhabitants of the Moon, if any ſuch there be. And firſt, we will ſuppoſe that the Hemiſphere, the Moon turns towards us, is always exactly the ſame. In this Caſe it is evident, that to ſuch as live in the Middle of that Hemiſphere, the Earth muſt always ſeem to be directly over their Heads; becauſe a Line drawn from the Center of the Moon through the Place where they are, would paſs through the Earth: and to ſuch as live at the Extremity of that Hemiſphere, the Earth muſt always appear in their Horizon. But ſince the Moon, as ſeen from the Earth, has theſe libratory Motions abovemention'd, it is impoſſible that the Earth ſhould always appear directly Vertical to the one, or in the Horizon of the other; but muſt ſeem always nearly at the ſame Degree of Elevation; continually ſhifting its Situation from Eaſt towards the Weſt, and from Weſt towards the Eaſt, and at the ſame time from North towards the South, and from South towards the North, according as the Libration of the Moon, as ſeen from the Earth, is made in that Time.

er Parts are thought by ſome to be Land, the other Water; becauſe leſs Light is reflected from Water than from Land. But *Keil* is of Opinion, that ſome of thoſe darker Places may be occaſioned by the Shadows of higher Places falling upon them, and others may conſiſt of a Soil of a darker Colour, that reflects leſs Light than the reſt *(b)*. However this be, it is moſt certain, that there are Mountains in the Moon, and thoſe very high ones. For when the Moon is viewed through a Teleſcope, the Line which ſeparates the illuminated Hemiſphere from the dark one does not appear ſtrait when the Moon is *halved*, or uniformly bent when it is *horned* or *gibbous*, as when we view it with the naked Eye; but ſeems jagged, broken, and uneven; which evidently ſhews that its Surface is ſo too: For if its Surface were ſmooth and even, no Reaſon could be given why the Light of the Sun ſhould not reach as far in one Place as another. But the Boundary of Light and Darkneſs in the Moon is not only thus jagged and uneven, but ſome Parts even within the darkened Hemiſphere, that lie near the Edge of it, are enlighten'd by the Rays of the Sun long before ſuch as lie nearer the illuminated Hemiſphere, and continue ſo till after the Parts about them are

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(b) See his Aſtronomy, Lect. X.

are in the *Dark.* Theſe therefore can be no other than the Tops of high Mountains, on which the Sun ſhines as it does on thoſe on Earth, both before and after it is viſible in the Vallies below. The perpendicular Height of a very remarkable Mountain in the Moon, called St. *Katharine (c)*, has been found by Menſuration to be nine Miles, which is three Times that of the higheſt Mountain we have upon the Earth *(d)*.

(c) See the Names Aſtronomers have diſtinguiſhed the Parts of the Moon by, in a Figure annexed to the Lecture referred to in the foregoing Note.

(d) The Method of Meaſuring the Height of a Mountain in the Moon is this: Let ABDE (Fig. 17.) repreſent the Moon, C its Center, SA a Ray of Light from the Sun, ABD the illuminated Hemiſphere, AED the dark one, F the Top of a Mountain beginning to be illuminated. Then with a Teleſcope in which there is a Micrometer fixed, let the proportionable Lengths of the Lines FA and AD be taken. Then becauſe the real Length of the latter, which is the Diameter of the Moon is known, the true Length of the former will alſo be had. Since then the Line SAF is a Tangent to the Moon, the Triangle FAC will be a right-angled one, and F C will be its Hypothenuſe, whoſe Square being equal to the Squares of the other two Sides, it may from thence be alſo known. Now if from this we ſubſtract the Semidiameter of the Moon, the Remainder is the perpendicular Height of the Mountain.

CHAP. IX.

Of the Eclipſes *of the* Sun *and the* Moon.

AN *Eclipſe* is a Deficiency of Light in ſome of the heavenly Bodies, occaſioned by the Interpoſition of ſome other between that and the Sun. Thus when the Moon paſſes through the Shadow of the Earth, it being by that means deprived of the Sun's Light, loſes its Brightneſs, and is then ſaid to ſuffer an Eclipſe. So when the Moon is interpoſed between the Sun and the Earth, the Sun being in Appearance deprived of its Light, is ſaid to be eclipſed; though it is the Earth which is properly ſo, that, and not the Sun, being the Body where the Light is wanting.

One primary Planet is never eclipſed by the Interpoſition of another; becauſe they are ſo ſmall with reſpect to the Magnitude of the Sun, that the Shadow of any one of them ends in a Point before it reaches the Body of another. Thus when the Earth is interpoſed between *Mars* and the Sun, its Shadow vaniſhes before it reaches to *Mars*. And even when the Moon is interpoſed between the Sun and the Earth, if the Earth be in its *Perihelion*, and the Moon in its *Apogeon* at that Time, the Shadow of the Moon will not reach the Earth, and ſo the Moon will not ſeem to cover the whole Body of the Sun, but appear as a large dark Spot, ſurrounded with a Circle of Light proceeding from the Limb or Edge of the Sun.

From what has been obſerved it appears, that there can be no Eclipſe of the Moon, but when the Earth is interpoſed between it and the Sun, that is, at the Time of its Oppoſition, or when it is *Full*; nor any of the Sun, except when the Moon is between the Earth and the Sun, that is, at the Time of its Conjunction with the Sun, or when it is *New*. And the Reaſon why theſe Eclipſes do not happen at every New and Full Moon, is, becauſe, as was obſerved above, the Plane of the Moon's Orbit is inclined to that of the Earth's, ſo that the Moon is ſometimes above the Ecliptic, and ſometimes below it, at the Time of its Conjunction with, or Oppoſition to the Sun, in which caſe the Shadow of the one paſſes above or below the other *(a)*.

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(a) It is not requiſite to conſtitute an Oppoſition or Conjunction of a Planet with the Sun, that it be in a right Line that paſſes through the Sun and the Earth; it may be in any Degree of Latitude it is capable of, and is nevertheleſs ſaid by Aſtronomers to be in Conjunction or Oppoſition with the Sun, provided it be in the ſame Part of the Zodiac, that is, if it have the ſame Degree of Longitude therewith.

The only Time therefore in which Eclipſes either of the Sun or Moon can happen, muſt be when the *Nodes* are in or near the Syzigies, that is, when the Line of the *Nodes* points towards the Sun. Now if the Line of the *Nodes* always pointed towards the ſame Points of the Heavens, or remained parallel to itſelf, then the Sun appearing by means of the Earth's Motion to run through the Ecliptic once a Year, it is obvious that one Extremity of this Line would point towards the Sun each half Year: but ſince this Line has a ſlow Motion contrary to that of the Sun, by which it revolves round in nineteen Years, as obſerved above, it meets the Sun with one or the other Extremity before it has got quite half round, and ſo points towards it twice in ſomewhat leſs than half a Year: So that it ſometimes happens that it points towards the Sun three times within the Space of a Year, which is as oft as poſſible. And it frequently happens, that the Sun being in its Approach towards one of the Nodes at the Time of the New or Full Moon, and near enough to occaſion an Eclipſe then, the Moon ſhall get about to the other Node, before the Sun be too far removed from the former, and ſo there will be another Eclipſe then. Upon which account it is poſſible there may be two Eclipſes, while the Sun is paſſing by either Extremity of the Line of the Nodes; the one of which (if it be at the New Moon) will be an Eclipſe of the Sun, and the other will happen at the next Full Moon, and will therefore be an Eclipſe of the Moon; or if the firſt be at the Full, the other will be at the New. But if the Moon be within a Day or two of paſſing thro' one of its Nodes at the Time of its Conjunction, it will paſs by the other without ſuffering an Eclipſe there.

When it happens that the Moon is in one of its Nodes at the Time of its Oppoſition to the Sun, then is it ſaid to be *centrally* eclipſed, its Center being then in a right Line with thoſe of the Sun and Earth. In this Caſe it is alſo *totally* eclipſed, every part of it being hid from the Sun's Rays. But the Shadow where the Moon paſſes through, is about three times as broad as the Face of the Moon, ſo that it may ſuffer a total Eclipſe, though it be at ſome Diſtance from the Node at the Time of its Opppoſition; an Eclipſe therefore may be *total*, when it is not a *central* One; but an Eclipſe of this Kind will be of ſhorter Duration than the former. Farther, if it be ſo far removed from the Node that a Part of its Diſk only falls into the Shadow of the Earth, it is ſaid to be a *partial* Eclipſe, and is of ſhorter Duration than the former. If the Moon be above 13 Degrees from the Node, there is no Eclipſe.

The Duration of a *central* Eclipſe of the Moon, is about 4 Hours; the firſt of which is taken up in entring the Shadow, two more it continues quite immerſed therein, and the fourth is ſpent in getting out.

What is here ſaid, may be illuſtrated in the following Manner. Let AB (Fig. 18.) repreſent a ſmall Portion of the Plane of the Earth's Orbit ſeen edgeways, CD a part of the Moon's Orbit, and let the Point where theſe Lines croſs each other be one of the Nodes, and E,E,&c. the Shadow of the Earth paſſing along the Ecliptic. If the Moon falls into the Shadow at N, it is both a total and central Eclipſe; if at O, it is total, but not central; if at P, the Eclipſe is only partial. And if the Moon is at Q at the Time of its Oppoſition, it then paſſes by the Shadow without being eclipſed at all *(b)*.

As all opake Bodies when illuminated by the Rays of the Sun caſt a Shadow from them, ſo they have alſo a *Penumbra*, which is an imperfect kind of a Shadow every where ſurrounding the former, and growing larger and larger,

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(b) We have hitherto ſuppoſed that the Shadow of the Earth reaches the Moon, as it would do, were it not for the Atmoſphere of the Earth, which by refracting the Sun's Rays as they paſs through it towards the Earth, throws them into its Shadow; which refracted Rays falling upon the Moon render it viſible to us, altho' placed within the Limits of the Shadow, and is the Occaſion of that reddiſh Colour, with which the Moon appears at that Time.

larger, as we recede from the Body. As the other is owing to a total Interception of the Rays of Light, this is occaſioned by a partial one. The 19th Figure will explain this better.

Let ABCL repreſent the Body of the Sun, KI the Earth, and MN the Moon; and let the Lines AM, BM, *&c.* be drawn as in the Figure; then will the Space MNFG, within which none of the Rays can come, be the Shadow thereof. But it is evident that beſides this there are other Spaces, *viz.* MDE and MEF, *&c.* within which but Part of the Sun's Rays can come; and the nearer thoſe Spaces are to the Shadow, the leſs Light they are capable of receiving. The Light which falls within theſe Spaces is called the *Penumbra.*

This being premiſed, we may proceed to conſider the different Kinds of Solar Eclipſes.

The Moon being ſmaller than the Earth, and having a conical Shadow, as being alſo leſs than the Sun, a very ſmall Part of the Surface of the Earth can be covered by the Shadow of the Moon at the ſame Time, though, as obſerved above, the whole Body of the Moon may be involved in that of the Earth; ſo that an Eclipſe of the Sun is viſible but to a few Inhabitants of the Earth, whereas one of the Moon may be ſeen by all that live on that Hemiſphere of the Earth that is turned towards it.

In order to conſtitute a *central* Eclipſe of the Sun, it is not neceſſary that the Moon ſhould be exactly in the Line of the Nodes at the Time of its Conjunction; for it is ſufficient to denominate an Eclipſe of the Sun *central*, that the Center of the Moon be directly betwixt the Center of the Sun and the Eye of the Spectator: for to him the Sun is then *centrally* eclipſed. But ſince the Shadow of the Moon can cover but a ſmall Portion of the Earth, it is obvious this may happen when the Moon is not in one of her Nodes. And not only this, but the Sun may be eclipſed *centrally*, *totally*, *partially*, and *not at all*, at the ſame time. For inſtance, when the Moon is interpoſed between the Sun and the Earth, to thoſe who live where the Center of the Shadow paſſes (as in the Middle between F and G in the laſt Figure) the Sun will be *centrally* eclipſed; to thoſe who live within the Shadow, but not in the Center (ſuppoſe nearer to F than G) *totally*; to thoſe who live in the *Penumbra*, as between F and D, or G and I, *partially*, and that more or leſs as they live nearer or farther from the Shadow; to thoſe who live without the *Penumbra*, as between D and K, *not at all*.

But as the Shadow paſſes along the Surface of the Earth, different Inhabitants, and thoſe very diſtant ones, will ſee the ſame kind and degree of Eclipſe, though not at the ſame Hour *(d)*.

(d) For the Method of predicting Eclipſes, ſee *Keil's* Aſtronomy, Lect. XIV. Or *Whiſton's* Aſtronomical Lectures, Lect. XII.

CHAP. X.

Of the Phænomena *of the* Satellites *of* Jupiter *and* Saturn, *their* Eclipſes *and* Occultations: *And alſo of* Saturn's Ring.

THE Satellites of *Jupiter* and *Saturn* are ſubject to the ſame kind of Motion with the Moon, except that their Orbits, as far as can be collected from the Obſervations of Aſtronomers, are circular, or very nearly ſo; and therefore, if we ſuppoſe them to be ſeen from their Primary ones, they will exhibit nearly the ſame *Phænomena*.

All the Satellites of *Saturn* revolve about it in almoſt the ſame Plane, *viz.* that of its Ring; excepting the fifth, the Plane of whoſe Orbit deviates a little therefrom. Thoſe of *Jupiter* move alſo in a Plane that is proper to themſelves, and nearly coincident with that in which *Jupiter* itſelf moves about the Sun.

Theſe Planets, *viz.* the Secondaries both of *Jupiter* and *Saturn*, when viewed from the Earth, appear to have their Progreſſions, their Stations, and Retrogradations different from thoſe of their Primaries; that is, when a Primary is progreſſive, its Secondary may be regreſſive, or otherwiſe; and *vice verſa.* For inſtance, when a Secondary is beyond its Primary, that is, in its ſuperior Conjunction with reſpect to the Earth, it ſeems to move according to the Order of the Signs; but when it is in its inferior one, it ſeems to move the contrary Way; and when its progreſſive Motion is changing into a regreſſive one, the Planet appears ſtationary.

To illuſtrate this; let S (Fig. 20.) repreſent the Sun, I the Planet *Jupiter* or *Saturn*, BCG the Orbit of one of its Satellites, and let EPF be the Orbit of the Earth. While the Satellite paſſes through OHB, the ſuperior Part of its Orbit, it will appear from the Earth to move the ſame Way with its Primary; but while it moves through the remaining Portion BLO, it will ſeem to go the contrary Way; as is evident if we conſider that its Motion in its Orbit is quicker than that of its Primary.

Theſe Satellites and their Primary ones mutually eclipſe each other in the ſame manner in which it has been ſhewn that the Earth and the Moon do. But there are three different Caſes in which the Secondaries diſappear to us. The One is, when a Secondary is directly behind the Body of its Primary with reſpect to the Earth; this is called an *Occultation* of the Planet: Another is, when it is directly behind its Primary with reſpect to the Sun, and ſo falls into its Shadow, and ſuffers an Eclipſe, as the Moon, when the Earth is interpoſed between that and the Sun. The laſt is, when it is interpoſed between the Earth and its Primary; for then it can-be diſtinguiſhed from the Primary itſelf. All which may be illuſtrated in the following Manner.

While the Planet deſcribes OHB the ſuperior Part of its Orbit, it is capable of diſappearing twice; once when its Primary is interpoſed between it and the Earth, as when it is at G or K, or ſome intermediate Point, according as the Earth is ſituated at that Time; and alſo at H, where it is involved in the Shadow of its Primary. When the Earth is between F and Q, or F and P, that is, when the Sun is to the Eaſtward of *Jupiter*, the Eclipſe of the Secondary happens firſt, *viz.* when it paſſes through the Shadow at H; and the *Occultation* afterwards, when the Planet lies hid at K. When the Earth is between E and Q, or E and P, the Occultation

tation is firſt, and the Eclipſe afterwards. While it paſſes through the inferior Part of its Orbit, it diſappears alſo, according to the Situation of the Earth, either at D or C; or when it is between thoſe Points, being directly between the Earth and its Primary, and therefore not diſtinguiſhable from the latter. When the Satellite is at L in Conjunction with the Sun, its Shadow falls upon its Primary, and eclipſes a Part of its Diſk, in like manner as the Moon, in the like Situation with reſpect to the Earth, eclipſes a Part of that (*a*). As

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(*a*) From the Occultations and Eclipſes of the Satellites of *Jupiter*, three notable Problems are determined: *viz.* 1. The Parallax of the Orbit of the Earth in that Planet, and thereby (as we obſerved Chap. VII. in the Notes) its Diſtance from the Sun. 2. The ſucceſſive Propagation of Light; and 3. The Longitude of Places upon the Earth.

1. The Method of finding the Parallax of the Orbit of the Earth, is this: Let the Interval of Time which lapſes between the Middle of an Occultation of one of the Secondaries, ſuppoſe at G, and the Middle of its Eclipſe afterwards at H, which is the Time the Planet deſcribes the Arch GH in, be obſerved. The Time then of its whole Revolution being known, that Arch itſelf may be had, which is the Meaſure of the Angle GIH, or its equal EIS, which is the Parallax of the Orbit of the Earth in that Planet.

2. If Light were *inſtantaneous*, a Spectator at P would ſee an Eclipſe of one of the Secondaries of *Jupiter*, at the ſame Time that he would do if on the contrary ſide the Sun at Q. But it appears from Obſervations, that when the Earth is at Q, thoſe Eclipſes happen ſooner, and that when it is at P they happen later than they ought to do by Computation. Which ſhews that Light takes up ſome time in paſſing from Q to P. From Obſervations of this kind it has been computed by *Romer*, that Light is eleven Minutes in paſſing from the

As to the *Phænomena* of *Saturn*'s Ring, it is obſervable, 1. That its Inclination to the Plane of the Ecliptic is about 31 Degrees: 2. That the Thickneſs of it is inſenſible; and therefore

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the Sun to us; which is upwards of 81 Millions of Miles: But it has ſince been found by the more accurate Obſervations of others, that it paſſes over that Space in about 7 Minutes.

3. The Longitude of a Place is its Diſtance Eaſt or Weſt of ſome other Place; and as Places lie more or leſs Eaſt or Weſt one of another, the Hour of the Day in one is more or leſs different from the Hour of the Day in the other. That is, as Places differ in point of Longitude, ſo they do proportionably in their Account of Time. For inſtance, if a Place lies fifteen Degrees to the Eaſtward of another, it is one Hour paſt Mid-day there, when it is but Mid-day at the other: The Reaſon is, becauſe the Sun is an Hour in paſſing over each fifteen Degrees of its daily Courſe. In like Manner, if a Place lies ſo many Degrees to the Weſtward of another, the Sun comes an Hour later to that Place than to the other; ſo that it is but Eleven of the Clock there, when it is Twelve at the other. And ſo proportionably for any other Diſtance. If therefore at two Places it be obſerved at what Hour the beginning of an Eclipſe of one of *Jupiter*'s Satellites happens at each of them, and if that Hour be not the ſame in both, the Difference will ſhew how far thoſe Places lie Eaſt or Weſt of one another. Or if inſtead of the Obſervation at one of the Places, an Almanack be made uſe of, that ſhews the Time when the Eclipſe will happen at that Place, and that Time be compared with the Time of its happening obſerved at the other Place, the Differences between thoſe Times, if there be any, will ſhew the ſame.

This Method very rarely ſucceeds at Sea, becauſe the Toſſing of the Ship is inconſiſtent with the Accuracy required in Obſervations of this kind.

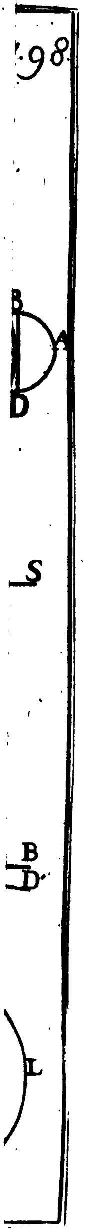
The like may be done by obſerving the Time when an Eclipſe of the Moon happens, or when it comes to a fixed Star: But theſe are *Phænomena* that are much leſs frequent than the other.

therefore when it is ſo ſituated, that the Plane of it, if produced, would paſs through the Earth, it becomes inviſible. Neither is it to be ſeen when the Plane of it paſſes between the Sun and the Earth; for then the Sun ſhines upon that Side of it which is turned from us; in which caſe, the dark Side being next the Earth, it is not perceptible to us. But in this caſe a black Liſt or Circle appears to encompaſs the Planet, which in all Probability is no other than the Shadow of the Ring thereon. The Extremities of the Ring which appear on either Side the Planet are by ſome called its *Anſæ*.

CHAP. XI.

Of the Comets.

THE Comets are opake, ſpherical, and ſolid Bodies like the Planets; and like them perform their Revolutions about the Sun in elliptical Orbits, which have the Sun in one of their *Foci*. The Particulars in which they differ from the Planets, are, that they move in various Directions, ſome the ſame Way with the Planets, others the contrary; neither are their Motions confined within the Zodiac, their Orbits admitting of any Inclina-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tion to the Ecliptic whatever: And the Excentricity of their Orbits is ſo very great, that ſome of the Comets perform the greateſt Part of their Motion almoſt in right Lines, tending in their Approach to the Sun almoſt directly towards it, after which they paſs by it; and when they leave it, march off again nearly in a right Line, till they are out of ſight, as if they were haſtening back to the fixed Stars; and return not till after a Period of many Years *(a)*.

As the Comets approach the Sun, their Motion grows proportionably ſwifter, for they deſcribe equal *Areas* in equal Times about its Center, as the Planets do. Hence it is, that when they are in their *Perihelia*, that is, at their leaſt Diſtance from the Sun, their Motion is immenſely ſwifter than when they are in their *Aphelia*, or greateſt Diſtance from it. This is the Cauſe that they are viſible to us for ſo ſhort a time; and when they diſappear, are ſo long before they come near enough to be ſeen again *(b)*.

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(a) See a Portion of the Orbit of a Comet deſcribed, Plate I. Fig. 1. at *abcd*.

(d) From the Obſervations of Aſtronomers, it ſeems probable, that the Comet which appeared in the Year 1682 was the ſame which was ſeen before in the Year 1607, and the Year 1531, and therefore may be expected again in the Year 1758, after a Period of about 151 Years. And that the great Comet which appeared in the Year 1680 was the ſame that was ſeen in the Time of K. *Henry* I. in the Year 1106, and in 531, and in

The Ancients were divided in their Opinions concerning them, ſome conſidering them as wandring Stars, others as Meteors kindled in the Atmoſphere of the Earth, ſubſiſting for a Time, and then diſſipated: others looked upon them as ominous Prodigies. But it is put beyond all doubt by the more accurate Obſervations of the late Aſtronomers, that they are a kind of Planets. That they are not Meteors is obvious, for if they were, they could not bear that vaſt Heat, which ſome of them in their *Perihelia* receive from the Sun. The great Comet which appeared in the Year 1680 was within a ſixth Part of the Sun's Diameter from its Surface, and therefore muſt acquire a Degree of Heat intenſe beyond all Imagination (*c*).

As

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in the forty fourth Year before Chriſt, in which *Julius Cæſar* was murthered. If ſo, then the Period of this Comet is about 575 Years. There are between 20 and 30 that have appeared ſince the Year 1337, but no two Appearances ſeem to belong to the ſame Comet, except thoſe abovementioned. See Dr. *Halley*'s Synopſis of the Aſtronomy of Comets, publiſhed at the End of *Gregory*'s Aſtronomy, in which there is a Table expreſſing the Motions of all the Comets that have been hitherto duly obſerved. The Comet of 1680 was in one Part of its Orbit within half the Sun's Breadth of the Way of the Earth.

(*c*) One Method by which Aſtronomers inveſtigate the apparent Courſe of a Comet, is this: They obſerve what two Stars are directly one on one ſide of the Comet, and the other on the other; which is done by holding up a Thread between the Eye and the two Stars, and extending it in ſuch manner, as that it ſhall ſeem to croſs each Star: then they look out two other Stars in

ſuch

As to thoſe *Phænomena* of them which ariſe from the Motion of the Earth; they agree in a great Meaſure with thoſe of the Planets. For inſtance, thoſe Comets which move according to the Order of the Signs, a little before they diſappear, become more than ordinarily *ſlow* or *retrograde*, if the Earth at that time be between them and the Sun; but more than ordinary ſwift, if the Earth be on the oppoſite Side: and the reverſe of this happens to thoſe which move contrary to the Order of the Signs. This is occaſioned by the Motion of the Earth, as was obſerved of the ſuperior Planets: For when the Earth goes the ſame Way with a Comet, but with a ſwifter Motion, the Comet ſeems *retrograde*; when with a ſlower Motion, the Comet's apparent Motion becomes ſlower; and when the Earth moves the contrary way, it becomes ſwifter.

Few of the Comets are to be ſeen in their Acceſs to the Sun, but in their Receſs appear with long fiery Tails, pointing directly, or nearly ſo, towards that Part of the Hea-

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ſuch Situation alſo, that the Comet ſhall appear in a Line that paſſes from one to the other; which are found as before: then they extend a Thread upon the celeſtial Globe, from one of the two firſt Stars to the other; and another Thread from one of the two laſt Stars to the other: and the Point on the Globe where the Threads croſs is the apparent Place of the Comet at the Time the Obſervation was made. This they do daily, and ſo trace out its apparent Courſe in the Heavens.

vens which with reſpect to the Comet is oppoſite to the Sun. Some are viſible before they reach the Sun, and begin to put forth their Tails, which at firſt are ſhort and thin, ſeldom exceeding fifteen or twenty Degrees in Length, but grow longer and denſer as the Comet comes nearer the Sun. If the Comet paſſes very near the Sun, it then ſends forth fiery Beams of Light every Way. After this it puts forth a Tail forty, fifty or ſixty Degrees long, which as the Comet recedes farther from the Sun, continually diminiſhes both in Length and Splendor; but is larger and longer at any Diſtance in its Receſs from the Sun, than at an equal Diſtance in its Acceſs to it.

This great Splendor and Length of the Tails, Sir *Iſaac Newton* thinks ariſes from the Heat which the Sun communicates to the Comet as it paſſes near it; and accounts for it after the following Manner. As the Aſcent of the Smoak in a Chimney is owing to the Impulſe of the Air, with which it is entangled (for the Air about a Fire being rarefied by the Heat thereof, has its ſpecifick Gravity thereby rendered leſs than that of the more diſtant Air: Upon this account it aſcends, and carries along with it the Smoak with which it is engaged) in like manner he thinks the Tail of a Comet may riſe from the Atmoſphere thereof into thoſe Parts which are oppoſite to the Sun, being carried up by the Æther about the

Comet, rarefied to a very great degree by the Heat thereof. And thinks his Opinion greatly confirmed by the Appearance of the Tails; for when accurately obſerved, they are found not to riſe always in a Direction preciſely oppoſite to the Sun, but to deviate or incline a little from thence towards thoſe Parts which the Comet has lately left; and not only ſo, but to be bent into a certain Curvature, the Extremities of the Tails deviating from the true Oppoſition more in proportion than the other Parts; and to be more denſe ſeemingly, and better defined on the convex, than on the concave Side: And farther, that the longer the Tail is, the more ſenſible is the Curvature, as being the greateſt at the greateſt Diſtance from the Body of the Comet. Upon theſe Accounts he thinks it evident, that the *Phænomena* of the Tails of Comets depend on the Motion of their Heads, and that the Heads furniſh the Matter which forms the Tails. For as in the Atmoſphere of the Earth, the Smoak of a Body aſcends perpendicularly if the Body be at reſt, and obliquely if the Body be moved any otherways than directly upwards, or downwards; ſo in the Heavens, where all the Bodies gravitate towards the Sun, Smoak and Vapour muſt aſcend from the Sun, and riſe perpendicularly, if the ſmoaking Body be at reſt; and obliquely, if the Motion of the Body be oblique to the Direction the Vapour aſcends with. And becauſe the Force by which the Vapour aſcends, is ſtrongeſt near the Body of the Comet, the Obliquity with which it aſcends will be the leaſt near the Body, and greater at greater Diſtances, and conſequently the Column will be incurvated or bent towards thoſe Parts the Comet leaves: And becauſe the Vapour in the preceding Side is ſomething more recent, that is, has aſcended ſomething more lately, it will be ſomething more denſe on that Side, and on that account muſt reflect more Light, as well as be better defined; the Vapour on the other Side languiſhing by degrees, and vaniſhing out of ſight (*d*).

This is the Sentiment of Sir *Iſaac Newton*; but I muſt beg Leave to differ from him in this Matter. For I think it ſomewhat improbable on account of the great Rapidity with which Comets move, that the Æther diffuſed throughout the heavenly Regions ſhould gravitate

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(*d*) Some have ſuppoſed, that the Heads of Comets are tranſparent, and that their Tails are no other than a Beam of the Sun tranſmitted through them: But were the Heads of Comets tranſparent, they themſelves would ſcarce be viſible. Others, that they ariſe from the Refraction of the Rays of Light in their Way from the Comets to us: But if ſo, then both the Planets and fixed Stars ought to have Tails alſo. *Kepler* aſcribed the Aſcent of the Tails to the Rays of the Sun carrying the Particles of the Comet's Atmoſphere away with them, that is, impelling them into the Regions oppoſite to it. But we have no Inſtance of any thing in Nature like this; this is therefore an *Hypotheſis* that cannot be ſupported.

gravitate ſufficiently towards the Sun (eſpecially at Diſtances equal perhaps to twice that of the Earth from it) to cauſe the Vapours, or the Atmoſphere of the Comets, to aſcend into thoſe Parts, towards which the Comets are tending. For when a Body aſcends in a Fluid, as being ſpecifically lighter than it, its Motion at or near the beginning of it, is very ſmall; ſo that if the Fluid have but a ſmall Degree of Motion the contrary Way, the Reſiſtance thereof will readily ſtop the Aſcent of the Body, and will carry it the ſame Way it moves itſelf. This is the Reaſon the Air, which cauſes the Flame of a Candle to aſcend, can ſcarcely blow downwards with ſo ſmall a Degree of Motion, or (which is all one) the Candle itſelf can ſcarcely be lifted up with ſo gentle an Hand, but the Flame will turn downwards. How is it then likely, that when a Comet moves with an incredible Velocity (*viz.* at the Rate of above a thouſand Miles in a Minute, as ſome of them do at the Time when their Tails riſe almoſt directly before them:) How is it then likely, I ſay, that the Æther ſhould by its Gravity alone raiſe the Vapour of the Comet with ſuch Force, as to cauſe it to overcome its Reſiſtance, when that Reſiſtance ariſes from ſo great a Rapidity of the Comet? Would it not rather carry it with it the other Way? And if we augment the Denſity of the Æther, and therewith its

Gravity, then the Reſiſtance of it will be the greater; and ſo the Caſe will be more deſperate ſtill. If we ſuppoſe it to be rarer, it will be lighter, and ſo leſs able to buoy the Vapours of the Comet up. The Queſtion then is, how are thoſe Tails to be accounted for? My Conjecture is as follows: It is well known that when the Light of the Sun paſſes through the Atmoſphere of any Body (as the Earth) that which paſſes on one Side is by the Refraction thereof made to converge towards that which paſſes on the oppoſite one; and this Convergency is not wholly effected, either at the Entrance of the Light into the Atmoſphere, or at its going out; but that beginning at its Entrance, it increaſes in every Point of its Progreſs. It is alſo agreed on all Hands, that the Atmoſpheres of the Comets are very large and denſe. I ſuppoſe therefore, that by ſuch time as the Light of the Sun has paſſed through a conſiderable Part of the Atmoſphere of a Comet, the Rays thereof are ſo far refracted towards each other, that they now begin ſenſibly to illuminate it (or rather the Vapours floating therein) and ſo render that Part which they have yet to paſs through, *viſible* to us; and that this Portion of the Atmoſphere of a Comet thus illuminated, appears to us in the Form of a Beam of the Sun's Light, and paſſes under the Denomination of a Comet's Tail.

Thus, when a Burning-Glaſs is expoſed to the Sun's Rays, if there be the leaſt Smoak or Duſt hovering about the Glaſs, the Rays of the Sun, as ſoon as they are brought near enough together by the refractive Power of the Glaſs, render ſo much of the Smoak or Duſt as they have yet to paſs through, very diſtinguiſhable from the reſt. Indeed, by the accurate Reflection of the Glaſs, the Light of the Sun is brought to a *Focus*, wherefore the illuminated Space ends in a Point; but it happens not ſo to Rays refracted by the Atmoſphere of a Body; becauſe the Extremities of an Atmoſphere do not refract near ſo ſtrongly as thoſe Parts which are nearer the Body. Let us now ſee how this Hypotheſis will anſwer the *Phænomena* of the Tails.

When a Comet has been in its *Aphelion*, by means of its long Abſence from the Sun, the Vapours with which its Atmoſphere was full, are now condenſed and fallen down, the Atmoſphere itſelf grown cold, ſettled into its natural State, and reduced to a ſmall Compaſs; therefore as the Comet approaches the Sun, the Tail of it is inviſible to us, or at leaſt appears very ſhort: When it comes into the hotter Regions near the Sun, the Atmoſphere begins to rarefy and dilate itſelf, and new Vapours riſing up into it, encreaſe the Quantity of Matter therein, on which account the Tail grows longer. When the Comet is in its *Perihelion*, if it be then very near the Sun, by means of the exceſſive Heat thereof, not only the hinder Parts of its Atmoſphere are ſtrongly illuminated by refracted Light, but the other Parts of it by the direct Light of the Sun, ſo that the whole Comet, Atmoſphere and all, is as it were on Fire; which cauſes an infinite Number of Beams and Irradiations to be ſent out from every Part; in which State the Comet is ſaid to be *Hairy*. Afterwards when the Comet has got to ſome Diſtance from the Sun, and this great Illumination ceaſes, and the Atmoſphere is extended by Rarefaction to a monſtrous Size, and now more replete with Vapours than ever, which like the Steam of a boiling Cauldron have been continually riſing into it, the Rays of the Sun have a great Way to go within it, after they are ſo far refracted towards each other, as to render the Atmoſphere viſible. So that the Tail now appears at its greateſt Length; but from this Time grows continually ſhorter and ſhorter (as the Atmoſphere by the Comet's loſing its Heat contracts its Dimenſions, and the Vapour floating therein ſubſides on to its Body) till the Comet is out of Sight.

But the greateſt Difficulty is yet behind, and that is, to account for the Deviation of the Tail of the Comet from thoſe Parts which are oppoſite to the Sun, towards thoſe which the Comet has left. For it will moſt certainly be thought, that, upon this Hypotheſis, the *Axis* of the refracted Rays, which is alſo the *Axis* of the Tail, ought to be in a right Line, that paſſes through both the Center of the Sun and of the Comet. Now if we conſider the exceeding great Rapidity with which a Comet moves, and that the Propagation of Light is not inſtantaneous, this Difficulty will alſo vaniſh. For ſince Light is propagated in Time, that which paſſes near the Body of a Comet will not, when it comes at the Extremity of its Atmoſphere, enlighten a Portion of it that lies in a right Line drawn through the Center of the Sun and the Place of the Comet where it now is, but where it was at the Inſtant when that Light paſſed by it: And therefore the illuminated Point of the Extremity of the Comet's Atmoſphere, that is, the End of its Tail, will not be oppoſite to the Sun, but always in a right Line, that paſſes through the Center of the Sun and a Point of the Orbit behind the Comet, or which it has lately left; as will readily appear to any one that ſhall duly attend to this matter. And becauſe this is true of every other Portion of the Tail, in proportion to its Diſtance from the Head, the whole Tail will decline from thoſe Parts which are oppoſite to the Sun, towards thoſe which the Comet has lately left. And thus is the abovementioned

tioned Deviation to be accounted for. But it muſt not be concealed here, that that very ſucceſſive Propagation of Light, which occaſions this Deviation, is capable of augmenting or diminiſhing it in Appearance, or even of entirely taking it away, according to the Situation the Comet is viewed from. For, on account of the ſucceſſive Propagation of Light, all diſtant Bodies that are in Motion appear not ſo far advanced as they really are; and the faſter they move, and the farther they are from us, the greater is the Difference between their real and apparent Place. Let us then ſuppoſe, that the Extremity of a Comet's Tail is farther from the Eye of a Spectator than the Body of the Comet, by a Space equal to the Semidiameter of the Earth's Orbit, then will Light be ſeven Minutes longer in coming from the Extremity of the Tail than from the Head; and therefore the Tail will appear to be farther behind its true Place than the Comet does, by a Space equal to that through which the Extremity of the Tail moves in ſeven Minutes; and conſequently the apparent Deviation in this Caſe will conſpire with the true one, and render it in appearance proportionably larger. Again, when the Extremity of the Tail is nearer the Earth than the Comet, then Light comes quicker from the former than from the latter, and therefore the Comet appears farther behind its true Place than the

Extremity of its Tail does, and ſo the true Deviation appears the leſs, or not at all, according as the Caſe may happen.

The apparent Deviation of a Comet's Tail depends alſo on another Circumſtance, *viz.* the Situation of the Spectator with reſpect to the Plane of the Comet's Orbit; for when a Spectator is in the Plane of that Orbit, the Deviation becomes inſenſible to him, as not being able to diſtinguiſh nicely the exact Diſtances of the ſeveral parts of the Tail from his Eye: the farther he is removed out of that Plane, the more ſenſible it becomes.

There is another Particular, *viz.* that the Extremities of the Comet's Tail deviates from the true Oppoſition more in proportion than the other Parts, ſo that they are bent into a certain Curvature: and farther, that they are more lucid and diſtinct on the convex than on the concave Side. All which may be accounted for, if we allow the heavenly Regions the leaſt imaginable Reſiſtance; for in that Caſe, the Atmoſphere of a Comet will be ſomewhat denſer on the Side towards which the Comet tends; and therefore the Light which paſſes on that Side will ſuffer a greater Degree of Refraction throughout its whole Progreſs along the Atmoſphere, that is, to the end of the Tail, than ſuch as paſſes on the other: ſo that, that Side of the Tail which moves firſt, will be more lucid and better defined than the other: And becauſe this Reſiſtance will make the greateſt Alteration in the extreme Parts of the Comet's Atmoſphere, the Extremity of the Comet's Tail will deviate from the true Oppoſition more in Proportion than thoſe Parts which are nearer the Body; ſo that the Tail (at leaſt that ſide of it which moves firſt) will become convex. And if the Refraction of thoſe Rays which paſs on that ſide towards which the Comet tends, and on that account are moſt refracted, be ſo great as to cauſe them to croſs thoſe which paſs on the other ſide the Comet, before they get out of its Atmoſphere, their Refraction being irregular, they will croſs the other diſperſedly and in ſeveral Places, which, as is obvious to conceive, will neceſſarily give that other ſide of the Tail a concave Form, and render it imperfectly defined.

Thus I have ventured to throw in a Conjecture of my own; but propoſe it rather as a *Quere*, than a Solution; leaving it to be farther examined into by thoſe who may have had better Opportunities of acquainting themſelves with *Phænomena* of this Kind. *Valeat quantum valere poteſt.*

CHAP. XII.

Of the Parallax *of the* Heavenly Bodies.

THE Difference between the apparent Places of the Heavenly Bodies, when viewed from the Center of the Earth, and when ſeen from the Surface thereof, is called their *Parallax*. To explain this,

Let AB (Fig. 21.) repreſent a Portion of the Earth's Surface whoſe Center is C, HV the Heavens, and let APN be the Horizon of a Spectator at A; and P the Situation of a Planet therein. The Place of this Planet among the fix'd Stars, as ſeen from the Center of the Earth, will be M, which is called its *true* Place: But when view'd by a Spectator upon the Surface of the Earth at A, it appears in the Horizon at N, which is called its *apparent* Place; and the Arch MN, which meaſures the Diſtance between the one and the other, is the *Parallax* of that Planet. If the Planet be above the Horizon, as at Q, its true Place as ſeen from the Center of the Earth is R, its apparent Place S, and its *Parallax* is RS, which is leſs than before. From whence we ſee, the more the Planet is elevated above the

Horizon, the leſs is its Parallax; and when it is got directly over the Place of the Spectator, it has no Parallax at all; for when it is at T, its apparent Place in the Horizon is at V, whether it be ſeen from C or A.

Farther, by how much the more remote a Planet is from the Earth, ſo much the leſs is its Parallax, at the ſame Heights from the Horizon. Thus if the Planet had been at the Diſtance C D from the Center of the Earth, its Parallax, when in the Horizon at D, would have been NO; and when in the Line AQ produced, as at E, its Parallax would have been meaſured by the Arch SF, both which are leſs than the former.

The Angles MPN, and RQS, are called the *parallactic* Angles, ſuppoſing the Planet at P or Q, and are equal, the firſt to the Arch MN, and the other to the Arch RS: They are alſo equal to the Angle APC, or AQC, which are thoſe under which CA, Semidiameter of the Earth that paſſes through the Place of the Spectator, would appear if ſeen from the Planet.

From hence it is obſervable, that the apparent Place of a Planet is always lower in the Heavens than its real one, except when the Planet is vertical, or at T.

As Aſtronomers, by means of the annual Parallax of a Planet, that is, the Parallax of the annual Orbit of the Earth, as explained above *(a)*, compute the Diſtance of the Planet from the Sun, with reſpect to that of the Earth from thence; ſo by means of this Parallax, they calculate the Diſtance of the Planet from the Center of the Earth with reſpect to the Diſtance of the Surface of the Earth from the ſame *(b)*.

The Sun is alſo liable to this Kind of Parallax; for it is evident that its Place among the fix'd Stars, when ſeen from the Surface of the Earth, may be different from that in which it would appear if ſeen from the Center of the Earth. Now could this Parallax be determined with Accuracy, the exact

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(a) See Chap. VI. and VII.

(b) The Parallactic Angle APC being known, and the Angle VAN, which is meaſured by the Arch of the Heavens VN, being taken by Obſervation, which gives its Complement to two right ones PAC, from theſe two the third Angle PCA of the Triangle APC is had. And conſequently by the Relation PC (the Diſtance of the Planet) bears to CA, the Semidiameter of the Earth may be found.

There are ſeveral Methods of finding the Parallactic Angle; one of which is as follows:

Firſt, let it be obſerved when the Planet is between two fix'd Stars which are both in the ſame vertical Circle; and afterwards, when theſe two Stars appear ſituated in a Line that is parallel to the Horizon, obſerve how much the Planet appears below them; for the Planet being depreſſed by the Parallax, will at that time be lower than a Line drawn from one Star to the other; becauſe the fix'd Stars are at too great a Diſtance to ſuffer any Parallax: The Arch of the Heavens intercepted between this Line and the apparent Place of the Planet is its Parallax; allowance being made for the Motion of the Planet between the two Obſervations.

Diſtance of the Sun from the Earth might be known, and from thence, by the Method explained above *(c)*, that of the other Planets from the Sun. But the Sun's Diſtance from the Earth is ſo very great, and of conſequence its Parallax is ſo very ſmall, that the Obſervations neceſſary for taking of it cannot be made with ſufficient Accuracy. Aſtronomers therefore have recourſe to the Parallaxes of *Mars* and *Venus*; which if they could be found, that of the Sun would alſo be had: Becauſe the Horizontal Parallaxes of the Heavenly Bodies, as we have juſt been ſhewing, are proportionably leſs as their Diſtance is greater. Now when *Mars* is oppoſite to the Sun, it is above twice as near to us, and its Parallax is above twice as great as that of the Sun. And *Venus*, when in its inferior Conjunction with the Sun, is almoſt four times nearer than the Sun, and therefore its Parallax is ſo many times greater. Aſtronomers find, that the horizontal Parallax of *Mars*, when it is in Oppoſition to the Sun, does not exceed 25 Seconds, and from thence conclude the Sun's to be about 10 Seconds. With which Parallax of 10 Seconds if a Calculation be made, the Sun's Diſtance will be found about 81 Millions of Miles from the Center of the Earth.

However, Aſtronomers need not be ſollicitous about the Parallax of *Mars*, ſince Dr. *Hal-*

(c) See Chap. VI, VII.

Halley has laid down a Method in the Philoſophical Tranſactions (*d*), whereby the Parallax of the Sun may be had to ſo great a Nicety, that its true Diſtance within a 50th Part of the Whole may be determined from it; *viz.* At the next Tranſit of *Venus* over the Sun's Diſk, which will happen on the 26th of *May*, in the Year 1761; when, at five Minutes before Six in the Morning, *Venus* will appear not above four Minutes of a Degree South of the Middle of the Sun's Diſk.

(*d*) Numb. 348.

CHAP. XIII.

Of the Refraction *of the* Atmoſphere, *and the* Crepuſculum *or* Twilight.

WE find by Experience, that when Light enters a tranſparent Medium obliquely, either denſer or rarer than that in which it was before, it does not paſs ſtrait on, but its Courſe is bent at the Point where it enters, which bending is call'd its *Refraction*; and if the Medium into which it enters be denſer than the other, the Light is then refracted towards a Perpendicular to the Surface drawn through the Point where it enters; but if the Medium be rarer, it is then refracted or bent the contrary way (*a*). It is from hence that a Staff, having one End under Water and the other above it, appears bent or broke at the Surface of the Water, the Part below the Water appearing above its natural Situation, becauſe the Light that comes from thence is refracted or bent downwards at the Surface of the Water, where it comes out. Thus alſo the Stars near the Horizon appear above their true Places, on account of the Refraction which the Light that comes from them ſuffers in paſſing through the Atmoſphere of the Earth (*b*). The Manner of which is now to be explained.

Let then AB (*Fig.* 22.) repreſent the Surface of the Earth, C its Center, DF a Portion of its Atmoſphere, and GI the Sphere of the fixed Stars, and let AH be the Horizon of a Spectator at A, and let there be a fixed Star or a Planet at K; and let KL be a Ray of Light proceeding from hence. Now this Ray paſſing out of a Vacuum, or empty Space, into the Earth's Atmoſphere at L, will be refracted towards the Line LC, that being a Perpendicular to the Surface of the Atmoſphere, at the Point where the Ray enters. And ſince the ſuperior Air is rarer than the inferior, the Ray is conti-

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(*a*) See the Reaſon of this aſſigned, and the Manner of it largely explained, Part III. Chap. II.

(*b*) This has been already taken Notice of in Part III. Chap. VII. in the Notes.

continually entring a denſer Medium all the way it goes; and ſo is refracted every Moment downwards towards C, as at firſt, which will cauſe it to deſcribe a Curve, as LA, and to enter the Spectator's Eye at A, as if it came from E, a Point above L; and ſince an Object appears always in that Line in which the Ray paſſes when it enters the Eye *(c)*, the Star will ſeem to be at G, and therefore higher than its true Place, and ſometimes above the Horizon, when its true Place is below it; as in the Caſe before us, where its Situation K is ſuppoſed to be below the Horizontal Line AH.

Thus it has often happen'd, that in an Eclipſe of the Moon, at which Time it is directly oppoſite to the Sun, they have both appeared above the Horizon at the ſame Time.

When a Star is in the Zenith, it is liable to no Refraction; for then the Light that comes from it to a Spectator's Eye, enters the Atmoſphere in a Direction perpendicular to its Surface: But the lower the Star is, the more obliquely the Light enters the Atmoſphere; ſo that the greateſt Refraction of all is when the Star is in the Horizon, and it is then 33 Minutes and 45 Seconds; but when the Star is 50 Degrees or more above the Horizon, its Refraction is inſenſible *(d)*.

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(c) See the Reaſon of this, Part III. Chap. VIII. at the Beginning.

(d) See Sir *Iſaac Newton*'s Table of Refractions. Philoſoph. Tranſact. Numb. 368.

From hence we ſee the Reaſon why the Sun and Moon, when near the Horizon, do not appear of a circular Form, as at other Times, but of an Oval one, having their longeſt Diameter parallel to the Horizon. For their lower Parts appear elevated by Refraction more than their upper ones, and conſequently nearer together than they ought to do; while their Sides are equally elevated, and keep their juſt Diſtance from each other.

It is not with regard to Refraction, as in the Parallax: The latter of which, as explained in the preceding Chapter, depreſſes the Sun only and the Planets; and that according as they are more or leſs removed from the Earth; while the former elevates all the Heavenly Bodies alike, at whatever Diſtance they are removed from us.

Farther; were it not for the Atmoſphere, which continually reflects the Light of the Sun from one Portion of it to another, and thereby occaſions ſome of it to enter our Eyes, which way ſoever they are turned, and alſo throws it upon thoſe Parts of the Bodies about us which are not expoſed to the direct Light of the Sun's Beams, the whole Heavens would appear totally dark; and ſuch Bodies, or ſuch Parts of Bodies, as the Sun did not directly ſhine upon, would be inviſible, and the Stars would appear at Noon-day. But the Atmoſphere of the Earth being ſtrongly illuminated by the Beams of the

Sun, they are reflected to us, and to other Bodies from all Parts; ſo that the whole Atmoſphere is enlightened, and every thing therein is render'd viſible; while the fainter Light of the Stars, which would otherwiſe appear, is obſcured and eclipſed thereby.

Beſides, though the Sun immediately before it ſets, would in this Caſe ſhine with the ſame Brightneſs as at Noon, yet the Moment it was ſet, it would be quite dark. Whereas, by means of the Atmoſphere it happens, that although none of the Sun's direct Rays can come to us after it is ſet, yet we ſtill enjoy its reflected Light for ſome Time, and Night approaches by Degrees. For after the Sun is hid from our Eyes, the upper Parts of the Atmoſphere remain for ſome time expoſed to its Rays, and from thence the whole is illuminated by Reflection. But as the Sun grows lower and lower, that Portion of the Atmoſphere which is above our Horizon becomes leſs enlighten'd, till the Sun has got eighteen Degrees below it, after which it ceaſes to be illuminated thereby (*e*), till it

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(*e*) Hence it is, that during that Part of the Year in which the Sun is never eighteen Degrees below the Horizon, there is a continued Twilight from Sun-ſetting to Sun-riſing. Now that Part of the Year, in the Latitude of *London*, is while the Sun is paſſing from about the fifth Degree of *Gemini* to the twentieth of *Cancer*, that is, from about the 15th of *May* to about the 7th of *July*.

Some Philoſophers tell us, that the Height of the Earth's Atmoſphere may be determined from the Duration of the Twilight, in the following Manner. Let ABD (Fig. 23.) repreſent the Earth it has got within as many Degrees of the Eaſtern Side of the Horizon, at which Time it begins

Earth ſurrounded by its Atmoſphere GEF. Through A, the Place of a Spectator, draw the Tangent AH paſſing through the Extremity of the Atmoſphere at E, from whence draw the Line ES, touching the Surface of the Earth in B, as alſo from the Center C, the Lines CA, CE and CB. Now, ſay they, ſince the Crepuſculum is owing to the Reflection of the Sun's Rays, by the Particles of which the Atmoſphere conſiſts, it muſt end at ſuch time as Rays of Light coming from the Sun, and paſſing cloſe by the Surface of the Earth, are reflected from the Top of the Atmoſphere to the Spectator's Eye; and therefore knowing how far the Sun is below the Horizon at that time, we may know the Height of the Atmoſphere. Thus let S be the Place of the Sun when the Crepuſculum ends, SE a Ray of Light paſſing cloſe by the Earth at B, and reflected from E (a Particle at the Top of the Atmoſphere) into the horizontal Line EA, to the Eye of the Spectator at A; then will the Angle HES be the Meaſure of the Sun's Diſtance below the Horizon. And becauſe EB and EA are Tangents to the Earth, the Angle ACB at the Center will be equal to HES, or eighteen Degrees; and half of the former, namely ACE, will be nine, from whence EI, the Height of the Atmoſphere, which is the Exceſs of the Secant EC above the Radius CA or CI, is had by Trigonometry: ſee *Keil's* Aſtronomy, Lect. 20. But this way of arguing is founded on a Suppoſition, that Rays of Light cannot come to a Spectator's Eye after more than one Refraction in the Atmoſphere; the contrary to which ſeems evident, as I ſhall ſhew in the following manner. Let ABD (Fig. 24.) repreſent the Earth, A the Place of a Spectator, HO his Horizon, MNR a Portion of the Earth's Atmoſphere, and TVW an Arch of the Heavens; and let S be the Place of the Sun leſs than 18 Degrees below the Horizon; and let the Line SAI touching the Earth be drawn. It is well known that in this Caſe, ſo much of the Heavens as is not intercepted by the Earth from the Spectator's Eye at A, will ſtill appear enlightened; that is, in other Words, Light will flow to the Spectator's Eye from every Part of that Portion of the Atmoſphere that lies between P and Q; ſome therefore will come from thoſe Particles that lie within the Space PAE. But theſe Particles cannot be enlightened by the direct Rays of the Sun; for they neceſſarily gins to illuminate the Atmoſphere again, and in Appearance to diffuſe its Light throughout the Heavens, which continues to encreaſe till the Sun be up. This Light, whether in the Morning before Sun-riſe, or in the Evening after Sun-ſet, is called the *Crepuſculum*, or Twilight; and is ſuppoſed to begin and end when the leaſt Stars that can be ſeen by the naked Eye (*viz.* thoſe of the ſixth Magnitude) ceaſe or begin to appear.

But as the Twilight depends on the Quantity of Matter in the Atmoſphere fit to reflect the Sun's Rays, and alſo on the Height of it, (for the higher the Atmoſphere is, the longer it will be before the upper Parts of it will ceaſe to be illuminated) the Duration of it will be various. For Inſtance in Winter, when the Air is condenſed with Cold, and the Atmoſphere upon that account lower, the Twilight will be ſhorter; and in Summer, when the Limits of the Atmoſphere are extended by the Rarefaction and Dilatation of the Air of which it conſiſts, the Duration of the Twilight will be greater. And for the like Reaſon the Morn-

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ceſſarily paſs above the Line AE, theſe therefore themſelves are illuminated by reflected Light. Since then theſe afford Light, and nearly as much as any of the reſt, it is evident, that Rays may come to a Spectator's Eye, and in great Plenty, after having ſuffered two Reflections. And therefore that Demonſtration, which is built upon a Suppoſition that ſuch Rays can ſuffer but one, muſt fall to the Ground.

ing Twilights, the Air being at that Time condenſed and contracted by the Cold of the preceding Night, will be ſhorter than the Evening ones, when the Air is more dilated and expanded.

Another Effect that the Atmoſphere has upon the Rays of the Sun is, that it intercepts a conſiderable Part of them in their Progreſs through it. By which Means it happens, that when the Sun is near the Horizon, it appears leſs luminous and bright than when it is more elevated; for the nearer the Sun is to the Horizon, the greater is the Portion of the Atmoſphere through which the Rays of the Sun muſt paſs to come to a Spectator's Eye. For Inſtance, in Fig. 23, where ABD repreſents the Earth, ſurrounded by its Atmoſphere GEF, and A the Place of a Spectator; it is obvious, that when the Sun is in or near the Horizon, ſuppoſe at H, its Rays muſt paſs through a greater Portion of the Atmoſphere to come to the Spectator's Eye at A, than they would do if the Sun were ſituated in any Part of the Heavens above the ſaid Point.

CHAP. XIV.

Of the Doctrine *of the* Sphere.

THE Subject of this Chapter relates chiefly to the Explanation of thoſe Circles which Aſtronomers, for the better diſtinguiſhing the Places and Motions of the heavenly Bodies, conceive to be deſcribed within the Sphere of the Heavens. Some of theſe have already been defined; but that the Reader may have a compleat View of their Definitions all together, I ſhall repeat them here in their proper Places.

Theſe Circles are of two Kinds, *viz.* the *greater* and the *ſmaller*.

A great Circle is that which divides the Sphere into two equal Parts, having the ſame Center therewith. A great Circle.

A leſs Circle divides the Sphere into unequal Parts, having a different Center from that of the Sphere. A leſs Circle.

Theſe are ſometimes called parallel Circles, and receive their Denominations from ſome great Circle to which they are parallel. Thus, leſſer Circles Parallel Circles.

running parallel to the Equator, are called Parallels of the Equator; and ſo of the reſt.

The Poles of a Circle. The Poles of a Circle are two Points in the Surface of the Sphere, every where equally diſtant from the Circle.

Axis of a Circle. The Axis of a Circle is an imaginary Line ſuppoſed to paſs through both its Poles: Or, that paſſes through its Center, and is perpendicular to its Plane.

Secondaries of a Circle. The Secondaries of a Circle are ſuch as paſs through both its Poles. Theſe therefore cut it at right Angles in two Points oppoſite to each other, and are therefore always great Circles.

The Circles of the Sphere are alſo diſtinguiſhed into fixed and moveable.

A fixed Circle. A fixed Circle is that whoſe Situation depends not upon that of the Spectator.

A moveable Circle. A moveable Circle is that whoſe Situation depends on that of the Spectator.

Of the former Sort are the Ecliptic and the Equator, with their Secondaries and Parallels.

Of the latter are the Horizon, with its Secondaries and Parallels.

I. *Of*

I. *Of the* Ecliptic, *and other Particulars relating to it.*

The Ecliptic is that Circle which the Plane of the Earth's Orbit would mark out, were it ſuppoſed to be extended to the Heavens. This therefore is that which the Sun ſeems to deſcribe in its annual Courſe. **The Ecliptic.**

The Axis of the Ecliptic is a right Line ſuppoſed to paſs through the Centre of the Sun, and to be perpendicular to the Plane of the Ecliptic; and the Points in the Heavens, to which this Axis points, are called the Poles thereof; and the great Circles paſſing through theſe Poles, will be perpendicular to the Ecliptic, and are therefore called its Secondaries, and ſometimes Circles of Longitude. By Means of theſe the Place of any Star is referred to the Ecliptic, and its Longitude and Latitude eſtimated. For the Longitude of a Star is an Arch of the Ecliptic, intercepted between the firſt Point of Aries, and that where one of theſe Secondaries that paſſes through the Star cuts the Ecliptic. And the Latitude of a Star **The Axis of the Ecliptic.** **Its Poles.** **Secondaries of the Ecliptic.** **Longitude of a Star.** **Latitude of a Star.**

Parallels of Latitude.

a Star is the Arch of the ſame Secondary, intercepted between the Star and the Ecliptic. Upon this account all Circles conceived to be drawn parallel to the Ecliptic, that is, Parallels of the Ecliptic, are ſometimes called Parallels of Latitude. The Latitude of a Star is either Northern or Southern, as the Star is placed on this or that Side the Ecliptic.

The Signs of the Ecliptic.

The Ecliptic is divided into twelve equal Parts, called Signs. They begin at that Point in which the Sun appears to be at the Vernal Equinox, and are reckoned from Weſt to Eaſt. The Names of the ſaid Signs, together with the Characters whereby they are commonly expreſſed, are theſe that follow:

Aries,	Taurus,	Gemini,	Cancer,	Leo,	Virgo,
♈	♉	♊	♋	♌	♍
Libra,	Scorpio,	Sagittarius,	Capricornus,	Aquarius,	Piſces.
♎	♏	♐	♑	♒	♓

The Zodiac.

The firſt ſix of theſe are called the Northern Signs, as poſſeſſing that Half of the Ecliptic which lies neareſt the North Pole of the Earth; the latter, the Southern, as being in that which is neareſt the South Pole. The Sun is always ſeen in ſome Part of this Circle;

but the Planets, by reaſon of the Inclination of the Plane of their Orbits to that of the Ecliptic, appear ſometimes above it, and ſometimes below it, but never deviate from it above ſeven or eight Degrees on either Side; So that if we imagine a Space ſurrounding the Heavens ſixteen Degrees in Breadth, and extended equally above and below the Ecliptic, this will include all their Ecliptics, if I may ſo ſay; that is, all the Circles they would appear to move in when ſeen from the Sun, and is called the Zodiac.

The Signs of the Ecliptic took their Names, as was obſerved above, from Twelve Conſtellations ſituated in the Heavens near thoſe Places. But it muſt be remembred, that the Signs are different from the Conſtellations which bear the ſame Name. For the *Sign* Aries for Inſtance, is not in the ſame Part of the Heavens with the Conſtellation Aries. The Sign Aries is only thirty Degrees of the Ecliptic, counted from one of the Equinoctial Points; whereas the *Conſtellation* of that Name is a Syſtem of Stars, the moſt of which are now ſituated between the firſt and laſt Degree of Taurus. And ſo of the other Signs. Theſe Conſtellations were in the Infancy of Aſtronomy ſituated within the Signs which now bear their Names; but by a ſlow retrograde Motion of the Equinoctial Points, called the *Preceſſion of the Equinoxes*, as explained above (*a*), the Conſtellation Aries has got into the Sign Taurus, and that of Piſces into the Sign Aries; and ſo of the reſt.

II. *Of the* Equator, *and other Particulars relating to that.*

The Axis of the Earth. The Axis of the Earth is a right Line ſuppoſed to paſs through its Center, about which its daily Rotation is performed.

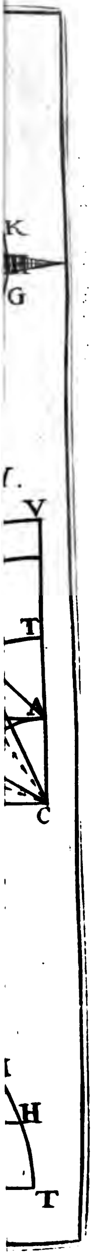
The Poles of the Earth. The Poles of the Earth are two Points in its Surface, where its Axis paſſes through; the one the North, the other the South Pole.

The Poles of the Heavens. The Poles of the Heavens are two Points therein, where the Axis of the Earth, if produced, would paſs through them.

The Equator. The Equator is a great Circle on the Surface of the Earth, equally diſtant from either Pole thereof. This is by Mariners called the *Line*.

A

(*a*) See Chap. V.


K
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Meridian.

A Secondary of the Equator paſſing through any particular Place upon the Surface of the Earth, is called the *Meridian of that Place.*

And as the Longitude and Latitude of the heavenly Bodies are determined by the Ecliptic and its Secondaries; ſo by the Equator and its Secondaries, Places upon the Earth's Surface are diſtinguiſhed as to their Longitude and Latitude.

The Longitude of a Place.

The Longitude of a Place is then an Arch of the Equator, intercepted between the firſt Meridian (*b*), and that which paſſes through the Place.

The

(*b*) By the *firſt Meridian* is meant that which paſſes through ſome remarkable and well-known Place, which Geographers pitch upon to eſtimate the Longitude of other Places from. The ancient Geographers called that the firſt Meridian which paſſed through the moſt Weſtern Part of the then known World; and ſo reckoned the Longitude Eaſtward from thence. And as they look'd upon the World to be longer from Weſt to Eaſt than from North to South, they gave the Name of Longitude to the longer Dimenſion, and that of Latitude to the ſhorter. But when it was diſcovered that the Earth was round, and conſequently that there was no extream Point of Longitude, Geographers began to neglect this Method, and every one choſe a firſt Meridian according to his Fancy and Inclination. Thus, an *Engliſh* Geographer ſhall tell you the firſt Meridian paſſes through *London*; a *Frenchman*, that it paſſes through *Paris*, &c.

This

The Latitude of a Place. The Latitude of a Place is an Arch of the Meridian, intercepted between the Place and the Equator; and is either Northern or Southern, as the Place lies on this or that Side the Equator.

The Equinoctial. The Equinoctial and the Equator are different, the former being a Circle in the Heavens, which the latter would mark out if ſuppoſed to be extended thither.

Northern and Southern Hemiſpheres. The Equinoctial divides the Heavens as the Equator does the Earth, into two equal Portions, called the Northern and Southern Hemiſpheres.

Right Aſcenſion and Declination. And as by the Secondaries of the Equator, Places upon the Earth are referred to the Equator; ſo by theſe of the Equinoctial, the Places of the heavenly Bodies are referred to the Equinoctial. But as an Arch of the Equator, intercepted between the firſt Meridian and that which paſſes through any Place, is the Longitude of that Place; an Arch of the Equinoctial intercepted between the firſt Point of Aries and that where it is cut by a Secondary

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This arbitrary Way of reckoning the Longitude from different Places, makes it neceſſary whenever Mention is made of the Longitude of any Place, to mention alſo the Place from whence that Longitude is counted.

dary of the Equinoctial that paſſes thro' the ſaid Body, is called its Right Aſcenſion. And an Arch of the ſame Secondary intercepted between the ſame Body and the Equinoctial, is called its Declination, which is alſo either Northern or Southern, as it happens to be in the one or the other Hemiſphere.

Declination.

From hence the Secondaries of the Equinoctial are ſometimes called Circles of Aſcenſion and Declination.

Circles of Aſcenſion and Declination.

The Equinoctial Points are thoſe where the Equator cuts the Ecliptic; and are ſo called, becauſe when the Sun is in theſe Points, it is alſo in the Equator, and therefore makes the Days and Nights of an equal Length all the World over. The Solſtitial Points are thoſe Points in the Ecliptic where the Equator is moſt diſtant from it; and are ſo called, becauſe when the Sun is in them, it can ſcarce be perceived to alter its Diſtance from the Equator for ſome time.

Equinoctial and Solſtitial Points.

There are two of the Secondaries of the Equinoctial more remarkable than the reſt, called the *Colures*; the one paſſes through the Equinoctial Points, and is called the Equinoctial Colure; the other is placed at right Angles with this, and paſſes through both the Poles of the Equator and of the Ecliptic too, and is called the Solſtitial Colure.

Hour Circles. Secondaries of the Equinoctial drawn through each 15th Degree thereof, will divide it into 24 Parts; each of which, ſince the Earth revolves once round its Axis in 24 Hours, will anſwer to one Hour; that is, each of them (ſuppoſing the Heavens to revolve as they appear to do) will ſucceed into the Place of a foregoing one in the Space of one Hour; and are for that Reaſon called Hour Circles. And the Inhabitants of any Part of the Earth, whoſe Meridian coincides with any one of theſe, will differ in their reckoning of Time one Hour from thoſe whoſe Meridian is coincident with the next; and ſo on.

Tropics and Polar Circles. Of the Parallels of the Equator, four are more remarkable than the reſt, and diſtinguiſhed by particular Names, *viz.* the Tropics and Polar Circles.

The Tropics. The Tropics are thoſe Circles which the Sun in its greateſt Declination ſeems to deſcribe by its diurnal Motion; and are therefore ſo far removed from the Equator, the one on the one Side, and the other on the other, as is the Inclination of the Equator to the Ecliptic, *viz.* 23 Degrees and an half. They are called Tropics, becauſe when the Sun arrives at either of them, it returns back again towards the Equator.

That which lies in the Northern Hemiſphere is called the Tropic of Cancer, becauſe it touches the Ecliptic in the firſt Degree of that Sign. The other, the Tropic of Capricorn, for a like Reaſon.

Tropic of Cancer, and Tropic of Capricorn.

The polar Circles are thoſe which the Poles of the Ecliptic ſeem to deſcribe by the diurnal Motion of the Earth; and are therefore each of them at the ſame Diſtance from the Poles of the Earth that the Tropics are from the Equator. That which lies in the northern Hemiſphere is called the Arctic Circle, from *Arctus*, the Bear, a Conſtellation near that Place; the other being in Poſition oppoſite to this, is called the Antarctic Circle. For the ſame Reaſon the North Pole of the Earth is ſometimes called the *Arctic Pole*, the other the *Antarctic*.

The Polar Circles.

Arctic and Antarctic Circles.

Arctic and Antarctic Poles.

By means of the Tropic and Polar Circles (or rather by certain imaginary Circles upon the Surface of the Earth correſponding to them, and called by the ſame Names) the Surface of the Earth is divided into five Parts, called *Zones*. The one of which is called the torrid Zone, and is that Space or Tract

Torrid Zone.

of the Earth which is comprehended between the two Tropics. The Ancients imagined this Tract of the Earth to be uninhabitable, on Account of the Heat of the Sun there. There are two frigid Zones, the one is that Portion of the Earth's Surface which is included within the arctic Circle, the other within the Antarctic. The remaining two are the temperate ones, lying one on one Side the Equator, and the other on the other, between the frigid and the torrid ones.

Frigid Zone.

Temperate Zones.

The Inhabitants of theſe Zones are diſtinguiſhed by the different Direction of their Shadows. Thus, they that live in the torrid Zone are called Amphiſcii; becauſe their Noon Shadows are caſt ſometimes towards the North, and ſometimes towards the South, according as the Sun at Noon is to the Northward or the Southward of them. But when the Sun paſſes directly over their Heads at Noon, they caſt their Shadows neither the one way nor the other, and are called Aſcii.

The Amphiſcii.

The Aſcii.

The Inhabitants of the temperate Zones are called Heteroſcii, becauſe they never caſt their Meridian Shadow but one way. Such as inhabit the frigid Zones are called Periſcii, becauſe, by

The Heteroſcii.

The Periſcii.

by reaſon that the Sun is ſometimes above their Horizon for a Day or more without ſetting, their Shadows turn quite round them.

The Inhabitants of the Earth have alſo been diſtinguiſhed into three Sorts, with regard to their Situation with reſpect to each other. They who live under the oppoſite Points of the ſame Parallel of the Equator, are called with reſpect to each other, Periœci. Periœci.

Theſe have the ſame Seaſons of the Year, but it is Midnight to the one when it is Noon or Mid-day to the other. Others are called Antœci, as Antœci. living under the ſame Meridian, but in oppoſite Parallels. Theſe have Day and Night at the ſame Time, but different Seaſons, it being Summer with one when it is Winter with the other. Others are Antipodes, with reſpect to Antipodes each other, that is, they live in oppoſite Parallels and Meridians too; ſo that they are on the oppoſite Points of the Globe; and therefore have both their Days and Nights, and their Seaſons of the Year reverſed, the one with reſpect to the other.

III. Of

III. *Of the* Horizon, *and other Particulars relating to it.*

The Horizon is a Circle in the Heavens, whoſe Plane touches the Surface of the Earth, in the Place of the Spectator. This Circle ſeparates that Part of the Heavens which is viſible from that which is not; and is called the ſenſible Horizon, in contradiſtinction to another Circle parallel to this, whoſe Plane paſſes through the Center of the Earth, and is called the rational Horizon. The Diſtances between theſe two Circles is ſo ſmall with reſpect to that of the fixed Stars, that it is uſually neglected, and the Circles are ſuppoſed to coincide.

The ſenſible Horizon.

The rational Horizon.

The Poles of the Horizon are two Points, the one of which is exactly over the Head of the Spectator, and is called the Zenith; the other directly under his Feet, and is called the Nadir.

Zenith.

Nadir.

The Secondaries of the Horizon paſs through the Zenith and Nadir, and are called Vertical Circles or Azimuths: Its Parallels, Almicantaraths, or commonly Almicantars (*c*).

Vertical Circles, or Azimuths.

Among

(*c*) Theſe are Terms introduced into Aſtronomy by the ancient *Arabian* Philoſophers, and ſtill retained.

Among the Vertical Circles of the Horizon there are two more remarkable ones, *viz.* the Meridian, and the prime Vertical, (for the Meridian of any Place is a Secondary of the Horizon of that Place, as well as of the Equator.) The Meridian, as obſerved above, paſſes through the Zenith and the Poles of the Equator. This cuts the Horizon in two Places, which are called the North and South Points thereof. The prime Vertical cuts the Meridian at right Angles in the Zenith and Nadir, and by interſecting the Horizon marks out the Eaſt and Weſt Points; and theſe two Circles cut the Horizon into four Quarters, each of which is ſuppoſed to be divided into eight, the whole therefore into thirty-two, which are called the Points of the Compaſs, of which the four firſt mentioned, *viz.* Eaſt, Weſt, North, and South, are called by way of Eminency, the Cardinal Points.

Meridian and prime Vertical.

Points of the Compaſs.

Cardinal Points.

The Altitude of a Star is an Arch of a Vertical Circle that paſſes through it, intercepted between the Star and the Horizon. If the Star be below the Horizon, it is then called its Depreſſion.

Altitude and Depreſſion of a Star.

When a Star is upon the Meridian of any Place, it is ſaid to culminate, or to be in its Meridian Altitude.

Culmination of a Star.

The Azimuth of a Star. The Azimuth of a Star is an Arch of the Horizon, intercepted between the North or South Point thereof, and the Vertical that paſſes through the Star; and is called Eaſtern or Weſtern according to the Situation of the Star.

The Amplitude of a Star. The Amplitude of a Star is an Arch of the Horizon, intercepted between that Point where the Star riſes or ſets, and the Eaſt or Weſt Point of the Horizon; and is either Northern or Southern. If the Amplitude be taken from the riſing of the Star, it is then called its riſing Amplitude; if when it ſets, its ſetting Amplitude. Riſing and ſetting Amplitude.

The Mid-heaven. That Point of the Ecliptic through which the Meridian of any Place paſſes, is called the Mid-heaven. And that Point of the Ecliptic which is diſtant a quarter of a Circle from the Points where it cuts the Horizon, is called the *Ninetieth Degree*. Ninetieth Degree.

The Elevation of the Pole. The Height of the Pole above the Horizon is called its Elevation. And this is equal to the Latitude of the Place; for either of them being added to the Diſtance between the Zenith and the Pole, makes juſt a quarter of a Circle *(c)*.

The

(d) From hence ariſes the Method of meaſuring the Cir-

The Inhabitants of the Earth are again diſtinguiſhed by the Sphere they are ſaid to live in. Thus, ſuch as live under the Equator, have their Horizon paſſing through both the Poles of the Earth, and cutting the Equator and all its Parallels at right Angles, and from thence are ſaid to live in a right Sphere. The Property of which Sphere is this, that the Days and Nights are of an equal Length all the Year long, and the Stay which any of the heavenly Bodies make

Right Sphere.

Circumference of the Earth. For, let two Places be pitched upon, lying North and South of each other, ſuch that the Elevation of the Pole in one ſhall exceed its Elevation in the other by one Degree ſuppoſe, then meaſure the Diſtance between the two Places in Miles: then as one Degree is to the Circumference of a Circle, or 360 Degrees, ſo is that Number of Miles to the Circumference of the Earth. By this Means the Circumference of the Earth is found ſuch, that the Length of one Degree therein is equal to 69 *Engliſh* Miles. This Meaſure was firſt found out by our Countryman *Norwood*, and afterwards attempted by the *French* Mathematicians, who twice applied themſelves to it by the King's Command, and diſcovered almoſt the ſame Meaſure that *Norwood* had done before. They took the Length of a paved Cauſey of near ſeven Miles, that lies between *Jews Village* and *Juvitum* in *Picardy*, by actual Menſuration; and then by a Survey from thence with the moſt accurate Inſtruments, they took the Diſtance between *Malcivin* and *Surdon*, which lies almoſt in a ſtrait Line from North to South; in both which Places they took the Height of the Pole, and from thence computed the Circumference of the Globe. The Mathematicians ſet over this Work were Monſieur *Picard* the firſt time, and Monſieur *Caſſini* the ſecond.

above the Horizon is equal to that which they make below it. The Reaſon is, that all the Parallels of the Equator, in which the heavenly Bodies ſeem to perform their diurnal Revolution, are cut into two equal Parts by the Horizon.

Such as live between the Equator, and either Pole, have their Horizon croſſing the Equator and all its Parallels, leſs or more obliquely, as they live leſs or more diſtant from the Equator. Hence thoſe are ſaid to live in an oblique Sphere, and have all the Parallels of the Equator cut into unequal Parts by their Horizon, ſo that neither the Sun nor any of the heavenly Bodies, unleſs they be in the Equator, make an equal Stay above and below their Horizon.

Oblique Sphere.

To thoſe who live at the Poles of the Earth, if any ſuch there be, the Equator is parallel to, or rather coincident with the Horizon, and all its Parallels are parallel to it; from whence this is called a Parallel Sphere. The Property of this Sphere is, that it is Day for half the Year together, and Night for the other half. For the Equator and Horizon being parallel, ſo long as the Sun continues on the ſame Side the Equator,

Parallel Sphere.

ſo long muſt its Stay be on the ſame Side the Horizon, and conſequently ſo long is it Day at one of the Poles, and Night at the other. And the fixed Stars do not riſe or ſet in this Sphere, becauſe they do not remove their Places from one Side of the Equator to the other, as the Sun and Planets are obſerved to do.

It is obvious, that in a right Sphere, the ſame Point of the Equator which comes to the Horizon with any Star, will come to the Meridian with the ſame; becauſe in this Sphere both the Horizon and the Meridian become Secondaries of the Equator. Now an Arch of the Equator intercepted between this Point and the firſt of Aries, is called the right Aſcenſion of a Star. But in an oblique Sphere it will not be ſo, but ſome other Point different from that which comes to the Meridian with it, will riſe with it, becauſe of the Obliquity of the Horizon to the Secondaries of the Equator. And the Diſtance between this Point and the firſt of Aries, is called the oblique Aſcenſion of a Star. And the Difference between right and oblique Aſcenſion is termed its Aſcenſional Difference.

Right Aſcenſion of a Star.

Oblique Aſcenſion.

Aſcenſional Difference.

In an oblique Sphere there is a Parallel of the Equator, ſo far diſtant from the Pole as is its Elevation above the Horizon, the Stars that are comprehended within which ſeem to revolve about the Pole without ever riſing or ſetting. This is called the Circle of perpetual Apparition. There is another oppoſite to this, and at the ſame Diſtance from the other Pole, called the Circle of perpetual Occultation; becauſe the Stars that are included therein never appear above the Horizon.

Circle of perpetual Apparition.

Circle of perpetual Occultation.

The ancient Geographers were wont to diſtinguiſh the different Regions of the Earth by Parallels and Climates, in ſuch manner, that in going from one Parallel to another, the Days at the Summer Solſtice ſhould be encreaſed a quarter of an Hour. So that ſuppoſing the Equator to be the firſt Parallel, under which the Days are always 12 Hours long, the next ſhould be where the longeſt Day of the Year ſhould be 12 Hours and a Quarter long, &c. And two ſuch Parallels made one Climate, till we come to the Polar Circles. And from thence to the Poles they reckoned the Climate ſuch, that the Stay of the Sun above the Horizon was a whole Month

Parallels and Climates.

Month different in one from what it was in the next contiguous one.

As the Year of the Ancients was not preciſely adapted to the annual Motion of the Sun, as we ſhall ſee hereafter, it happened that the Seaſons did not always fall on the ſame Part of their Year, ſo that it was not accurate to denote the Time of the Year by the Day of the Month. It was therefore uſual with them to diſtinguiſh the Seaſons by the Coſmical, Achronical, and Heliacal riſing and ſetting of the Stars. And becauſe we meet with theſe Diſtinctions moſt frequently among the Poets, they are called the Poetical Riſings and Settings of the Stars.

The poetical riſing and ſetting of the Stars.

A Star is ſaid to riſe or ſet coſmically, when it riſes or ſets at Sun-riſing; and when it riſes or ſets at Sun-ſetting, it is ſaid to riſe or ſet achronically. A Star riſes heliacally, when it emerges out of the Sun's Rays, and is got ſo far from it as to become viſible. And it is ſaid to ſet heliacally when it firſt immerges into the Rays, and becomes inviſible (*d*).

(*d*) *Kepler* will have it, that theſe Words are to be taken in another Senſe, ſo as that to riſe or ſet coſmically ſhall ſignify the ſame as to aſcend above, or deſcend below the Horizon; but that to riſe or ſet achronically, is the ſame as to riſe or ſet in the Sun's

Oppoſite, or in the other Achron, or Extreme of the Night: In which Senſe *Ptolemy*, and to this Day moſt Aſtronomers ſay, a Planet is achronical, when it is oppoſite to the Sun, and ſhines all Night; ſo that to riſe achronically is, as above, the ſame as to riſe when the Sun is ſetting; but to ſet achronically, is to ſet when the Sun is riſing; which is commonly called the Coſmical Setting.

A
COMPENDIOUS SYSTEM
OF
Natural Philoſophy,
With NOTES
Containing the MATHEMATICAL DEMONSTRATIONS, and ſome occaſional REMARKS.

PART IV. Continued.

CONTAINING THE

DESCRIPTION
OF THE
ORRERY *and* GLOBES,
The PRINCIPLES of
CHRONOLOGY;
Together with the Phyſical Cauſe of the Motion of the
HEAVENLY BODIES.

By *J. ROWNING*, M. A.
Rector of ANDERBY in *Lincolnſhire*, and late
Fellow of MAGDALEN College in *Cambridge*.

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CHAP. XV.

The Deſcription *and* Uſe *of the* Orrery *and the* Globes.

I. THE Orrery is a Machine deſigned to repreſent at one View the real Motions of the Heavenly Bodies. In order to this, in the middle of a large Circle deſigned to repreſent the Ecliptic, is fixed a Globe repreſenting the Sun. Next to the Sun a ſmall Ball repreſenting *Mercury*. Next to this is *Venus* repreſented by a larger. At a greater Diſtance ſtill from the Sun, is the Earth repreſented by an Ivory Ball. Round the Earth, at ſome Diſtance from it, is a Ring to expreſs the Orbit of the Moon, making an Angle with the Circle that repreſents the Ecliptic, and thereby ſhewing the Inclination they have to each other in the Heavens, and alſo the Line of the Nodes. Within this Ring is a ſmall Ivory Ball repreſenting the Moon, and having a black Cap or Caſe, which always covers that Hemiſphere which is turned from the Sun; and thereby diſtinguiſheth the enlightned Part from the dark one. Beyond theſe is ſeen *Mars*, then *Jupiter* and its four Moons, and outermoſt of all is *Saturn* with its Ring and five Moons.

All theſe are fixed upou ſmall Stems which ſeverally repreſent their Axes, each of which has its peculiar and proper Inclination to the Plane of that Circle which we ſaid was to repreſent the Ecliptic: And when the Machine is put in Motion, all theſe Bodies move round that which repreſents the Sun, and at the ſame time both that and all thoſe which repreſent ſuch of the Planets as have been obſerved to have a Rotation about their Axes, turn round upon the ſaid Stems, and in their proper Times. The Moons alſo revolve about their Primaries at the ſame Time, and the Ring that repreſents the Orbit of the Moon has likewiſe its proper Motion, whereby that of its Nodes is alſo expreſſed.

Theſe Bodies are all kept in Motion by Wheelwork conceal'd in a Caſe underneath, and turned by a ſmall Winch in the ſide of the Caſe; one Turn anſwering to a Revolution of the Ball which repreſents the Earth, about its Axis; and conſequently to 24 Hours of Time. So that 27 Turns cauſes the Sun to turn once about its Axis, 365 carries the Earth about the Sun, 27 the Moon about the Earth, *&c.* So that not only the Motion of the Heavenly Bodies may be repreſented, but their Situations with reſpect to each other may be ſhewn for any Time either paſt or to come; (*a*)

by

(*a*) This and the next Paragraph muſt be underſtood with ſome Limitation, Machines of this kind being ſufficiently adapted to common Uſe, though not accurate enough for this purpoſe; none of the Machines of this kind which have yet been made, being ſo contrived as to ſhew this with Accuracy.

by giving the Winch a proper Number of Turns backwards or forwards as the Caſe requires.

There is uſually alſo a Lamp in a Caſe, with one or more convex Glaſſes in the ſide of it, to throw a ſtrong Light directly forwards; this Lamp being put in the Place of the Sun, and that ſide of the Caſe which has the Glaſs in it directed towards the Earth, it being alſo contrived to turn according to the Motion of the Earth, a ſtrong Light is thereby continually thrown upon it, and the Moon, in whatever part of its Orbit it is, and ſo not only the Times in which the Eclipſes of the Sun and Moon, that is, properly of the Earth and Moon will happen, are ſhewn, but they themſelves are duly repreſented. (*a*)

(*a*) Artificers generally erect upon the Ecliptic ſome Semicircles, to repreſent ſome of the principal Circles of the Heavens. But this is wrong, and tends to Confuſion; becauſe theſe Circles being only imaginary, and ariſing from the *apparent* Motions of the Heavenly Bodies, ought to have no Place in the Orrery: Which as it expreſſes their *real* Motions, ſerves to *explain*, yet is by no means intended to *repreſent* their *Phænomena*; the Phænomena and real Motions being in ſeveral Caſes directly contrary to one another. To be particular, they have in ſome affixed the two Extremities of a Semicircle, which they call a moveable Horizon by two Hinges to the Ecliptic, at the firſt Degree of *Aries* and *Libra*; this is as if all Horizons neceſſarily paſſed through theſe Points, which is impoſſible, becauſe there are as many Horizons as Points upon the Earth. So that the putting on this Circle is not only an Impropriety common to the reſt, but is attended with a neceſſary Blunder. A learned Expoſitor of the Orrery ſays, ‘ when this Machine is ſet to any Latitude, ‘ (for they have a Contrivance for that too) ſet the moveable ‘ Horizon to the ſame Degree upon the Meridian, and you may

II. As the Orrery is a Machine proper to repreſent the real Motions of the Heavenly Bodies, ſo the Globes are adapted to their apparent Motions: And that I may give the Reader a juſt and adequate *Idea* of them, I ſhall premiſe the following Deſcription of a Machine, which though imaginary, will greatly tend to the true Conception both of their Nature and Uſes.

In Order to this, let us conceive a large hollow Sphere repreſenting the inſide of the Heavens, and on the inner Surface of this Sphere let the Sun, Moon and Stars, be ſuppoſed to be delineated in their proper Places; and let the Sphere be put into a Frame, and hang there in ſuch Manner, as that it may be capable of turning about an Axis: And let it be ſo contrived, that either end of the Axis may be raiſed or lowered at Pleaſure,

‘ form an Idea of the reſpective Altitudes, or Depreſſions of the ‘ Planets above or below the Horizon, according to their reſpective Poſitions, with regard to the Meridian.’

Now the Idea which the Motion of the Orrery in this Poſition gives me, is that the whole Earth is one half of the Year above the Horizon, and the other half below it. For the Ball which repreſents it in the Orrery is manifeſtly ſo, while it performs its Revolution about the Sun.

Again, ſome Artiſts, in order to make the Orrery repreſent the ſeveral Phaſes of the Moon, make one half of the Ball that repreſents her black; in which caſe they are obliged to contrive it ſo, that the ſame ſide of the Moon ſhall not be always turned towards the Earth; which is contrary to her real Motion, or that the dark ſide ſhall ſometimes be turned towards the Sun, which is abſurd.

In the midſt of this Sphere, let there be a ſmall Ball to repreſent the Earth, but ſo fixed there, as not to turn round with the Sphere, and let a Spectator be ſuppoſed to be placed on the uppermoſt Point of this Ball; and to prevent the lower half of the Sphere from being viſible to him, let a Plane, that is, a thin broad Subſtance, a Plate of Braſs for Inſtance, be ſuppoſed fixed on that Point of the Ball where the Spectator ſtands, and to reach on all ſides as far as the inner Surface of the Sphere: This will repreſent the Horizon of that Spectator; becauſe it ſeparates the upper half of the Sphere, which we are to ſuppoſe viſible to this Spectator, from the lower, which we muſt ſuppoſe to be out of his Sight. Then, as is moſt obvious, by turning this Sphere once round, while the Ball, the Spectator and the aforeſaid Plane remain unmoved, the riſing and ſetting of the Sun, Moon and Stars, by their paſſing by the Edges of this Plane, will be duly repreſented: (*a*) And thoſe Stars which do not go down ſo low as the Plane (as is evident if one End of the Axis of the Sphere be above it, ſome will not,) will be thoſe which never ſet to a Spectator on that Part of the Earth, which correſponds with the

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(*a*) The Reader is to ſuppoſe here, that in this caſe the Sphere is not hung in its Frame in ſuch manner, that one Extremity of its Axis ſhall be directly over the Spectator; becauſe in that caſe none of the Heavenly Bodies in turning the Sphere, will paſs by the Edge of the Plane.

uppermoſt Point of the Ball; thoſe which during the whole Revolution, do not riſe ſo high as to come above it, will be thoſe which never riſe.

Again, if we ſuppoſe a Circle as large as the inſide of the Sphere will admit of, to be placed within it, in ſuch manner as to have the Axis of the Sphere paſſing through two oppoſite Points of it, and alſo to have one Point of it directly over the Head of the Spectator, and the oppoſite one directly under his Feet, and to remain unmoved while the Sphere turns: This will repreſent the Meridian of that Spectator; and ſo the Culminations of the Heavenly Bodies, that is, their paſſing the Meridian in the Heavens, will be exhibited by their paſſing by this Circle.

Farther, if we alſo imagine an Index like the Hand of a Clock, to be fixed upon one of the Extremities of the Axis of the Sphere, in ſuch manner as to turn with the Sphere, and to point continually to a Circle fixed on the Frame, with the 24 Hours delineated upon it; and if this Index be ſet to point to the Hour 12, when the Sun's Place upon the Sphere for any given Day, is at the Circle that repreſents the Meridian, then by turning the Sphere this way or that, till the Place of the Sun or any of the Stars comes to the Plane abovementioned, the Index moving along with the Sphere, will point to the Hour at which they riſe or ſet that Day: And if we obſerve to what

Hour the Index points, when any of them croſſes the Meridian within the Sphere, the Time of their Culmination, or Southing, will be expreſſed: And ſo by this Machine all Problems relating to the riſing, ſetting and ſouthing of the Sun, Moon and Stars, may be ſolved. The Reaſon why the Index muſt be ſet to point to the Hour 12, when the Place of the Sun is brought to the Meridian, is that it may point to that Hour, which the real Sun indicates, when the Heavens ſtand in a Poſition correſponding with that of our imaginary Sphere.

But ſince by means of the annual Motion of the Earth round the Sun, the Sun continually ſhifts its apparent Place in the Heavens in ſuch manner, as to ſeem to deſcribe a great Circle once a Year; it is evident that its Place cannot be delineated on the inſide of this Sphere as we have ſuppoſed, unleſs it be for a Day or a few Hours, during which Time its change of Place is ſo ſmall that it may be neglected. In like manner the Planets are continually changing their apparent Places in the Heavens, by means of their Motions in their reſpective Orbits; neither therefore can their Places be aſſigned them, unleſs for a very ſmall interval of Time. Let us then imagine both the Sun and the Planets to be repreſented by little Balls adhering to the inner Surface of the ſaid Sphere; but let it be ſo contrived, that each Ball ſhall ſhift its Place with every Turn of the Sphere, juſt ſo much, and in the ſame Direction that the Sun or other heavenly Body the Ball repreſents, appears to ſhift its Place in the Heavens in one Day: And let the Index ſhift with the Sun, ſo that it may always point to 12, when the Place of the Sun comes to the Meridian.

Then, as is moſt obvious, will our Machine be ſo adapted, that upon turning it round continually, the Change of the Time of the riſing and ſetting of the Sun, its different Meridian Altitude, the viciſſitude and length of Day and Night, the alteration of the Time of the riſing and ſetting of the fixed Stars and of the Planets, for ſo many Days running, as is the Number of Turns given to the Globe, will all be exhibited to View: And conſequently by turning the Sphere 365 Times round, the Succeſſion of the Seaſons will be repreſented throughout the Year with abundance more, or rather will all the remaining Phænomena of the Heavens.

This Machine, as hitherto deſcribed, ſerves only to exhibit the Phænomena of the Heavens as they appear on one Point of the Earth's Surface, *viz.* that which is repreſented by the uppermoſt Point of the Ball, or that where the Spectator was ſuppoſed to ſtand. But if we imagine the Plane which was put as a Boundary to the Spectator's Sight, to be applied to ſome other Point of the Ball and fixed there, then the Motion of the Sphere will exhibit the Phænomena, as they would appear to a

Spectator on that Part of the Earth, and ſo for the reſt; or which comes to the ſame thing, if while the Plane remains unmoved, one of the Poles of the Sphere be elevated above it as many Degrees as the correſponding Pole in the Heavens is above the Horizon of a Spectator, on any Part of the Earth; then will the Revolution of the Sphere repreſent the Phænomena as they appear to that Spectator. For it is the ſame Thing whether the Pole be lifted up from the Horizon, or the Horizon put down below the Pole. (*a*)

Now this Machine (except in ſome few Particulars, in which it vaſtly excels the celeſtial Globe) is exactly the ſame with it as to its Uſes. Thus, the Globe turns upon an Axis, either End of which may be raiſed or depreſſed at Pleaſure; and on one of them is an Index that turns with the Globe, and points to a fixed Circle that has the Hours upon it. There is no Ball in the middle, as we ſuppoſed in the Machine, but the Place of the Spectator is al-

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(*a*) By means of an Orrery to repreſent the real Motions of the heavenly Bodies, and ſuch a Machine as above deſcribed, to ſhew afterwards how the apparent Motions of thoſe Bodies depend on, and are connected with their real ones, the Elements of Aſtronomy might be taught much better than by any thing that has as yet been contrived; or rather, if I may ſo ſay, the Science itſelf might thereby be exhibited to View. For the Orrery, though it admirably well illuſtrates ſome Things, particularly the Eclipſes, and in general what relates to the Motions of the Planets about the Sun, yet it ſhews the Appearances or Phænomena ariſing from thence to a Spectator, ſuppoſed to be at reſt upon the Surface of the Earth, but in a forced and unnatural Way.

ways ſuppoſed to be there; and inſtead of the Plane we ſuppoſed to touch that Part of the Ball on which the Spectator ſtands, and to be extended every way as far as the inner Surface of the Sphere, there is a broad wooden Circle on the outſide of the Globe, (which is Part of the Frame of it) exactly where that Plane would paſs, was it continued beyond the Surface of the Globe, and ſo anſwers the Uſes of that Plane. In the next Place, inſtead of the Circle we ſuppoſed fixed within the Sphere and paſſing through the *Zenith* and *Nadir* of the Spectator, and through which the Axis of the Sphere paſſes, there is a brazen Circle on the outſide of the Globe, and in which it ſwings. And inſtead of the Stars being on the inner Surface, they are here on the outer one, and the Conſtellations are delineated in their proper Forms. Then as to the Places of the Sun and Planets, Artificers are content to leave them quite out, and only draw a great Circle repreſenting the Path of the Sun in its annual Courſe through the Heavens, *viz.* the Ecliptic, and diſtinguiſh it into 12 Parts by the Signs thereof, and each Sign into 30 leſs, which are the Degrees thereof. When therefore a Problem is to be determined by the Globe, requiring that the Sun ſhould be delineated in its proper Place, you are to look out the Place of the Sun for the given Time in ſome aſtronomical Tables, Ephemeris or Almanack for that Purpoſe, or on the broad Surface of the

Horizon of the Globe, where there is uſually a Table ſhewing the Sun's Place in the Ecliptic for every Day of the Year, and then to mark its Place on the Line which we ſaid repreſented the Path of the Sun upon the Globe. In like manner, if a Problem is to be ſolved requiring the Place of any of the Planets, the Place of the Planet for the Time given muſt be looked out in ſome Ephemeris or Almanack, of which there are many publiſh'd yearly by *Parker*, and others, for that Purpoſe, containing Tables expreſſing the Longitude and Latitude of each Planet for every Day of the Year; and marking the Place of the Planet upon the Globe, by ſticking on a Patch, or the like, and ſetting the Index according to the Place of the Sun for the given Time, and turning the Globe about, the Index will ſhew the Time of that Planet's coming to either Side of the Horizon, to the Meridian, or to any Height above or below the Horizon, *&c.*

If then the Reader will form to himſelf a true Idea of the Machine I have deſcribed, and of the manner in which ſuch a Machine is capable of repreſenting the apparent Motions of the heavenly Bodies, he cannot fail of having a thorough Notion of the Nature of the celeſtial Globe; the main Difference between them being only this, *viz.* that what we have ſuppoſed in the one to be on the Inſide, is either actually on the other on the Outſide, or to be fixed on, or to have its Place marked there, as Occaſion requires. The uſes both of the celeſtial and terreſtrial Globe, as to particular Caſes, will be further illuſtrated by the Inſtances at the End of this Chapter, after we have conſidered ſome other Particulars relating to each.

The Circles delineated on the celeſtial Globe are uſually, beſides the Ecliptic abovemention'd, the Equinoctial Circle at an equal Diſtance from either Pole, and two Parallels of it, *viz.* one on either Side, to repreſent the apparent diurnal Motion of the Sun, when at its greateſt Diſtance from the Equator, called the Tropics; and at the like Diſtance from either Pole, two more Parallels call'd the polar Circles. Both the Ecliptic and the Equinoctial are divided into Degrees, beginning at the Point that repreſents the vernal Equinox, which is one of thoſe where they croſs each other; the Equinoctial into 360, but the Ecliptic into 12 Signs, diſtinguiſhed by their proper Characters, every one of which is ſubdivided into 30 Degrees (*a*). There are alſo ſix Secondaries of the Equator, and as many

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(*a*) As to the Diviſions of the Ecliptic, it is to be obſerved, that the Degrees belonging to each Sign are not to be found where the Conſtellation whoſe Name it bears is delineated, but where the Character peculiar to that Sign is placed. The Reaſon of which is, becauſe the firſt Degree of *Aries*, from whence all the reſt are counted, being where the Equinoctial cuts the Equator, is not in this Age in any Part of the Conſtellation that bears that Name, as was accounted for above.

of the Ecliptic cutting it in the firſt Degree of each Sign; two of which are alſo Secondaries of the Equator, *viz.* the Equinoctial and Solſtitial Colures. Further, the Globe is ſo hung in its Frame, that that Side only of the brazen Meridian which has the Degrees marked upon it, paſſes through the Axis of it, which Side only, in all Caſes where that Meridian is conſidered, is therefore ſuppoſed to repreſent it. Upon the broad Surface of the wooden Horizon, beſides the innermoſt Circle divided into its proper Degrees, there is a Table ſhewing what Degree of the Ecliptic the Sun is in for every Day of the Year, as was obſerved above; in order to which the 12 Signs of the Ecliptic, with the Degrees of each, and the 12 Calendar Months, are ſo placed over-againſt one another, both according to the *Julian* and *Gregorian* Account, that is, according to the old and new Stile, as that the Days of the one may duly anſwer to the correſponding Places of the Sun upon the other. There is alſo a Circle, repreſenting the Winds or Points of the Compaſs.

There is alſo belonging to the Globe a *Quadrant of Altitude.* It is a long and narrow Plate of Braſs, made ſo thin and plyant, that the whole Length of it may be applied cloſe to the Surface of the Globe, and having the Degrees upon it, (*a*) ſerves to meaſure the Diſtances

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(*a*) That Edge, which has the Degrees neareſt it, is called its *graduated Edge,* and is the only Part of it that is conſidered in the uſing it.

ces of Places on the Globes; or being fixed on to the uppermoſt Point of the Meridian of the celeſtial Globe, (for which Purpoſe it has a Screw at one End) and having the other End thruſt in between the Horizon and the Globe, meaſures the Height of a Star, or other Heavenly Body above the Horizon, *&c.* From which latter Uſe it has its Name.

The terreſtrial Globe differs from the celeſtial principally, in that inſtead of the Conſtellations, the ſeveral Countries of the Earth are delineated thereon; and the Circles on this Globe are uſually the Ecliptic, and the Equator, on which are numbered the Degrees of Longitude, (*a*) the two Tropics, the two polar Circles, Secondaries of the Equator cutting it at every tenth Degree, and Parallels alſo of it at every tenth Degree on each Side. There are alſo drawn upon it Rhumb Lines, or thoſe which a Ship, keeping the ſame Courſe, that is, ſteering continually in the ſame Point of the Compaſs, deſcribes upon the Surface of the Sea. The Property of a Line of this kind, is that it cuts all the Meridians it paſſes through, under the ſame Angle, and therefore is a kind of Spiral continually approaching towards ſome Point. This is called a *Loxodromic* Curve.

(*a*) On ſome Globes, the Degrees of Longitude are number'd both ways from the firſt Meridian, till you come to the oppoſite Point, and the Longitude is accordingly diſtinguiſhed into Eaſt and Weſt.

The Lines of Variation deſcribed upon the terreſtrial Globe, ſhew the Direction of the magnetic Needle, in thoſe Places in which it does not point directly towards the North. For it is obſerved, that the Needle in ſome Parts of the World, declines from the North to the Eaſt, and in others to the Weſt.

Further, on the Pedeſtal of ſome Globes there is a Mariner's Compaſs fixed, by the help of which, the brazen Meridian of the Globe may be ſet North and South. (*a*)

After this Deſcription of the Globes, and general Idea of the Uſe thereof, it may now be proper to exemplify the latter by the following Inſtances.

And firſt of the celeſtial Globe.

Problem I. *To rectify the* Globe *to the Latitude of any Place upon the Surface of the Earth.*

If the Place be in the Northern Hemiſphere of the Earth, ſlide the Meridian within the Notches of the Horizon, till there be as many Degrees

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(*a*) The Armillary Sphere comes the neareſt of any we have to that imaginary Machine deſcribed above, for it is hollow, and has a Ball in the middle to repreſent the Earth; but then in all other Reſpects it is like a Globe, except that the Surface is cut away every where, (unleſs where a Circle is deſcribed) that you may ſee the Ball in the middle. There is one Circle left broad enough to repreſent the Zodiac, on which the Places of the Sun and Planets may be marked, as Occaſion requires.

Degrees of the Meridian between the North Pole and the Horizon, as are equal to the Latitude of the Place (*a*). If the Place be in the Southern Hemiſphere, the Southern Pole muſt be raiſed to a like Height above the Horizon.

PROB. II. *To find what* Stars *never riſe or never ſet in any* Place *or* Latitude *given.*

The Globe being rectified to the given Latitude of the Place by Problem the firſt, thoſe Stars which paſs not under the wooden Horizon of the Globe, during its whole Revolution, never ſet to a Spectator in that Latitude: And ſuch as do not come up above it, never aſcend above that of the Spectator, and ſo never are ſeen to riſe.

(*a*) The Reaſon of this is, becauſe to a Spectator, upon any Part of the Earth, the Elevation of the Pole is equal to the Latitude of the Place; and the Globe is ſaid to be *rectified* by this Operation, becauſe, by this means, that Point of the little Ball, (for ſuch we muſt always ſuppoſe to be in the middle of the Globe, to repreſent the Earth) that correſponds to the Place of the Spectator upon the Earth, is brought to the uppermoſt Point thereof; and ſo the Horizon thereof becomes coincident with the Plane of the wooden Horizon that ſurrounds the Globe, and the Poles of the Globe having the like Situation with reſpect to the uppermoſt Point of the Ball, that the Poles of the World have with reſpect to the Place of the Spectator; it is evident that the Revolution of the Globe is fitly diſpoſed to exhibit that of the Heavens, as it would appear to that Spectator.

PROB. III. *To find the* Sun's *Place in the Ecliptic, for any Day given.*

If the Day given be according to the *Julian* Account, or old Stile, then in the wooden Horizon of the Globe, in the *Julian* Kalendar, otherwiſe in the *Gregorian*, find the Day given, over-againſt which in a Circle, which contains the ſeveral Signs and Degrees of the Ecliptic, is that Degree in which the Sun is in that Day. Though it may be found more accurately by an Ephemeris calculated for that Year. Then in the Ecliptic, delineated on the Surface of the Globe, look for the ſame Degree, and that is the Place of the Sun upon the Globe for that Day.

PROB. IV. *The Latitude of a Place being given, to find the Time of the* Sun's *riſing and ſetting, and conſequently the Length of Day and Night in that Place.*

Having rectified the Globe according to the Latitude, (by *Prob.* 1.) and found the Sun's Place in the Ecliptic (by *Prob.* 3.) bring the latter to the graduated Edge of the brazen Meridian, and holding the Globe in that Poſition, ſet the Hour Index to 12 at Noon; (*a*) then turn the Globe till the Sun's Place in the Ecliptic comes to the eaſtern Part of the Horizon, and the Index will point to the Hour at which the Sun riſes; and if you turn the Globe till the Sun's Place cuts the weſtern Side of the Horizon, the Index will ſhew the Time of its ſetting.

Prob. V. *To find the Hour of the Day, by the Height of the* Sun.

Firſt, by the Help of a Quadrant or ſome other Inſtrument for that Purpoſe, obſerve how many Degrees the Sun is elevated above the Horizon. Then the Globe being rectified to the Latitude of the Place, and the Sun's Place in the Ecliptic found, and the Hour Index duly placed, that is, ſo as to point to 12 at Noon, when the Sun's Place is under the Meridian; and ſcrewing the Quadrant of Altitude on to the Meridian at the Zenith, (*i. e.*) at ſo many Degrees from the Equator, as is the Latitude of the Place. Turn the Quadrant of Altitude and the Globe backwards and for-

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(*a*) Becauſe when the Place of the Sun is in the Meridian, the Globe repreſents the Poſition of the Heavens, in reſpect of the given Place, as they are at Noon, or 12 o' Clock of the Day given. And conſequently, if the Hour Index be put to 12, when the Globe is in this Poſition, as it turns with the Globe, it will point to the Hour that correſponds with any other Poſition of the Globe that Day.

wards, till the Sun's Place in the Ecliptic lies under that Degree on the graduated Edge of the Quadrant of Altitude, which anſwers to the Sun's Height found as above, then will the Hour Index point out the Time of the Day. (*a*)

PROB. VI. *To find the Hour of the Night, by the Height of a* Star.

The Height of the Star being found by an Inſtrument, the Globe rectified, and the Quadrant of Altitude fixed, as in the foregoing Problem, move the Globe and the Quadrant, till the Star comes under its graduated Edge, at that Degree which expreſſes the Height of the Star above the Horizon, then will the Hour Index point to the Hour of the Night.

PROB. VII. *To find the Place of a* Star *or* Planet *upon the* Globe, *its Longitude and Latitude being given.*

Lay the firſt Degree of the Quadrant of Altitude, upon that Degree of the Ecliptic, which expreſſes the given Longitude, and the ninetieth Degree thereof on the Pole of the Ecliptic

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(*a*) Becauſe when the Sun's Place is brought to this Point of the Globe, it is ſo many Degrees above the Horizon, as the Sun itſelf is above that of the Spectator.

Ecliptic, (that is, where its Secondaries croſs each other upon the Globe) either Northern or Southern, as the given Latitude is North or South: Then look upon the Quadrant for the Degree of Latitude given, and the Point of the Globe which is under that, is the Place of the Star or Planet ſought. (*a*)

PROB. VIII. *To repreſent the Appearance of the* Heavens *at any Time of the Night, that is, to ſhew the Situation of the fixed* Stars *at that Time.*

Rectify the Globe, and ſet the Hour Index to the Sun's Place in the Ecliptic, and turn the Globe till the Index points to the Hour of the Night. (*b*)

(*a*) The Reaſon of this is, becauſe the Quadrant of Altitude in this Situation, repreſents a Secondary of the Ecliptic, an Arch of which interſected between the Star and the Ecliptic is its Latitude; and an Arch of the Ecliptic between the Point where it is interſected by a Secondary paſſing through the Star, and the firſt Degree of *Aries*, is its Longitude.

(*b*) For then the Situation of the Stars upon the Globe, with reſpect to the Horizon and Meridian, will be ſimilar to that of the Stars in the Heavens; *viz.* thoſe which appear on the Eaſtern Side the Globe in the Horizon, will then be riſing, and thoſe in the Horizon at the other Side will be ſetting, thoſe which are under the Meridian, will then be culminating, &c.

And if by means of a Compaſs upon the Pedeſtal of the Globe, you ſet the brazen Meridian North and South, ſo that the South

PROB. IX. *The Day of the Year and the Latitude of the Place being given, to find the beginning of the Morning, and End of the Evening* Twilight.

The Globe being rectified to the Latitude of the Place, and the Hour Index ſet to the Place of the Sun in the Ecliptic, elevate that Degree of the Ecliptic which is diametrically oppoſite to the Sun's Place, 18 Degrees above the Weſt Side of the Horizon, (*a*) and the Hour Index will ſhew the Time of Day-break

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Part of it ſhall reſpect the South Part of the Heavens, the Situation of the Stars upon the Globe, with reſpect to the Points of the Compaſs, will alſo be ſimilar to that of the Stars in the Heavens at that Time; ſo that by comparing the Situation of the Stars upon the Globe, with thoſe in the Heavens, you may eaſily make yourſelf acquainted with all the Stars, that are viſible in this Part of the World.

If the Situation of the Moon or the other Planets is alſo to be repreſented, their Places in the Zodiac for that Time muſt be aſſigned them by Problem the laſt, their Longitude and Latitude being firſt found by an Ephemeris for that Purpoſe.

(*a*) Becauſe then the Place of the Sun, will be ſo many Degrees below on the other Side, in which Situation of the Sun, we obſerved above, that the Twilight begins. The Quadrant of Altitude will ſhew when the Point oppoſite to that of the Sun is 18 Degrees above the Horizon. If this be tried for any Time while the Sun is paſſing from about the 5th Degree of *Gemini*, to the 20th of *Cancer*, that is, from about the 15th of *May*, to about the 7th of *July*, it will be found that there is no Beginning or End of Twilight, the Sun never being 18 Degrees below the Horizon (in the Latitude of *London*) during that Time.

Of

or the Beginning of Morning Twilight. And if the ſame Point of the Ecliptic be raiſed 18 Degrees above the Eaſt Side of the Horizon, the Index will point out the Time when the Evening Twilight ends.

PROB. X. *To account for the Phænomena of the* Harveſt Moon.

About the autumnal Equinox, when the Moon is at or near the *Full*, it riſes almoſt at the ſame Hour for ſeveral Nights together; which Phænomenon is called the *Harveſt Moon*. (*a*) In order therefore to ſhew by the Globe how this comes about, let us ſuppoſe the Place of the Sun to be in the firſt Degree of *Libra*, as it always is at that Equinox, and that of the Moon to be in the Point oppoſite to the Sun, or the firſt Degree of *Aries*, becauſe the Moon muſt be conſidered as being at the *Full*. In this Caſe it is obvious that the Moon will riſe when the Sun ſets, which will be at Six of the Clock; becauſe both Luminaries are in the Equator and at oppoſite Points. Now ſince the Moon performs an entire Revolution in its Orbit, (which we will at preſent conſider as coincident with the Ecliptic) in the Space of one Month, it therefore advances forwards 12 Degrees each Day, and conſequently

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(*a*) Mr. *Johnſon* calls this Phænomena *Luna autumnalis*, and is the only Author that I know of that has accounted for it. Quæſtiones Philoſoph. Edit. 2da. Page 124.

the next Night it will be in the 12th Degree of *Aries*. Let us then enquire at what Time the Moon will riſe this Night. To this End, rectify the Globe to the Latitude of the Place, bring the Sun's Place, which is now the ſecond Degree of *Libra*, to the brazen Meridian, and put the Hour Index to 12; and if the Globe be turned till the Place of the Moon, *viz.* the 12th Degree of *Aries*, cuts the eaſtern Side of the Horizon, we ſhall find the Hour Index will point nearly to the Hour of Six, which, as obſerved above, was the time of the Moon's riſing the Evening before. And thus if we inveſtigate the Time of the Moon's riſing for ſeveral Nights together, both before and after the Full, it will be found much the ſame in all.

The Reaſon of this is, that about the Time of the full Moons, which happen at this Part of the Year, the Moon being in the aſcending Signs of the *Zodiac*, appears in its nocturnal Courſe to deſcribe a Parallel of the Equator one Night, much nearer the North Pole than another, and ſo riſes every Night more and more to the Northern Parts of the Horizon, and that conſiderably, as paſſing quite from the Southern to the Northern Tropic in a Fortnight's Time. Now it is eaſy to conceive that the nearer any of the Heavenly Bodies is to the North Pole, the ſooner that Body (*cæteris paribus*) aſcends the Horizon: Thus, if the Body be within a little of the *Circle of perpetual Apparition*, that Body is no ſooner ſet but it riſes again. In every Lunation therefore, while the Moon is paſſing from the Southern to the Northern Tropic, ſome Minutes are on that Account to be deducted from thoſe by which it would riſe later and later each Night, did the Plane of its Orbit coincide with that of the Equinoctial; and as much is to be added while ſhe paſſes from the Northern to the Southern Tropic. And for the like Reaſon in the former Caſe ſomething is to be added to the Time it would ſet at; and in the latter a like Quantity is to be deducted.

Now about the time of the autumnal Æquinox, the Sun being then in *Libra* or thereabouts, the Moon is in the midſt of the aſcending Signs, when at the *Full*. Hence it is that the *full* Moons at that Time of the Year, riſe ſo little later the one than the other.

In like manner, if any one thinks it worth while to obſerve it, he will find that the *new* Moons in the Spring, riſe nearly at the ſame Hour for ſeveral Nights ſucceſſively, while the full Moons ſhall riſe one later than another, by a greater Difference than at any other time of the Year. The Reaſon is, becauſe at the time of the new Moons, which happen at that Part of the Year, the Moon is in the aſcending Signs, but at the full, in the deſcending ones. (*a*)

(*a*) It is here to be obſerved, that this Phænomenon is not the ſame as to Degree, in different Years; becauſe as the lunar Orbit is inclined to the Ecliptic 5 Degrees, and the Line of its Nodes is continually revolving round, the Inclination of that Orbit to the Equator will be ſometimes 10 Degrees greater than at others. So that the Moon does not haſten to the North, or deſcend to the South in each Revolution with equal Pace.

PROB. XI. *To determine the Time of the Year in which a Star riſes or ſets* Coſmically, *or* Acronically.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Horizon on the Eaſtern Side of the Globe, and obſerve what Degree of the Ecliptic riſes with it. Look for that Degree in the wooden Horizon, and right againſt it in the Kalendar, you will find the Month and Day when that Star riſes coſmically *(a)*. And if you bring the Star to the Weſtern Side of the Horizon, the Degree of the Ecliptic which cuts the Eaſtern Side of the Horizon in that Situation of the Globe, will give the Day of the Month when the ſaid Star ſets coſmically *(b)*. So likewiſe againſt the Degree which ſets with the Star, you will find the Day of the Month of its *Acronical* ſetting; and if you bring it to the Eaſtern Part of the Horizon, the Degree of the Ecliptic which then cuts the Weſtern Part of the Horizon will be the Sun's Place when the Star riſes *Acronically*.

(a) For that will be the Day when the Sun and the Star riſe at the ſame time, which is its coſmical Riſing.

(b) The coſmical ſetting of a Star being its ſetting at Sun-riſe.

PROB. XII. *To find the Time of the* Heliacal *riſing and ſetting of a* Star.

Having rectified the Globe to the Latitude of the Place, bring the Star to the Eaſtern Side of the Horizon, and apply the Quadrant of Altitude to the Weſtern Side in ſuch manner that its twelfth Degree may cut the Ecliptic, if the Star be of the firſt Magnitude (*a*). Then will the Point of the Ecliptic oppoſite to that which is cut by the Quadrant, be 12 Degrees below the Horizon. Look for this Degree on the wooden Horizon, and over-againſt it you will have the Day of the Year, when the Star riſes Heliacally. To find the Heliacal ſetting, bring the Star to the Weſtern Side of the Horizon, and turn the Quadrant of Altitude about to the Eaſtern Side, till the 12th Degree of it cuts the Ecliptic; then that Degree of the Ecliptic which is oppoſite to this Point, is the Sun's Place at that Time of the Year when the Star ſets Heliacally.

(*a*) Becauſe a Star of the firſt Magnitude being above the Horizon, may be ſeen when the Sun is 12 Degrees below it. If the Star be of the 2d Magnitude, that Degree of the Ecliptic muſt be taken, which is 13 Degrees below the Horizon when the Star riſes, if of the 3d, then the 14th, *&c.*

PROB.

PROB. XIII. *The Day of the Month, the Time of the Day, and the Latitude of the Place being given, to find thereby the Hour of the Day, according to the* Babylonic, Italic, *and the* Judaical *Way of reckoning.*

1. To find the *Babylonic* Hour (which is reckoned from the Time of the Riſing of the Sun) having found the Time of Sun's riſing in the given Latitude, the Number of Hours between that and the Time given, is the Hour of the Day according to the *Babylonic* Way of Reckoning.

2. To find the *Italic* Hour (which is reckoned from Sun ſetting,) firſt find the Time at which the Sun ſets on the given Day.* The Time elapſed from that to the Time given, will give the *Italic* Hour ſought.

3. To find the Hour according to the *Jewiſh* Way of Reckoning (one of which Hours is a twelfth Part of the Time that the Sun continues above the Horizon, and they are reckoned from Sun riſing,) firſt find the Hour the Sun ſets at, this being doubled will give the Number of Hours the Sun continues above the Horizon on the Day given. Then as that Number of Hours is to 12, ſo is the Number

of Hours ſince Sun riſing, to the *Judaical* Hour required.

Thus in the Latitude of *London*, when the Sun riſes at 4 of the Clock and ſets at 8; if the Hour given be 5 in the Evening, the *Babylonic* Hour will be the 13th, the *Italic* the 21ſt, and the *Jewiſh* Hour will be Nine and three Quarters.

PROB. XIV. *To find the Place of any of the Heavenly Bodies upon the* Globe, *from their right Aſcenſion and Declination.*

Bring that Point of the Equator which expreſſes the right Aſcenſion given, to the brazen Meridian, and look upon the Meridian towards the North or South Pole, according as the Declination is North or South, for the Degree of Declination given, under which is the Place ſought (*a*).

(*a*) For in this Caſe the Meridian is made to repreſent a Secondary of the Equator paſſing through the Place, an Arch of which lying between the Place and the Equator is the Declination, and the Diſtance between the Place of the Equator through which it paſſes, and the firſt Degree of *Aries*, is the right Aſcenſion.

Of the Terreſtrial Globe.

PROB. XV. *From the Longitude and Latitude of a Place, upon the Surface of the Earth being given, to find the Place upon the* Globe.

This Problem is much the ſame with the laſt, only inſtead of the celeſtial, we uſe the terreſtrial Globe; and inſtead of reckoning the Longitude from the firſt Degree of *Aries*, the Point from which right Aſcenſion is reckoned, it muſt be counted from the firſt Meridian.

Bring the Degree of the Equator, which numbers that of the Longitude given, to the brazen Meridian, upon which count towards the North or South Pole, according as the Latitude is North or South, till you come to the Number of the Latitude, under which is the Place ſought. *(a)*

(a) In ſome Globes, the Degrees of Longitude are numbered both Ways from the firſt Meridian, till they meet in the oppoſite Point, and the Longitude is diſtinguiſhed into Eaſt and Weſt as obſerved above: In ſuch Caſe it muſt be known which Kind the given Longitude is of.

PROB. XVI. *To determine the Difference of Time in different Places.*

Find the Longitude of each Place, by the Converſe of the laſt Problem, and reduce the Difference into Time; which is done by allowing an Hour for every 15 Degrees, and ſo proportionably for leſſer Parts, the Number of Hours, &c. thus found, will be the Difference between the Time of the Day at one Place, and that at the other (*a*).

PROB. XVII. *To find the Diſtance between two given Places, and the Angle of Poſition, that is, the Bearings of them, or the Situation of one from the other, with reſpect to the Points of the Compaſs.*

Rectify the Globe to the Latitude of one of the Places, and bring that Place to the brazen Meridian; then fix the Quadrant of Altitude

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(*a*) The Reaſon of this is, becauſe as the Earth by turning round its Axis, brings all Places to the Sun once in 24 Hours; that Place which lies 15 Degrees to the Weſtward of another, comes to it an Hour later than the other; and ſo the Inhabitants of this Place are an Hour later in their reckoning of Time than at the other Place. And the contrary holds of ſuch as live towards the Eaſt.

to the uppermoſt Point of the Meridian, and putting the lower End of it between the Horizon and the Globe, ſlide it about till it paſſes through the other Place, and the Number of Degrees upon the Quadrant between Place and Place, turned into Miles by reckoning 60 to each Degree, will give the Diſtance between them; and the Number of Degrees upon the Horizon, between the Meridian and the Foot of the Quadrant, will give the Bearing of the Second, or its Situation from the firſt, with reſpect to the Points of the Compaſs.

PROB. XVIII. *To find a Meridian Line, partly by the Help of the celeſtial Globe.*

Set the Hour Index to the Place of the Sun in the Ecliptic, and turn the Globe till the Pole Star, that is, the Star which is the neareſt the Axis of the Globe, comes under the Meridian, either above or below the Pole; above, if you find that the Index in that Situation of the Star, points to one of the Hours of the Night, otherwiſe below; and mark what Hour the Index points to. Then at that Hour of the Night hang up two Plummets, letting their Weights hang in Water to prevent their vibrating, in ſuch manner that the String of one of them may be directly between the Pole Star and the String of the other. Then will a Line drawn from one String to the the other, or the Shadow of the one when it falls upon the other, be a Meridian Line. And this I take to be a much more accurate, as well as an eaſier Method, than the common one of fixing up a Pin upon a Plane, drawing a Circle about the Foot of the Pin, and obſerving where the Extremity of the Shadow of that Pin enters the Circle in the Morning and goes out of it in the Afternoon, and drawing a Line from the Foot of the Pin through the middle Point between thoſe Places for a Meridian Line. For this latter Method requires more Care and Trouble to do it accurately, than they who have not tryed it, would imagine.

CHAP. XVI.

Of the Equation *of* Time.

WHereas Time conſtantly paſſes on with an uniform and regular Flux, it is evident there is no poſſible Way accurately to meaſure and compare the ſeveral Intervals of it one with another, but by the Motion of ſome Body whoſe Progreſs is as uniform and regular as itſelf. To this End it was natural to pitch upon the Sun, whoſe Motion is obvious to all, and ſeemingly regular enough for that Purpoſe. But by the Circumſpection and accurate Obſervations of Aſtronomers, it is diſcovered not to be ſo; and that neither the Days, nor even the Hours themſelves, as meaſured by the Motion of the Sun, are of an equal Length. And this upon two Accounts, as will appear from the following Conſiderations.

I. A natural Day, or 24 Hours, is that interval of Time, in which the Sun ſeems to paſs from the Meridian of any Place to the ſame again: Had therefore the Earth no Motion but about its own Axis, and conſequently the Sun no apparent one along the Ecliptic, all the Days would be of an equal Length, as depending only on the Motion of the Earth about its Axis, which is uniform: But ſince the Earth is conſtantly moving in its Orbit, the ſame Way it turns about its Axis, the Sun is always in appearance advancing forwards. When the Earth therefore is turned round, and a Spectator upon its Surface brought again to the Place he was at the foregoing Day at Noon, it is not yet Noon with him, the Sun in appearance having advanced to the Eaſtward in the mean time; ſo that the Spectator muſt be carried farther ſtill, *viz.* ſo many Degrees as the Sun has advanced that Way, before he can be brought to the Meridian the Sun is in (*a*). Now by Reaſon of the unequable and

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(*a*) This is the Reaſon that the Length of the Day is not exactly equal to the Time, in which the Earth performs its Rotation about its Axis, which is, as was obſerved in Note (*f*), Chap. I. but 23 Hours 56 Minutes and 4 Seconds.

irregular Motion of the Earth in its Orbit, or, the irregular apparent Motion of the Sun along the Ecliptic depending thereon, the Number of Degrees the Sun advances Eaſtwards each Day, is ſometimes greater and ſometimes leſs; and conſequently, upon this Account, the Days muſt differ one from another in Length; and therefore the Hours alſo, ſince each Day is ſuppoſed to conſiſt of 24, be it long or ſhort.

II. Was the apparent Motion of the Sun through the Ecliptic uniform and regular, yet as the Ecliptic is oblique to the Equator, and therefore equal Portions of the one do not correſpond to equal Portions of the other; the Motion of the Sun, when referred to the Equator, would not be uniform; or, which is the ſame Thing, thoſe Points of the Equator which come to the Meridian with the Place of the Sun on different Days, would not be at equal Diſtances one from another; (*a*) which as to the Time of the Sun's appearing in the Meridian, or its ſhewing the Hour of the Day upon a Dial, is the ſame as if it had an irregular Motion along the Equator; and therefore muſt cauſe it to render the Days unequal among themſelves.

As theſe Cauſes are independent of each other, they ſometimes conſpire together; ſometimes they are contrary the one to the other.

(*a*) This is eaſily tried upon a Globe, where if we bring every tenth Degree of the Ecliptic to the brazen Meridian, we ſhall find that each tenth Degree of the Equator, will not come thither with it.

The Time which an uniform Motion would mark out, is called *true Time*; and that which is indicated by the Sun, is *apparent Time*; and the Difference between theſe is termed *the Equation of Time.* When the abovementioned Cauſes have continued for ſome time, to render the apparent Motion of the Sun too quick or too ſlow, this Difference is conſiderable. About the 13th of *October*, it is the greateſt of all; at which Time the Sun is 16 Minutes and 11 Seconds too faſt.

CHAP. XVII.

Of the Diviſion *of* Time.

THE ſeveral Parts of Time are principally diſtinguiſhed by Days, Hours, Weeks, Months, Years, Cycles, Periods and Æras. Of which in their Order.

By a Day, according to the moſt natural and obvious Senſe of the Word, is meant that Space of Time during which it continues to be *Light*, and thus is oppoſed to that wherein it is *Dark*, which is called the Night. But this Space of Time being ſomewhat vague and indeterminate, the Time between the Sun's Riſing and Setting is uſually looked upon as the Day, and the Time which lapſes from its Setting to its Riſing again, the Night.

But the Word Day is frequently taken in a larger Senſe, ſo as to include alſo the Night, and to denote the Time of an whole apparent Revolution of the Sun round the Earth. In this Senſe it is called by ſome a natural Day, and by others an artificial one; but what ſome call the Artificial, others call the Natural. To avoid which Confuſion, it is uſual to call it in the former Senſe, ſimply the Day, and in the latter a *Nuchthemeron*; by which Term that Acceptation of it is aptly denoted, as being a Word that implies both Day and Night *(a)*.

The *Nuchthemeron* is divided into 24 Parts, called Hours, which are of two Sorts, *Equal*, and *Unequal* or *Temporary*. The common Diviſion of the equal Hour is into Halves and Quarters: But Aſtronomers, and thoſe who are more accurate in their Account of Time, divide the Hour into ſixty Parts, called Minutes, and theſe again into ſixty Parts called Seconds, &c. And here we may obſerve, that the Word Minute is taken in a double Senſe, either to denote the ſixtieth Part of an Hour, and is therefore by Way of Diſtinction ſometimes called an *Horary* Minute; or elſe to ſignify the ſixtieth Part of a Degree, in which Caſe it may be called a *graduary* Minute, or Minute of a Degree.

Different Nations and People begin their Day at a different Hour. Thus the *Egyptians*

(a) A Greek Compound of *νὺξ Night*, and *ἡμέρα Day.*

began their Day at Midnight; from whom *Hipparchus* introduced that Way of reckoning into Aſtronomy, and *Copernicus* and others have followed him. But the greateſt Part of Aſtronomers reckon the Day to begin at Noon, and ſo count 24 Hours till the Noon of the next Day, and not twice twelve, according to the vulgar Computation.

The Method alſo of beginning the Day at Midnight prevails in *Great-Britain*, *France*, *Spain*, and moſt Parts of *Europe*.

The *Babylonians* began their Day at Sunriſing, reckoning the Hour immediately before its Riſing again, the 24th Hour of the Day. From whence the Hours reckoned in this Manner are called the *Babylonic*.

In ſeveral Parts of *Germany*, they begin their Day at Sun-ſetting, and reckon on till it ſets the next Day, calling that the 24th Hour. Theſe are uſually called the *Italian* Hours.

The *Jews* alſo began their *Nuchthemeron* at Sun-ſetting, but then they divide it into twice twelve Hours as we do, reckoning 12 for the Day, be it long or ſhort, and 12 for the Night (*a*). So that their Hours continually varying

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(*a*) According to this Way of reckoning, the Hours of the Day mentioned in Scripture are to be underſtood. The *Jews* alſo divided the Night into four Quarters, called *Watches*, each Watch containing three of their Night Hours. And diſtinguiſhed them ſometimes according to the Order of their Succeſſion. Thus we find in Scripture Mention of the *ſecond*, *third* and *fourth* Watch, ſometimes the firſt Watch was called the *beginning of the Watches*; the ſecond, the *middle Watch*, as not being ended till Midnight; and the fourth, the *Morning Watch*. The firſt alſo was ſometimes termed the *Evening*; the ſecond *Midnight*; the hird, the *Cock-crowing*; and the fourth, the *Dawning of the Day*.

varying with the Length of the Day and Night, the Hours of the Day were longer than thoſe of the Night one half of the Year, and the contrary the other. From whence their Hours are called *Temporary*: Theſe at the Time of the Equinoxes become equal, becauſe the Days and Nights are ſo then. The *Romans* alſo reckoned their Hours after this Manner, as do the *Turks* at this Day (*a*).

A Week conſiſts of the ſeven Days in moſt Countries, called after the Names of the Planets as above. But becauſe Eaſter Week

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(*a*) This kind of Hours are alſo called *Planetary*, becauſe the ſeven Planets were anciently looked upon as preſiding over the Affairs of the World, and to take it by turns each of theſe Hours, according to the following Order, *Saturn* firſt, then *Jupiter*, *Mars*, the *Sun*, *Venus*, *Mercury*, and laſt of all the *Moon*. Hence they denominated each Day of the Week from that Planet whoſe Turn it was to *reign* or preſide the firſt Hour of the *Nuchthemeron*. Thus aſſigning the firſt Hour of *Saturday* to *Saturn*, the ſecond will fall to *Jupiter*, the third to *Mars*, and ſo the twenty-ſecond of the ſame *Nuchthemeron* will fall to *Saturn* again, and therefore the twenty-third to *Jupiter*, and the laſt to *Mars*. So that on the firſt Hour of the next Day, it will fall to the Sun to preſide. And by the like Manner of reckoning the firſt Hour of the next will fall to the Moon, of the next to *Mars*, of the next to *Mercury*, of the next to *Jupiter*, of the next to *Venus*. Hence the Days of the Week came to be diſtinguiſhed by the Latin Names of *Dies Saturni*, *Solis*, *Lunæ*, *Martis*, *Mercurii*, *Jovis*, and *Veneris*; and among us by the Names of *Saturday*, *Sunday*, *Monday*, *Tueſday*, *Wedneſday*, *Thurſday* and *Friday*. For, as *Saturday*, *Sunday* and *Monday* plainly denote the Day of *Saturn*, the *Sun* and the *Moon*; ſo *Tueſday*, *Wedneſday*, *Thurſday* and *Friday*, denote the Day of *Tuiſco*, *Woden*, *Thor* and *Friga*, which are the *Saxon* Names reſpectively anſwering to *Mars*, *Mercury*, *Jupiter* and *Venus*. *Well*'s Chronol. Chap. II.

was formerly eſteemed the firſt or principal Week of the Year, and each Day thereof was a *Feria* or Holyday; hence the ſeveral Days were diſtinguiſhed among the primitive Chriſtians, by the Names of *Feria prima, ſecunda*, &c. that is, the firſt, ſecond Holyday, *&c.* But the Sunday, or *Feria prima*, was otherwiſe ſtiled by them the *Lord's Day*, as being the Day of our Lord's Reſurrection (*a*).

Months are diſtinguiſhed principally into two Sorts; *viz. Solar* and *Lunar*. The Solar Month is either *Aſtronomical* or *Civil* (*b*). The Lunar Month is divided into *Periodical*, *Synodical* and *Civil*. The *Aſtronomical* Solar Month is the Time the Sun takes up in paſſing through one of the Signs of the Ecliptic.

The *Periodical* and *Synodical* lunar Months have already been obſerved; the former to be equal to the Revolution of the Moon about the Earth, *viz.* 27 Days 7 Hours and 43 Minutes; the latter, the Interval of Time that lapſes between two ſucceeding Conjunctions of the Moon with the Sun, *viz.* 29 Days, 12 Hours, 44 Minutes and 3 Seconds.

(*a*) The Hebrew Word which in the old Teſtament is rendered *Week*, ſignifying a Collection of ſeven, is there ſometimes uſed for ſeven Years; and in Conformity thereto, our *Engliſh* Word *Week* in thoſe Places muſt be underſtood to imply not a *Week of Days*, but a *Week*, or *Syſtem of ſeven* Years.

(*b*) A Month or Year is denominated *Civil*, from its being of common Uſe in any Republic or Society of People.

Now becauſe in common Uſe it would be inconvenient to have the Months to conſiſt of odd Pieces of Days; it was uſual among the *Jews*, *Greeks* and *Romans*, (till *Julius Cæſar* made an Alteration) in order to adapt their Months to the apparent Motion of the Sun, to have their Months conſiſt of 29 and 30 Days alternately, one of theſe Numbers being about half a Day above, and the other about half a Day leſs than the Synodical Month; ſo that by this Means the ſame Appearances of the Moon would happen nearly on the ſame Day of the Month for a long Time together. This Month thus adapted to the laſt mentioned lunar Month, is called the *Civil* lunar Month; and is ſtill made uſe of by the *Turks*.

The *Egyptian* Months conſiſted of 30 Days each, 12 of theſe make a Year wanting 5 Days, which they added to the End of the Year, and from thence called them Epagomenæ, or Supplimental.

Now becauſe 12 civil lunar Months want 11 Days of a Year, it is evident that the ſeveral Seaſons of the Year muſt in Time fall upon different Months; to prevent which, *Julius Cæſar* ordained, that the Month ſhould not be taken from the Motion of the Moon, but from that of the Sun; that they ſhould conſiſt alternately of 30 and 31 Days, and that *February* ſhould contain 29, and every fourth Year 30. But it happened afterwards that the Name of the ſixth Month, which was formerly

Sextilis, was in Honour of the Emperor *Auguſtus* changed for that of his, as that of the foregoing Month *Quintilis* had before been, in Honour of his Father *Julius*; and therefore ſince this Month conſiſted but of thirty Days according to the Inſtitution of *Julius Cæſar*, one more was added to it; to make it equal to the foregoing one, that the Honour paid to the Son might not ſeem to fall ſhort of that which was paid to the Father. Upon this, the next Month, *viz. September*, was reckoned but 30, the next 31, and ſo on; that the alternate Order abovementioned might not be deſtroyed, till *January*, which, according to that Order, ought to have had but 30. But this Month being ſo named in Honour of *Janus*, the God of Time, it was though improper to rob the God of a Day, wherefore *February* was reduced to 28, and every fourth Year to 29. And ſo the Months came to ſtand as they are in our Calendar, from whence they are called *Calendar* Months. Theſe are properly *civil Solar Months*.

The principal Diviſion of the Year is alſo into two kinds, *viz. Solar* and *Lunar*; each of which are again divided into *Aſtronomical* and *Civil*.

The Aſtronomical Solar Year is of two kinds, *viz. Tropical* and *Sydereal*. The former is the Space of Time which lapſes while the Sun appears to move from either of the Soltitial or

Equinoctial Points to the ſame again. Now by Reaſon of the continual Preceſſion of the ſaid Points, whereby they do as it were meet the Sun, this kind of Year is ſomething leſs than the *Sydereal* one; which is the Time in which the Earth performs an entire Revolution in its Orbit, or in which the Sun ſeems to paſs from any fixed Star to the ſame again. The former conſiſts of 365 Days, 5 Hours, 48 Minutes, and 57 Seconds; the latter of 365 Days, 6 Hours, 9 Minutes, and 14 Seconds, and is alſo called *Periodical*, or *Anomaliſtical*. The Aſtronomical Lunar Year conſiſts of 12 Synodical Months; and is therefore 354 Days, 8 Hours, 48 Minutes, and 36 Seconds.

The *Civil* Lunar Year is alſo of two kinds, *viz. moveable* and *fixed*; the moveable civil Lunar Year conſiſts of 12 civil Lunar Months, which being but 354 Days, falls ſhort of a Year by 11 Days, ſo that the beginning of this Year varies with reſpect to the Seaſons, till it runs through them all, which it does in about 32 Years Time; and from hence it is denominated *Moveable*, or the *wandring Lunar Year*; As the *Turks* now make Uſe of the civil Lunar Month, ſo they alſo do of this civil Lunar Year.

In order to render this kind of Year *fixed*, that is, to prevent the Seaſons from ſhifting from one Part of it to another, the *Greeks*, and the *Romans*, till *Julius Cæſar*'s Time, after every three Years added a Month of thirty-three Days, which they called *Embolimæus*, or the *Intercalary* Month.

But becauſe this Method is not ſufficient, (for the civil Lunar Years fall ſhort of the Aſtronomical Solar Years above 11 Days,) others adapt ſuch Intercalatians to them as do in a manner render the beginning of them fixed to the ſame Part of the Solar Year. Hence theſe Years properly intercalated are called *fixed* Lunar Years, as alſo from their Dependency on the Motion both of the Sun and Moon, *Luni-Solar* Years. Years of this kind are uſed by the *Jews*, and the Clergy of the Church of *Rome* in their Eccleſiaſtical Affairs.

The civil Aſtronomical Year is alſo either *moveable* or *fixed*. The moveable one was in Uſe among the *Egyptians*, and is from thence called, the *Egyptian* Year. It conſiſted of 365 Days, and therefore fell ſhort of the tropical Year, with which all the Seaſons return, near ſix Hours; ſo that four ſuch Years are leſs than four Aſtronomical Solar ones, by almoſt a whole Day, and therefore in 1460 Years, the beginning of the Year would *paſs* through all the Seaſons.

Although the *Romans* did not uſe this kind of Year, yet theirs, taking one Year with another, was of the ſame Length (*a*); *Julius Cæ-*

(*a*) For having as obſerved above, ſix Months of 29 Days each, and ſix of 30, and an *Intercalary* Month of 33 Days, at the End of every third Year; their Years conſiſted one with another of 365 Days.

ſar therefore to *fix* this Year, when he rectified the Calendar, and threw out the civil Lunar Months, to make up the ſix Hours which this Year wants of the true Aſtronomical Solar Year, and with which only the Seaſons keep pace, ordered that every fourth Year ſhould have an intercalary Day, *i. e.* that a Day ſhould be added to the Calendar, over and above thoſe it contain'd the other three, or that every fourth Year ſhould contain 366 Days. Hence it was that as we obſerved above, *February* came to have 29 Days every fourth Year. But he did not add it to the End of the Month, but ordered that the ſixth of the Calends of *March*, which according to our way of Reckoning is the 24th of *February*, ſhould be reckoned twice. Upon which Account that Year was wont to be called *Biſſextilis*, and is ſtill retain'd by us under the Name of *Leap* Year. From hence, Years reckoned after this Manner are called *Julian* Years.

But however the Year was not rendered *fixed* by this means, for it was now too long by about eleven Minutes, and therefore varied from the Sun about a Day in 131 Years. So that from the Time of the *Nicene* Council, (in which the Rule was eſtabliſhed for the Time of celebrating *Eaſter*) to the Year 1582, in which Pope *Gregory* the XIIIth undertook a farther Reformation of the Calendar, there was found to have roſe a Variation of 10 Days, the vernal Equinox being obſerved to fall on the 11th of *March*, which at the Time of the *Nicene* Council, fell on the 21ſt. The Pope therefore in order to bring back the Equinoxes to the Time of the Year they fell upon at the Time of that Council, took thoſe ten Days out of the Kalendar by ordering the 5th of *October* 1582, to be reckoned the 15th, which neceſſarily removed the following Equinox from the 11th to the 21ſt of *March*. And to prevent the Seaſons of the Year from going backwards for the future, he ordered that every hundredth Year, which according to the Form inſtituted by *Julius Cæſar*, was to be a *Biſſextile* or *Leap* Year, ſhould be a common one, and conſiſt only of 365 Days, but that ſince too much would be taken away by this Means, every fourth hundred Year ſhould remain *Biſſextile*.

This Method being eſtabliſhed by Pope *Gregory*, is called the *Gregorian* Account, or the *New Stile*, as being new in compariſon of that eſtabliſhed before by *Julius Cæſar*, which is now called the *Old*; and is obſerved in all Places where the Papal Authority is acknowledged; as alſo, towards the End of the laſt Century was received by many of the reformed People in *Germany*. In *Great-Britain* and *Ireland*, and in ſome reformed Nations of the Northern Parts of *Europe*, the old *Julian* Form is ſtill retained. But above an hundred Years being now lapſed ſince the abovementioned Reformation of the Calendar, and the hundredth Year not being made a common one, as in the *Gregorian* Method; the *Gregorian* Account differs now from the *Julian* one Day more than it did at its firſt Inſtitution; ſo that that Day which according to the *Old Stile* is the firſt of the Month, is the 12th of the ſame according to the *New.*

As the Form of the Year in different Nations is various, ſo alſo is the Beginning of it. Thus the *Jews* begin their Eccleſiaſtical Year with the *new* Moon of that Month whoſe *full* Moon happens next after the vernal Equinox. The Church of *Rome*, with the *Sunday* that falls upon the ſaid *full* Moon, or that happens next after it; that is, with the Feaſt of the Reſurrection of our Lord. The *Grecians* began theirs with the new Moon that happened next after the Summer Solſtice. And the *Romans* anciently began theirs with the new Moon next after the Winter Solſtice. The *Venetians*, *Florentines*, *Piſans* in *Italy*, and the Inhabitants of *Triers* or *Treves* in *Germany*, make the *vernal* Equinox the Beginning of their Year. The ancient Clergy made the 25th of *March*, or the Feaſt of the Bleſſed Virgin, the Beginning of the Year; which Method the Church of *England* ſtill retains; and according to our Law the Year begins here too, though in Things that do not require a *legal Date*, we, with the neighbouring Nations, look upon the firſt Day of *January* to be the Beginning of the Year.

The *Perſians* ſtill retain the *Egyptian* Year, and the Period in which the firſt Day of it paſſes through all the Seaſons of the Aſtronomical Solar Year, *viz.* 1460 Years, they call the *Sothiacal Period*, or *great Dog-Star Year*; as being reckoned to begin in that Year in which the *Dog-Star* (or *Sothis*) riſes Heliacally on the firſt Day of the Month *Thoth*, which is the firſt Day of their Year.

A *Cycle* is a continual and ſucceſſive Revolution of a certain Number of Years. The Cycles are four, *viz.* the Cycle of the Sun, the Cycle of the Moon, the Cycle of *Dionyſius*, the Cycle of the Indiction.

In order to apprehend the Foundation and Nature of the Cycle of the Sun, it muſt be obſerved that to each Day of the Year is aſſigned in the Calendar, ſome one of the firſt ſeven Letters of the Alphabet A, B, C, D, E, F, G, the firſt Letter A being affixed to the firſt Day of *January*, to the ſecond B, and ſo to G. After which the Letters are repeated again in the ſame Order, A falling to the 8th, B to the 9th, *&c.* By which means, becauſe the common Year contains 52 Weeks and one Day over, the laſt Day of *December* has alſo the Letter A affixed to it. The Letters being appropriated in this manner to each Day of the Year, it follows, that which ever of the Letters belongs to any particular Day in the firſt Week of *January*, that Letter will belong to the ſame all the Year long. But ſince the ſame Letter, *viz.* A, is affixed to the laſt, which is affixed to the firſt Day of the Year, it is obvious, that at the End of one Year and beginning of the next, two ſucceeding Days will have the ſame Letter A; ſo that if *Monday* had an A the laſt Year, *Tueſday* has it this, and *Monday* the G: And therefore the Letters will move backwards with reſpect to the Days of the Week; for Inſtance, that Day which has G one Year, will have F the next, after that E, then D, and ſo on to A; and conſequently the ſame Day will run through the ſeven Letters in ſeven Years Time. But before the ſeven Years are expired, a Leap Year will intervene, having one Day more than common, which Day is added at the 24th of *February* (*a*), and has the ſame Letter that the 24th has, ſo that in this Caſe, as there was a Change or Regreſſion of the Letters, with reſpect to the Days of the Week, upon the Account of two Letters coming together at the End of one Year, and the beginning of the next, there will be another like Change in *February* alſo; and therefore, as in the former Caſe, the ſame Letter would have run through all the Days of the Week in ſeven Years; this ſecond Change happening once in four Years,

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(*a*) Some ſay it is added after the 28th, and that it has the ſame Letter with that. From this Uncertainty there ariſes a Difficulty in fixing the Day of the Celebration of the Feſtival of St. *Matthias*, which has occaſioned a great Diſpute among the Learned.

Years, the Courſe of the Letters is ſo oft interrupted, that it does not become the ſame again, till after four times 7, or 28 Years.

Thus, ſuppoſe a Leap Year begins with a *Sunday*, it will end with a *Monday*; in which Caſe, A and G will be the *Dominical* or *Sunday* Letters, becauſe A is appropriated to the firſt of *January*, and G to the 30th of *December*: But ſuch another Year cannot return till after 28 Years, as any one that tries will readily find; the Conſequence of which is, that thoſe two Letters cannot be *Sunday* Letters again in one Year, till after that ſpace of Time. And becauſe the ſame *Sunday* Letters do return after every 28 Years, that Term is called the *Cycle of the Sunday Letter*, or otherwiſe, though not ſo properly, *the Cycle of th Sun* (*a*).

The Cycle of the Moon is a Revolution o 19 Years; after which Time, as was anciently ſuppoſed, the new and full Moons would fall upon the ſame Days of the Year again (*b*).

This Cycle is therefore of great Note in the

(*a*) The ninth Year before our Saviour's Birth, was the firſt of this Cycle; from whence may eaſily be known what Year of it the preſent is. The firſt of this Cycle is always Leap Year, and has for its Dominical Letters G and F. And the Letter A is as already obſerved, always affixed to the firſt Day of *January*. From which *Data* may at any Time be computed what is the *Sunday* Letter for the preſent Year, and from thence what Day of the Week each Day of the Year falls on.

(*b*) This is alſo called the *Metonic* Cycle from *Meton*, the Author of it.

Chriſtian Church, a Method founded upon this Suppoſition, being eſtabliſhed by the Fathers of the *Nicene* Council, for finding the Time of celebrating *Eaſter*, and the other moveable Feaſts of the Church for any Time to come.

In order to underſtand which, we muſt obſerve, that as the Chriſtian Paſſover, or Feſtival of *Eaſter*, ſucceeded the *Jewiſh* one, the Time of its Celebration was regulated by that upon which the *Jewiſh* Paſſover was at its Inſtitution, by God himſelf appointed to be kept, which was on the fourteenth Day of the firſt Month, according to the manner of reckoning among the *Jews*, *Exod.* 12. Now the *Jewiſh* Months being Lunar ones, each Month began upon the Day of the new Moon, or however at the Time of its *Heliacal* riſing, and ſo the full Moon fell upon the fourteenth of each Month. And further, that Month was called the firſt of the Year, whoſe full Moon or fourteenth Day, either fell upon the vernal Equinox, or was the firſt that ſucceeded it. And at the Time of the *Nicene* Council, when the Rule for finding *Eaſter* was drawn up, the Equinox fell, or was thought to fall, on the 21ſt of *March*. But becauſe our Saviour's Reſurrection, which the Feſtival of *Eaſter* is deſign'd to commemorate, happened on a *Sunday*; it was ordered that it ſhould not be kept on the full Moon, but the *Sunday* following. Accordingly the Rule then eſtabliſhed for finding *Eaſter*, and which is ſtill made uſe of by the Church, runs thus; *Eaſter-Day is always the firſt* Sunday *after the firſt full Moon, which happens next after the One and twentieth Day of March. And if the full Moon happens upon a* Sunday, Eaſter-Day *is the* Sunday *after.*

The Foundation of the Rule being thus explained, it remains now to ſhew the Meaning of it, which is not ſo obvious as at firſt Sight it appears to be.

It is to be obſerved then, that in order to find on what Days of the Year the new and full Moons would happen for the Time to come, the ancient Method was to obſerve on what Day of each Month, the new Moons fell in each Year of the Moon's Cycle, and to the ſaid Days reſpectively to ſet the Number of the Year in their Calendar. Thus, obſerving that the new Moons in the firſt Year of the Cycle fall on *January* the 23d, *February* 21, *March* 23, *&c.* to thoſe Days they affixed the Number 1. And in like manner obſerving, that, in the ſecond Year, they fell on *January* 12, *February* 10, *March* 12, *&c.* to each of them they put the Number 2; and ſo for each Year of the Cycle. And on Account of their great Uſefulneſs, or becauſe they were wrote in Letters of Gold, the Number thus ſet to the Days on which the new Moons fell, were called the *Golden Numbers* (*a*).

(*a*) In the Table for finding *Eaſter* for ever in ſome Common Prayer-Books, they are ſtiled Prime Numbers, probably as being placed in the Prime or firſt Column of the Calendar.

The Numbers being thus affixed in the Calendar, it is only looking for the Golden Number of any Year; and over-againſt it you have the Day in each Month of that Year, in which the new Moon happens.

Thus, at the Time of the *Nicene* Council the Time of the new Moon's might be accurately enough found, but not at this Time, becauſe the new Moons do not happen at the ſame Time of the Year every 19 Years, but fall ſhort an Hour and half, which in 304 Years comes to a Day, ſo that now the new and full Moons fall almoſt 5 Days ſooner than they are ſhewn to do by the Golden Numbers, affixed to the Calendar.

However, as no proper Authority has interven'd to alter the Method of finding the *Eaſter* full Moon by the Golden Numbers, eſtabliſh'd at the *Nicene* Council, that Method is ſtill retained in the Church. And therefore to underſtand the abovementioned Rule aright, it muſt be ſuppoſed, that by the full Moon is meant the Time of the full Moon as found by the Golden Numbers affixed to the Calendar in the Common Prayer Book, and not by an Almanac, or by Aſtronomical Obſervation (*a*).

(*a*) The firſt Year of Chriſt was in the 2d of this Cycle. By the abovementioned Method the true new Moon may at any Time be found, if we deduct five Days in Conſideration of the abovementioned Deficiency in the Moon's Cycle, and ſuppoſe the new Moon to fall five Days ſooner than that Method ſhews it to do.

By multiplying the Cycle of the Sun and Moon together, we have a third Period, containing the Properties of both the other two, ſo that at the End of this Cycle not only the Days of the Week have the ſame Letters belonging to them, and ſo fall on the ſame Days of the Month, but the new and full Moons alſo fall on the ſame Days of the Year, ſo that the Time of celebrating *Eaſter* and the other Feſtivals of the Church, return to the very ſame Days of the Year. From whence this is called *the great Paſchal Cycle*, and ſometimes from its Author, the *Dionyſian Cycle* (*a*). It conſiſts of 532 Years, and the firſt Year of it was 457 Years before Chriſt.

The *Cycle of the Indiction* is a Revolution of 15 Years made uſe of by both the *Greeks* and *Romans*, probably upon ſome political Account. It has no Relation to the Motions of the Heavenly Bodies. The Nativity of our Saviour fell upon the 4th of this Cycle.

By multiplying the three Cycles of the Sun, Moon, and Indiction together, ariſes a Term of 7980 Years, called the *Julian Period*; which as it began ſeveral hundreds of Years before the Creation, and is not now ended, and in all probability will not, ſo long as the World ſtands, is of ſingular Uſe in Chronology, as

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(*a*) This is alſo by ſome called the *Victorian* Period.

taking in all Events, both paſt, preſent and to come.

This Period was invented by *Joſeph Scaliger*, and began 4713 Years before Chriſt.

An *Æra*, otherwiſe called an *Epoch*, is a Continuation of Time beginning from ſome certain Point, as from a Root, and continually proceeding forwards without beginning again.

That which is of principal Note among Chriſtians, is the Æra of Chriſt.

The Author of this Æra was *Dionyſius Exiguus*, who flouriſhed about 500 Years after Chriſt. He began it on the Annunciation of the Virgin *Mary*, or the 25th of *March*: the 25th of *March*, as he ſuppoſed, immediately preceeding the Nativity of our Lord: but it is now generally thought that our Saviour was born the *December* before that, wherefore others, beginning the Æra from the Nativity it ſelf, or the 1ſt of *January* following it, reckon almoſt a quarter of a Year before thoſe who adhere to the *Dionyſian* Account.

The *Engliſh* and *Iriſh* ſtill adhere to the *Dionyſian* Account in their eccleſiaſtical and civil Affairs, which all the reſt of the World, and even they themſelves in common Account, have laid aſide for that which begins at *January* the firſt. This Æra is frequently reckoned backwards, as well as forwards: Thus, it is uſual to ſay an Event happened ſo many Years before Chriſt.

The firſt Year of this Æra, agreeably to what was obſerved above, anſwers to the 4714th of the *Julian* Period.

There are ſeveral *Epochs* or Æra's of leſs Note, which I ſhall but juſt mention; this Chapter having already run out to a much greater Length, than I at firſt intended.

The Æra of the *Creation* according to the *Greek* Church, and which is now of Uſe in ſome Eaſtern Nations, begins before the *Julian* Period, *viz.* 5508 Years before Chriſt. Tho' others, among whom are the greateſt Aſtronomers, begin it ſomething leſs than 4000 Years before our Saviour's Birth.

The Æra of the *Deſtruction of Troy*, according to *Dionyſius Halicarnaſſeus* and *Diodorus Siculus*, begins 1181 Years before Chriſt.

The Æra of the *Olympiads*, or *Olympic Games* of Uſe among the *Greeks*, 776 before Chriſt.

The Æra *Urbis conditæ*, or *of the Building of* Rome, made Uſe of by the *Romans*, according to *Varro*, began 753 Years before Chriſt; according to the *Faſti Capitolini*, 752.

The Æra of *Nabonaſſar* made Uſe of by the *Chaldeans* and *Egyptians*, famous among Aſtronomers, as conſiſting of *Egyptian* Years, which are diſturbed by no Intercalation. It begins 747 Years before Chriſt.

The Æra of the *Death of Alexander the Great*, 324.

The Æra of the *City of Antioch*, 49.

The Æra of the *Julian Reformation of the Calendar*, 45.

The Æra *Actiaca*, 30.

The *Diocleſian Æra*, 284 Years after Chriſt.

The Æra of the *Hegira*, or *Flight of Mahomet*, uſed by the *Turks* and *Arabs*, 622 after Chriſt.

The Æra of *Yeſdegird* or *Perſian Æra*, 632 after Chriſt.

N. B. A Year in the Æra's of the Death of *Alexander*, and of *Nabonaſſar*, conſiſts but of 365 Days; and a Year of that of the *Hegira* is only 354.

As the Subject of this Chapter, though founded in *Aſtronomy*, relates principally to the Science of *Chronology*, of which this *Compendium* is not deſigned to treat; I have little more than thrown a few Definitions together, leaving it to thoſe who would ſee more on the Subject, to conſult ſuch Authors as have wrote more largely thereon; as *Gregory*, *Keil*, *Wells*, *Whitby* on the Common-Prayer, and many others.

CHAP. XVIII.

Of the Forces *neceſſary to retain* Bodies *revolving in* circular *and* other Orbits.

And firſt of circular Orbits.

IN Order that a Body may move round in the Circumference of a Circle, it is requiſite, that as ſoon as the Body begins to move, ſome Power or Force continually act upon it, in ſuch manner as to make it bend its Courſe every Moment towards the Center of the Circle; becauſe, as is obvious to conceive, it would otherwiſe only move right forwards (*a*); for in Conſiderations of this kind, we do not ſuppoſe the revolving Body to be affected even by its own *Weight*, or any other moving Cauſe whatever, except that whereby its Motion is made circular: And becauſe the Circumference of a Circle is every where equally diſtant from the Center thereof, it is obvious there muſt be the moſt exact *Adjuſtment* imaginable between the Power that retains the Body in the Circle, whatever that Power be, and the Endeavour the Body has to move right on in a ſtraight Line: For in Caſe the former be an *over-balance* for the latter, it will bring the Body *nearer*

(*a*) See Part I. Chap. IV.

and *nearer* to the Center of the Circle it ſhould deſcribe; and on the other hand, if it be too *weak*, it permits it to move *farther* and *farther* off. So that the Body, inſtead of a Circle, will deſcribe ſome other Figure, which will be of this or that Form, according as the Force the Body moves with, compared with the Power which bends its Courſe, is greater or leſs; and not only ſo, but as this latter Force (which is ſometimes the Caſe) acts more or leſs powerfully, as the revolving Body comes nearer or goes farther from the Point it revolves about. And here we may have recourſe to a Fact obvious to every one, which will in ſome Meaſure illuſtrate what we have been ſpeaking of. Let a Ball be hung up by a String: This Ball, when drawn a little to one Side, will endeavour to return back to the loweſt Point, as if there were ſome Power ſeated in that Point to draw it thither from all Sides. But the Body may have a Caſt given it ſideways in ſuch Manner, as that by Virtue of the Caſt, and by Virtue of its Tendency to the loweſt Point together, it ſhall deſcribe a Circle parallel to the Horizon; or inſtead of that it may be made to deſcribe other Figures, as Ellipſes ſuppoſe, which will be of various Forms according to the Force and Direction of the Caſt given to the Ball. And if we ſuppoſe ſeveral Balls hung up, which when removed to *equal* Diſtances from their loweſt Points, ſhould tend thither again with *unequal* Forces, and theſe

Forces urging them by different Laws and in different Manners, the Figures deſcribed by them, when thrown Sideways with this or that Force and Direction, will be proportionably different one from another. Now in this Suppoſition, the Caſt which is given to the Ball Sideways, correſponds with the Endeavour a Body revolving in the Circumference of a Circle or other Figure, has to move right on continually, and ſo to leave the Center it ought to revolve about; and the Endeavour, Struggle or Tendency the Ball when drawn to one Side, has to move towards the loweſt Point, or rather the Power which is, as it were, ſeated in that loweſt Point, and occaſions the abovementioned Endeavour or Tendency in the Ball to move thither, correſponds with the Power we ſuppoſed above neceſſary to retain a Body in the Circumference of the Circle or other Figure it deſcribes, and which by bending the Courſe of the Body inwards, that is, towards the Center of the Figure, prevents it from going off in a ſtraight Line.

We are therefore in the next Place, to enquire what Forces or retaining Powers are neceſſary to bend the Courſe of Bodies in ſuch Manner that they ſhall deſcribe this or that Orbit (*a*); and ſince the Planets both primary and

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(*a*) For unleſs we can *meaſure* the Forces we aſſign, and ſay they are exactly equal to the Effects we ſuppoſe them to produce, it would be in vain to aſſign them at all. And this is what makes the Difference between true and falſe Philoſophy. The and ſecondary, as alſo the Comets, deſcribe Orbits either circular or elliptical, as obſerved above, we will confine ourſelves chiefly to the Conſideration of the Forces neceſſary to retain revolving Bodies in ſuch Orbits. This Subject has been already touched upon in the firſt Part of this *Compendium* (*a*), under the Title of *Centripetal and Centrifugal Forces*; but little more is taken Notice of there, than what was requiſite to ſhew the Poſſibility of ſuch a Motion as we are ſpeaking of, and to explain the Terms; which are theſe that follow, *viz. Centripetal* Force, or that whereby the revolving Body is prevented from going on in a right Line: The *Centrifugal* Force, or that by which it endeavours to go on in ſuch a Line. Theſe, like Action and Reaction, are equal in all Caſes, and therefore called by one common Name, *Central* Forces: That whereby the Body continues its Motion, is the *Projectile* Force; and the Time in which it revolves once round,

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The former by a mathematical Scrutiny, (as it were by a kind of Menſuration) finds that the Cauſes aſſigned are juſtly adapted to the Facts accounted for; while the other, by a conjectural kind of Method, aſſigns Cauſes which perhaps are plauſible at firſt Sight, but when meaſured by the Rules of Geometry, are found either too ſmall or too great for the Effects they are ſuppoſed to produce.

This was the Caſe of *Des Cartes*'s Vortices, the Doctrine of which captivated the whole Philoſophical World, and continued to do ſo, till they were weighed in that unerring Balance, and thereby diſcovered to be inadequate to the Purpoſes for which they were contrived.

(*a*) Chap. VIII.

round, is called the *Periodical Time.* The ſeveral Caſes proper to be conſidered, ſhall be compriſed in the following Propoſitions.

PROPOSITION I.

When two or more Bodies revolve at *equal* Diſtances from the Center of the Circle they deſcribe, but with *unequal* Velocities, the central Forces neceſſary to retain them, will be to each other, as the Squares of their Velocities. That is, if one revolves twice as faſt as the other, it will require four times the retaining Force the other does; if with three times the Velocity, it will then require nine times the Force to retain it in its Orb. *&c.* (*a*)

N. B. We do not here conſider the Magnitude of the revolving Body, becauſe we ſuppoſe the Power at the Center, to act upon every Part of it alike, ſo that the larger it is, the more forcibly in Proportion it is acted upon; and therefore it is the ſame thing, whether it be large or ſmall.

(*a*) Let BDF Fig. 25. repreſent a Circle whoſe Center is S, draw ABC a Tangent to the ſame at the Point B, and let there be a Body moving from A towards C, and let it be ſuppoſed to be deſtitute of Weight, or whatever elſe might alter its Courſe. In this Caſe it would deſcribe the right Line ABC; but when it comes at B, let it begin to be acted upon by ſome central Force, or retaining Power ſeated in S, ſuch as ſhall *bend* its Courſe at B from a rectilineal to a circular one, and acting upon it afterwards in every Inſtant of Time in like manner, ſhall cauſe it to deſcribe the Circle BDFB. Let us then proceed to eſtimate the Quantity of Force exerted in producing one of theſe *Bendings*; in order to which let us conceive a Point of the Circle as D to be the very next to the Point B, though taken at ſome Diſtance in the Figure to avoid Confuſion of Lines. Parallel to the Diameter BF draw the Line CD, and Parallel to the Tangent BC the Line DG, draw alſo the Line BD, which will be the Diagonal of the Parallelogram CG. Now it is well known that ſuppoſing a Body would move from B to C, and by Virtue of ſome other Force acting upon it at the ſame Time, would move from B to G, it will neither move to C nor G, but to D; that is, its ſtraight Courſe will be *bent* into a circular one at B, for that is what is meant by its moving from B to D when B and D are contiguous, as is here ſuppoſed. BG therefore is the Space over which the revolving Body would move by that Action of the central Force which cauſes the Bend at B, or any other Point of the Circle. And as BG is equal to the Space over which the revolving Body would move by Virtue of the central Force, it will always be proportional to the Force it ſelf. The Value of that Line is therefore now to be determined. Join the Points D and F, then (by 31. El. 3.) the Triangle BDF will be rightangled at D, and conſequently (by 8 El. 6.) BG : BD :: BD : BF, from whence $BG = \frac{BDq}{BF}$; but when B and D are contiguous (as is here ſuppoſed) the Difference between the Chord BD and the Arch BD vaniſhes, ſo that $BG = \frac{\text{the Arch } BDq}{BF}$ Having thus found a Quantity proportionable to the Act of the central Force requiſite to retain a revolving Body in a Circle, we may proceed to eſtabliſh the following fundamental *Lemma.*

LEMMA I.

The Space that a Body retained in a Circle, by Virtue of any central Force, would move over in a given Time, by Virtue of that Force only, is equal to the Square of the Arch deſcribed in that Time, divided by the Diameter of the Circle.

Dem. The Action of a central Force upon a revolving Body is analogous to that of Gravity upon falling Bodies, and therefore the Space a Body would move over by Virtue of that Force only, is as the Square of the Time that Action continues: as then the Square of the Time the Arch BD is deſcribed in (the Points B and D being contiguous as before) is to $\frac{BDq}{BF}$ which by what was ſhewn above, is a Quantity equal to the Space the Body would move over towards the Center in that Time, ſo is the Square of the Time given to the Space the Body would move over by Virtue of that Force in that Time. If we then put *t* to expreſs the Time the Arch BD would be deſcribed in, T for the Time given, and S the Space the Body would move over towards the Center in that Time, we ſhall have this Proportion.

Viz.	1	$t^q : \frac{BD^q}{BF} :: T^q : S$
Let the Arch the Body would deſcribe in the Time T, be the Arch BN, then becauſe the Motion of the Body in the Circumference of the Circle is uniform, we have	2	$t : T :: BD : BN$
Squaring the laſt Proportion, we have	3	$t^q : T^q :: BD^q : BN^q$
Altering the Poſition of the Means in the firſt Proportion, we have	4	$t^q : T^q :: \frac{BD^q}{BF} : S$
Comparing the third and fourth Step together.	5	$BD^q : BN^q :: \frac{BD^q}{BF} : S$
Turning the 5th Step into an Equation.	6	$S \times BD^q = \frac{BN^q \times BD^q}{BF}$
Dividing the laſt by BD^q	7	$S = \frac{BN^q}{BF}$ Q. E. D.

Coroll. From this *Lemma* it follows, that the central Force requiſite to retain a revolving Body in the Circumference of a Circle, muſt be ſuch as ſuppoſing the projectile Motion ſtopp'd, would cauſe the Body in the Time it would otherwiſe deſcribe any Portion of that Circumference, to move towards the Center of the Circle over a Space or Diſtance of ſuch Length, as would ariſe from the Diviſion of the Square of that Portion of the Circumference by the Diameter of the Circle. And conſequently, it will in all Caſes be proportionable to the Square of the Arch the Body would deſcribe in a given Time divided by the Diameter of the Circle: Becauſe the Force exerted in any Time is proportionable to the Space it cauſes a Body to move over in that Time.

To illuſtrate this, ſuppoſe the Diameter of the Circle 25 Feet, and the Arch BN 10 Feet, and that the revolving Body moves over thoſe 10 Feet in a Minute. Then the Square of BN, which is 100, divided by the Diameter, or 25, gives 4 Feet: The central Force therefore at S, neceſſary to make the Body revolve in the Circle BNF at the Rate of 10 Feet in a Minute, muſt be ſuch, as ſuppoſing the Body left to itſelf at B, (that is, without any Motion at all towards C, or any Weight or Gravity of its own) would make it move over 4 Feet towards S in a Minute. Again, ſuppoſing Things as before; only that the Body revolves with but half its former Velocity, that is, that it deſcribes but 5 Feet of the Circumference of the Circle in a Minute; then the Square of that 5 Feet, which is 25, divided by 25, the Diameter gives 1; ſo that the central Force in this Caſe muſt be ſuch, that by Virtue thereof a Body deſtitute both of Weight and projectile Force, ſhall move over a Space 1 Foot long in a Minute. And converſely, if there be a Circle, whoſe Diameter is 25 Feet, and there be a Power lodged in the Center thereof capable of cauſing a Body to move over the Space of 4 Feet in a Minute; then in order that a Body may be made to revolve round in the Circumference of that Circle, it muſt be thrown forwards in a Direction at right Angles with the Radius of the Circle, with ſuch Velocity as would carry it over the Space of 10 Feet in that Time. And if the central Force be capable of cauſing it to move over but 1 Foot in a Minute, then it muſt be thrown forwards with a Velocity that ſhall make it move at the Rate of only 5 Feet: Or, ſince the Velocity is uniform, and therefore the Arch deſcribed is proportional to the Time it is deſcribed in, inſtead of an Arch as BN to calculate with, we may take the whole Circumference, and ſay, that *the Space a Body would move over by Virtue of the Central Force alone, in the Time it deſcribes the whole Circumference, is equal to that which would ariſe from the Diviſion of the Square of that whole Circumference by the Diameter of the Circle.* And this is that *exact* Adjuſtment between the central and projectile Force we have been ſo long in Search of.

Suppoſe then we would calculate, how far a Stone would fall by its own Weight, in the Time in which, if thrown forwards with a ſufficient Velocity, it would move quite round the Earth in a Circle. Let us call the Diameter of the Earth, that is, the Diameter of the Circle it deſcribes, 1, then the Circumference thereof would be 3 nearly, the Square of that 3, is 9, that divided by the Diameter 1, gives 9; ſo that a Stone would fall by Virtue of its own Weight through a Space equal to about 9 Diameters of the Earth in the Time it would revolve once round it. But it is here to be obſerved, that the Stone muſt be ſuppoſed to be of the ſame Weight during its whole Fall, that it is of at firſt; and ſo in all other Caſes, we ſuppoſe the Energy of the central Force to be the ſame at all Diſtances whatever, unleſs when the contrary is expreſſed.

We have hitherto been conſidering the Energy of the central Force, by comparing the Space it would cauſe a Body to move over in any certain Time, with the Space the Body ought to deſcribe along the Circumference of the Circle in the ſame Time: Let us now take it in another View, and compare the Velocity a Body ought to move with to deſcribe a Circle, with the Velocity it would acquire in moving over ſome certain Space by Virtue of the central Force only; from whence will ariſe a Propoſition very uſeful in Matters of this kind; and which for Diſtinction Sake we will call

LEMMA II.

The Velocity a Body ought to revolve with in the Circumference of a Circle, is ſuch as it would acquire in moving by Virtue of the central Force only, over half the Radius of that Circle. To explain this, let the Body be ſuppoſed at Reſt at B, and let it be acted upon by a central Force at S. Upon this the Body will proceed towards S with a Motion, which will increaſe every Moment (becauſe as in the Caſe of falling Bodies we ſuppoſe the central Force acting upon it with the ſame Intenſeneſs, whether the Body moves or not) and the Inſtant it gets half Way to the Center S, it will have acquired ſome certain Degree of Velocity; that's the Velocity it ought to move with along the Circumference of the Circle to render its projectile Force an exact Counterbalance to the centripetal one of the Power at S. Thus, ſuppoſe there were a Loadſtone fixed at S, and a Piece of Steel at B, diveſted of its own Weight, ſo that it might be at Liberty to move freely towards the Loadſtone, and to make the Inſtance parallel to the Caſe before us, let it be ſuppoſed that the Loadſtone attracts the Steel with the ſame Degree of Force at all Diſtances, then whatever Velocity that Steel would have when it had got half way to the Loadſtone, that's a Velocity with which if the Steel were thrown from B along the Tangent BC, it would move round in the Path BDFB, not approaching towards or receeding from the Loadſtone. But the Steel muſt be ſuppoſed to meet with no Reſiſtance from the Air, for if it does it will continually loſe ſomewhat of its Velocity, and ſo after ſome Revolutions would fall upon the Body of the Loadſtone. Again, ſuppoſe it were required to find the Velocity a Ball ought to be thrown forwards with at a ſmall Diſtance from the Surface of the Earth, that it ſhould move quite round in a Circle, keeping the ſame Diſtance from the Surface of it all the Way. It muſt be ſuch, as that Ball would acquire by falling half way from the Place it is thrown from to the Center of the Earth (for that is ſuppoſed to be the Seat of the Power that gives it its Weight, and thereby hinders it from moving forwards in the Direction it is thrown in.) And that Velocity is eaſily calculated from what was demonſtrated, Part I. Chap. V. § 4, or 5, ſuppoſing as *Hugens* has determined by very accurate Experiments, that a Body near the Surface of the Earth falls 15 $\frac{1}{12}$

Paris Feet in the firſt Second of Time. But enough of this by way of Explication, it is Time to demonſtrate what is here affirmed.

Dem. Let the Diameter of the Circle be 16 Feet, in which Caſe half the Radius will be 4 Feet, then by *Coroll.* of *Lemma* 1. the Arch the Body muſt run over in the Time it would paſs through thoſe 4 Feet by Virtue of the central Force, muſt be ſuch that the Square of it divided by the Diameter may be 4; that is, it muſt be 8 Feet, (becauſe the Square of 8, which is 64 divided by 16 the Diameter, gives 4.) but if it deſcribes an Arch of 8 Feet, in the Time it would fall through 4 Feet, it then moves with that very Velocity it might acquire by the Fall; becauſe as has been demonſtrated Part I. Chap. 5, § 7. if a Body moves uniformly with the Velocity it might acquire by a Fall, it will move over a Space equal to *twice* the Height of the Fall, in the Time thereof. Since then the Body in the Caſe before us, deſcribes an Arch of the Circumference of 8 Feet in the Time it would fall through 4 Feet, it moves therein with a *Velocity* it would acquire by falling through 4 Feet, that is, half the Radius. Q. E. D. To avoid a Circumlocution in the midſt of the Argument, I call the Space the Body would move over by Virtue of the central Force only, from its Analogy therewith, a *Fall.*

Univerſally thus, let the Diameter be equal to 4 times ſome certain Space, which Space let us call *d*; the Diameter then will be equal to 4 *d*, and half the Radius will be equal to the Space *d*, in which Caſe, the Arch the Body deſcribes in the Time it would fall through the Space *d* muſt by the abovementioned Corollary, be ſuch as that the Square of it divided by 4 times *d* may be once *d*, that is, the Arch muſt be twice *d* (becauſe the Square of twice *d*, which is 4 times *dd* divided by the Diameter 4 times *d*, gives once *d*) but as before, if the Body move uniformly through *twice* the Space *d*, in the time it would fall through *once* that Space, it then moves with that very Velocity it would acquire by falling through that Space, that is, *ex Hypoth*, through half the Length of the Radius. Q. E. D.

From what has been demonſtrated above, we may eaſily learn the Relation which the central Force that is requiſite to retain one Body in the Circle it deſcribes, has to that which is required for the ſame, or another Body revolving in a larger or leſs Circle. Which Relation is expreſſed in the following *Lemma.*

LEMMA III.

The central Force whereby one Body is retained in the Circle it deſcribes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the Radius of the Circle it deſcribes, to the Square of the Velocity of the latter divided by the Radius of the Circle which it deſcribes. If then we call the central Forces whereby the two revolving Bodies are retain'd in their reſpective Circles C and *c*, their Velocities V and *v*, and the Diſtances they revolve at, that is, the Radii of the Circles they deſcribe, R and *r*, we ſhall always have this Proportion, $C : c :: \frac{VV}{R} : \frac{vv}{r}$.

Dem. By Corollary of Lemma 1, the Energy of the central Force is proportionable to the Square of the Arch the Body deſcribes in a given Time, divided by the Diameter of the Circle. And Arches deſcribed in a given Time, are as the Velocities whereby they are deſcribed, and the Diameter of a Circle is as its Radius; conſequently the Action of the central Force is proportionable to the Square of the Velocity the Body moves with divided by the Radius of the Circle it deſcribes: The central Force therefore whereby one Body is retained in the Circle it deſcribes, is to that whereby a Body moving with a different Velocity or in a different Circle might be retained therein, as the Square of the Velocity of the former divided by the Radius of the Circle it deſcribes, to the Square of the Velocity of the latter, divided by the Radius of the Circle which it deſcribes. Q. E. D.

Theſe Things which are the Foundation of what follows being eſtabliſhed, we may now proceed to demonſtrate each Propoſition of this Chapter in the Order they lie in; the firſt of which is, That when two or more Bodies revolve at *equal* Diſtances from the Center, but with *unequal* Velocities, the central Forces neceſſary to retain them will be to each other, as the Squares of their Velocities.

Dem. Let the Circle which one of the revolving Bodies is ſuppoſed to deſcribe in this and the following Caſes, be BDF, in Fig. 26. and that which the other deſcribes be *b d f*, and let us call the Diſtances they revolve at from the common Center, that is, the Radii of the Circles they deſcribe, R and *r*; the Velocities they revolve with, V and *v*; and the central Forces neceſſary to retain them C and *c*. Then by Lemma 3. we ſhall

have for the firſt ſtep of this Demonſtration,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
But by the Propoſition R is equal to *r*, therefore	2	$C : c :: VV : vv$. Q. E. D.

PROP. II. When two or more Bodies move with *equal* Velocities, but at *unequal* Diſtances from the Center they revolve about, their central Forces muſt be inverſely as their Diſtances. That is, by how many times greater the Diſtance a Body revolves at is from the Center, ſo many times leſs Force will retain it (*b*).

PROP. III. When two or more Bodies perform their Revolutions in *equal* Times, but at *different* Diſtances from the Center they revolve about; the Forces requiſite to retain them in their Orbs will be to each other as the Diſtances they revolve at from the Center. For Inſtance, if one revolves at twice the Diſtance the other does, it will require a double Force to retain it; if it deſcribes a Circle three times as large as the other does, then a treble Force is requiſite, &c. (*c*)

(*b*) *Dem.* By Lemma 3, as before,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By the Propoſition V is equal to *v*, therefore,	2	$C : c :: \frac{1}{R} : \frac{1}{r}$. Q. E. D.

(*c*) In order to demonſtrate this the following Lemma will be of uſe.

LEMMA IV.

The periodical Time of a Body revolving in a Circle, is as the Diſtance it revolves at from the Center *directly*, and as its Velocity *inverſly*. And therefore in comparing the periodical Times of two Bodies together, if we call their periodical Times T and *t*, the Diſtances they revolve at, R and *r*; and their Velocities V and *v*, as before, we ſhall have this Propor-

Prop. IV. When two or more Bodies revolving at *different* Diſtances from the Center, are retained by *equal* centripetal Forces, their Velocities will be ſuch, that their periodical Times will be to each other as the Square Roots of their Diſtances. That is, if one revolves at four times the Diſtance another does, it will perform a Revolution in twice the Time the other does; if at nine times the Diſtance, it will revolve in thrice the Time, &c. (*d*)

Proportion, *viz.* $T : t :: \frac{R}{V} : \frac{r}{v}$.

Demonſtration of the Lemma. The periodical Time of a revolving Body, *cæteris paribus*, depends on the Diſtance it revolves at from the Center, becauſe the greater that Diſtance is, the greater is the Circle it deſcribes, and ſo the longer it is in coming round. Again, *cæteris paribus*, the greater its Velocity is, the ſooner it comes round; and therefore the periodical Time is directly as the former, and inverſely as the latter: Or, as the former divided by the latter; and conſequently $T : t :: \frac{R}{V} : \frac{r}{v}$ Q. E. D.

Demonſt. of the Propoſition. By Lemma 3.	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By Lemma 4,	2	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying the 2d Step by the Ratio of V to *v*, we have,	3	$TV : tv :: R : r$
By the Propoſition, T is equal to *t*, therefore,	4	$V : v :: R : r$
Comparing the 1ſt and 4th Steps together,	5	$C : c :: \frac{VV}{V} : \frac{vv}{v}$
That is,	6	$C : c :: V : v$. Q. E. D.

(*d*) *Dem.* By Lemma 3.	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by $R : r$	2	$CR : cr :: VV : vv$
By the Propoſition, C is equal to *c*, therefore	3	$R : r :: VV \;\; vv$

By

Prop. V. And in general, whatever be the Diſtances, the Velocities, or the periodical Times of the revolving Bodies, the retaining Forces will be to each other in a Ratio compounded of their Diſtances directly, and the Squares of their periodical Times inverſely. Thus for Inſtance, if one revolves at twice the Diſtance another does, and is three times as long in moving round, it will require $\frac{2}{9}$, that is, *two ninths* of the retaining Power the other does. For as, if the retaining Power was in a Ratio compounded of the Diſtance *directly* and *directly* alſo of the Square of the periodical Time, the latter Body would require *twice* the retaining Power the other would *multiplied by* 9, the Square of the periodical Time; ſo, as the latter Ratio is an *inverſe* one, it requires twice that Power *divided by* 9; that is, $\frac{2}{9}$ or *two ninths* of it. For in compounding of Ratio's the Way is, if they are both *direct* to *multiply* them both together; if one be *direct* and the other *inverſe*, to divide the *direct* by the *inverſe*; if both are *inverſe*, to multiply them both together, and to divide *Unity* by them: Thus, if the retaining Power had been

By Lemma 4.	4	$T : t :: \frac{R}{V} : \frac{r}{v}$
Squaring the laſt Step	5	$TT : tt :: \frac{RR}{VV} : \frac{rr}{vv}$
Comparing the 3d and 5th,	6	$TT : tt :: \frac{RR}{R} : \frac{rr}{r}$
That is,	7	$TT : tt :: R : r$
Extracting the Roots of the 7th Step,	8	$T : t :: \sqrt{R} : \sqrt{r}$ Q.E.D.

as the Diſtance *inverſely* and *inverſely* too as the Square of the periodical Time, then in the Caſe before us, the latter Body would have required a retaining Power, which ſhould have been as the Number 1 divided by 2, multiplied by 9; that is as 1 divided by 18: That is, if we ſuppoſe 1 to expreſs the central Force requiſite for the former Body, the Fraction $\frac{1}{18}$ would have expreſſed that which would have been neceſſary for the latter; or in other Words, their retaining Powers would have been related to each other, as 1 is to $\frac{1}{18}$; that is, the former would have been 18 times greater than the latter (*e*).

PROP. VI. If ſeveral Bodies revolve at different Diſtances from one common Center, and the retaining Power lodged in that Center decreaſes as the Squares of the Diſtances increaſe; that is, if at a double Diſtance it be

(*e*) *Dem.* By Lemma 3.	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by R to *r*,	2	$CR : cr :: VV : vv$
By Lemma 4,	3	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying the laſt by V to *v*,	4	$TV : tv :: R : r$
Dividing the laſt by T to *t*,	5	$V : v :: \frac{R}{T} : \frac{r}{t}$
Squaring the laſt,	6	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
Comparing the 2d and 6th,	7	$CR : cr :: \frac{RR}{TT} : \frac{rr}{tt}$
Dividing the laſt by R to *r*	8	$C : c :: \frac{R}{TT} : \frac{r}{tt}$. Q. E. D.

four times weaker; and at a treble Diſtance, 9 times weaker, &c. the Squares of the periodical Times of thoſe Bodies will be to each other as the Cubes of their Diſtances from the common Center. That is, if there be two Bodies, whoſe Diſtances when *cubed*, that is, multiplied by themſelves twice, are *double* or *treble*, &c. of each other, then their periodical Times will be ſuch, as that when *ſquared* only, that is, multiplied by themſelves *once*, they ſhall be alſo *double*, or *treble*, &c. of each other (*f*).

(*f*) *Dem.* By Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
By the Propoſition,	2	$C : c :: \frac{1}{RR} : \frac{1}{rr}$
Comparing the 1ſt and 2d Step,	3	$\frac{VV}{R} : \frac{vv}{r} :: \frac{1}{RR} : \frac{1}{rr}$
Multiplying by R to *r*,	4	$VV : vv :: \frac{1}{R} : \frac{1}{r}$
By Lemma 4,	5	$T : t :: \frac{R}{V} : \frac{r}{v}$
Multiplying by V to *v*,	6	$TV : tv :: R : r$
Dividing by T to *t*,	7	$V : v :: \frac{R}{T} : \frac{r}{t}$
Squaring the laſt,	8	$VV : vv :: \frac{RR}{TT} : \frac{rr}{tt}$
Comparing the 4th and the 8th,	9	$\frac{1}{R} : \frac{1}{r} :: \frac{RR}{TT} : \frac{rr}{tt}$
Multiplying by TT to *tt*	10	$\frac{TT}{R} : \frac{tt}{r} :: RR : rr$
Multiplying by R to *r*.	11	$TT : tt :: RRR : rrr$. Q.E.D.

And after like manner, we may eaſily demonſtrate the ſeventh Corollary of the fourth Propoſition of the firſt Book of Sir *Iſaac Newton*'s *Principia*. Which becauſe it is of more than ordinary Difficulty to Beginners, I here add. It is to this Effect.

When Bodies revolve in ſuch Manner that their periodical Times are to each other as ſome Power (which call *n*) of their

Diſtances, the centripetal Forces requiſite to retain them, will be to each other inverſely as twice that Power wanting one of their Diſtances; that is, as their Diſtances raiſed to the Power $2n-1$.

Dem. By Lemma 3,	1	$C : c :: \frac{VV}{R} : \frac{vv}{r}$
Multiplying by the Ratio of R to r,	2	$CR : cr :: VV : vv$
By the Suppoſition,	3	$T : t :: R^n : r^n$
By Lemma 4,	4	$T : t :: \frac{R}{V} : \frac{r}{v}$
Comparing the 3d and 4th,	5	$R^n : r^n :: \frac{R}{V} : \frac{r}{v}$
Multiplying by V to v,	6	$VR^n : vr^n :: R : r$
Dividing by R^n to r^n,	7	$V : v :: \frac{R}{R^n} : \frac{r}{r^n}$
Dividing the Numerators in the laſt Step by $R : r$, gives $1 : 1$, and dividing the Denominators $R^n : r^n$ by the ſame, brings them one Power lower, that is, to $R^{n-1} : r^{n-1}$ therefore the laſt Step is reducible to	8	$V : v :: \frac{1}{R^{n-1}} : \frac{1}{r^{n-1}}$
Squaring both Sides of the 8th Step, which in the Denominators R^{n-1} and r^{n-1} is done by doubling their Index $n-1$, we have	9	$VV : vv :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$
Comparing the 2d and 9th Steps,	10	$CR : cr :: \frac{1}{R^{2n-2}} : \frac{1}{r^{2n-2}}$
Dividing by R to r, which in the Denominators R^{2n-2} & r^{2n-2} is done by adding 1 to their Index $2n-2$, by which means it becomes $2n-1$, we have	11	$C : c :: \frac{1}{R^{2n-1}} : \frac{1}{r^{2n-1}}$ Q. E. D.

Scholium. The foregoing Demonſtrations would have proceeded with equal Succeſs, had we only taken the Antecedents, or the Conſequents in each Step; that is, only the large or only the ſmall Letters. Which Method, becauſe I ſhall make uſe of it hereafter, I will here give a Specimen of. Let it then be required to demonſtrate the firſt Propoſition of this Chapter over again in this latter Method.

By Lemma 3. The central Forces are as the Squares of the Velocities divided by the Radii of the Circles deſcribed, which put thus	1	$C : \frac{VV}{R}$
By the Propoſition, the Circles deſcribed have one common Radius, call it 1. Then	2	$R = 1$
From the 1ſt and 2d Step compared, we have	3	$C : VV.$

That is, the central Forces of the revolving Bodies are to each other as the Squares of their Velocities, which is the Senſe of the Propoſition.

Again, let it be required to demonſtrate the Corollary taken from Sir *Iſaac Newton*'s *Principia*, over again in this Method.

By Lemma 3. ——	1	$C : \frac{VV}{R}$
By the Suppoſition ——	2	$T : R^n$
By Lemma 4. ——	3	$T : \frac{R}{V}$
Comparing the 2d and 3d Steps	4	$\frac{R}{V} : R^n$
Multiplying by V	5	$R : VR^n$
Dividing both Sides by R, reduces R to 1, and brings R^n one Power lower, that is to R^{n-1}, ſo that we ſhall then have	6	$1 : VR^{n-1}$
Dividing by R^{n-1}	7	$\frac{1}{R^{n-1}} : V$
Squaring the laſt Step, which in the Quantity R^{n-1} is effected by doubling the Index $n-1$, we have	8	$\frac{1}{R^{2n-2}} : VV$
Multiplying the firſt Step by R	9	$CR : VV$
Comparing the two laſt	10	$CR : \frac{1}{R^{2n-2}}$
Dividing by R, which in the Quantity R^{2n-2} is done by adding 1 to the Index, we have	11	$C : \frac{1}{R^{2n-1}}.$

Q. E. D.

Secondly,

Secondly, Of the Forces *neceſſary to retain Bodies revolving in* other Orbs.

PROP. VII. If a Body be turned out of its rectilineal Courſe by Virtue of a central Force, which decreaſes as you go from the Seat thereof, as the Squares of the Diſtances increaſe; that is, which is inverſely as the Square of the Diſtance, the Figure that Body ſhall deſcribe, if not a Circle, will be a Parabola, an Ellipſe or an Hyperbola, and one of the Foci of the Figure will be at the Seat of the retaining Power. That is, if there be not that exact Adjuſtment between the projectile Force of the Body and the central Power neceſſary to cauſe it to deſcribe a Circle, it will then deſcribe one of thoſe other Figures, one of whoſe Foci will be where the Seat of the retaining Power is *(g)*.

(g) In order to ſhew this, it will be neceſſary to premiſe the following Lemma's.

LEMMA V.

The Velocity of a Body deſcribing any Curve, (if its Courſe is regulated by a retaining Power) is in one Point of the Curve to its Velocity in any other Point thereof, inverſely as a Perpendicular drawn from the Seat of the retaining Power to a Tangent of the Curve at the firſt Point, to a Perpendicular drawn from the ſame Place to a Tangent at the other Point. That is, in fewer Words, the Velocity of the Body is inverſely as a Perpendicular let fall from the Seat of the retaining Power to a Tangent at the Point where the Body is. Thus, if the Body be at D, a Point in Curve DEF, Fig. 27, and PD be a Tangent thereto, and SP a Perpendicular to that Tangent, drawn through S the Seat of the retaining Power, the Velocity of the Body at D, compared with its Velocity in other Parts of the Figure, will be inverſely as SP, that is, as the Quantity $\frac{1}{SP}$.

Dem. It has been demonſtrated, Part I. Chap. VIII. that revolving Bodies deſcribe equal Areas in equal Times; that is, if a Body deſcribes the Curve Line DEF, and the Arches DE, EF, *&c.* be run over in equal Times, the Areas DES, EFS, *&c.* will be equal; which Areas if we ſuppoſe the Times as ſmall as poſſible, may be conſidered as ſo many right-lined Triangles, becauſe in that Caſe the Lines DE, EF, *&c.* looſe their Curvity, and being produced are Tangents to the Curve. Produce the Line DE to P, and let fall the Perpendicular SP, then the Area of the Triangle DES is had by multiplying its Baſe DE into half SP; and the Area of EFS is equal to its Baſe EF multiplied by half a Perpendicular drawn from S to EF produced, *&c.* But theſe Triangles being deſcribed in equal Times, are all equal, the longer therefore their Baſes, the ſhorter their Perpendiculars. But the Baſes being run over in equal Times by the revolving Body, they are as the Velocity of the Body wherewith they are deſcribed; conſequently the greater the Velocity of the Body, the ſhorter is the Perpendicular to the Tangent where the Body is. Which is the Senſe of the Lemma.

LEMMA VI.

Whatever Curve a revolving Body deſcribes by Virtue of a centripetal Force, whether the Seat of that Force be within or without the Curve, the Action of that Force upon the Body when at any one Point of the Curve, muſt be to the ſame upon the Body when at another Point of the Curve, directly as the Diſtance of the Body from the Seat of the retaining Power and inverſely as the Cube of a Perpendicular let fall from the Seat of the retaining Power to a Tangent to the Curve at the Point where the Body is, multiplied by the Radius of a Circle of equal Curvature with the Figure at that Point. That is, if AX in Fig. 28, be the Curve deſcribed, and MDN, whoſe Center is C, be a Circle of equal Curvature with the Figure at the Point D, and PD be a Tangent to that Point, and SP a Perpendicular thereto drawn thro' S the Seat of the retaining Power, the central Force requiſite to act upon the Body at D muſt be as $\frac{DS}{SP^3 \times CD}$, that is, if we put R for the Radius CD, as $\frac{DS}{SP^3 \times R}$.

Dem. By Lemma 5. the Velocity of the revolving Body when at D is as $\frac{1}{SP}$, calling then that Velocity V,

we have —	1	$V : \frac{1}{SP}$
Squaring both Sides —	2	$VV : \frac{1}{SPq}$

Call CD the Radius of the Circle MDN, R; then by Lemma 3, the central Force neceſſary to retain the Body in that Circle, (or which is the ſame thing, at the Point D in the Curve ADX, becauſe they are both of equal Curvature there) would, if placed at C, be as $\frac{VV}{R}$. But whereas it is placed at S it acts obliquely to DC, and therefore to produce the ſame Effect, muſt act more forcibly in the Proportion of SD to DT, ſuppoſing ST perpendicular to DC; that is, becauſe SP is equal to DT, in the Proportion of SD to SP.

But by the Golden Rule	3	$SP : SD :: \frac{VV}{R} : \frac{VV \times SD}{SP \times R}$
Conſequently the central Force neceſſary if placed not at C but at S, is expreſſible by	4	$\frac{VV \times SD}{SP \times R}$
Putting then C for the Action of the central Force neceſſary to be placed at S, we have	5	$C : \frac{VV \times SD}{SP \times R}$ or $VV \times \frac{SD}{SP \times R}$
Comparing the 2d and 4th Steps together	6	$C : \frac{1}{SPq} \times \frac{SD}{SP \times R}$
That is,	7	$C : \frac{SD}{SP3 \times R}$. Q. E. D.

LEMMA VII.

Let AD Fig. 29 be a Parabola, whoſe Axis is AG, and its Focus S, and let PDF be a Tangent at the Point D, through the Focus S and Point of Contact D draw the Lines PS and DG both perpendicular to the Tangent PD, then will PS be equal to half DG.

Dem. Produce DP till it meets the Axis in X, and draw the Diameter DO, and join the Points D and S: Then becauſe DO, as being a Diameter is parallel to G X, the Angle FDQ is equal to DXS; 'tis alſo equal to the Angle XDS (Miln. Conic. Sect. Part IV. Prop. 4.) the Triangle XSD is therefore an Iſoſceles one, and SP being perpendicular to the Baſe XD, XP is equal to half XD; but PS and DG being parallel, the Triangles XPS and

XDG are ſimilar, and therefore PS is alſo equal to half DG. Q. E. D.

LEMMA VIII.

Let AD Fig. 30, be a Portion of a Parabola, an Ellipſe or an Hyperbola, one of whoſe Foci is S, its longer Axis AG, and its Parameter or Latus rectum L; and let PF be a Tangent at the Point D, join the Points D and S, through the latter of which draw the Line SP perpendicular to the Tangent, and through the former the Line DG perpendicular alſo to the Tangent, and croſſing the longer Axis in G, then will DG be equal to $\frac{L\times SD}{2SP}$.

Dem. Through the Point G draw the Line GT perpendicular to DS, then will the right-angled Triangles DGT and DSP be ſimilar, becauſe the Angles DSP and SDG are alternate, conſequently

	1	$DG : DT :: SD : SP$
But by Miln. Conic. Sect. Part IV. Prop. 6.	2	$DT = \frac{1}{2} L$
From the firſt and 2d Step compared together	3	$DG : \frac{1}{2} L :: SD : SP$
Turning the 3d Step into an Equation.	4	$DG = \frac{L\times SD}{2SP}$. Q. E. D.

LEMMA IX.

Things remaining as before, produce DG to C in Fig. 31, 32, 33; ſo that the Line DC may be equal to $\frac{L\times SD^3}{2SP^3}$, then will a Circle whoſe Center is C, and Radius CD, be of the ſame Curvature, and coincident with the Figure AD at the Point D, whether it be a Parabola, an Ellipſe or an Hyperbola.

Demonſtration.

I. *For the Parabola.* The ſame Lines being drawn as in the foregoing Figure, draw alſo through D, Fig. 31. the Diameter DY, and take a Portion of it as DI equal in Length to the Parameter of that Diameter, and let fall the Perpendicular IH meeting DG produced in H, and let L be the Latus rectum or Parameter of the Figure, then by the 3d Step of Lemma 8.

	1	$DG : \frac{1}{2} L :: SD : SP$
The Angle HDI wants IDF to make it a right one, the Angle PSD wants SDP to make it a right one, but IDF and SDP are equal, Miln. Conic. Sect. Part IV. Prop. 4. therefore HDI and PSD are ſo alſo, the Triangles therefore HDI and SDP are ſimilar; conſequently	2	$SD : SP :: DH : DI$.

From

From the 1ſt and 2d Steps compared together	3	$DG : \frac{1}{2} L :: DH : DI$
By Miln. Conic. Sect. Part IV. Prop. 10. Corol.	4	$DI = 4\,DS$
From the 3d and 4th Step compared	5	$DG : \frac{1}{2} L :: DH : 4\,DS$
Multiplying the Antecedents of the firſt Step by 4.	6	$4\,DG : \frac{1}{2} L :: 4\,SD : SP$
Compounding the fifth and ſixth Steps, that is multiplying them together	7	$4\,DG^2 : \frac{LL}{4} :: DH : SP$
Lemma 7.	8	$\frac{1}{2}\,DG = SP$
Comparing the 7th and 8th Steps	9	$4\,DG^2 : \frac{LL}{4} :: DH : \frac{1}{2}\,DG$
Turning the 9th Step into an Equation	10	$2\,DG^3 = \frac{LL \times DH}{4}$
Taking half the 10th Step	11	$DG^3 = \frac{LL \times DH}{8}$
From Lemma 8,	12	$DG^3 = \frac{L^3 \times SD^3}{8SP^3}$
Comparing the 11th and 12th Steps	13	$\frac{DH \times LL}{8} = \frac{LLL \times SD^3}{8SP^3}$
Dividing the 13th Step by $\frac{LL}{8}$	14	$DH = \frac{L \times SD^3}{SP^3}$
Halving the 14th Step	15	$\frac{1}{2}DH = \frac{L \times SD^3}{2SP^3}$
But by Suppoſition	16	$CD = \frac{L \times SD^3}{2SP^3}$
Comparing the two laſt Steps	17	$CD = \frac{1}{2}DH$

But DH being perpendicular to PF, and the Angle at I a right one, it is obvious a Circle whoſe Diameter is DH, (and conſequently by the laſt Step, whoſe Radius is DC,) will paſs through the Point I, 31. El. 3. and will have the Line PF for a Tangent to it at D; but by Miln. Conic. Sect. Part III. Prop. 11. Corol. 1. ſuch a Circle will be of equal Curvature with the Parabola at the Point D; and the Circle and the Parabola will be alſo coincident with each other at that Point, becauſe the Line PF is a Tangent common to both. Q. E. D.

II. *For the Ellipſis and Hyperbola:* Let AD in Fig. 32, be a Portion of an Ellipſis, in Fig. 33, a Portion of an Hyperbola, one of whoſe Foci is S, its Latus Rectum L, and its longer Axis AB; let PF be a Tangent at the Point D, and draw the Lines SP, SD, and DG as in Fig. 30; draw alſo the Diameter D*d* and produce it to I, ſo that DI may be equal to the Parameter of that Diameter; let fall the Perpendicular IH meeting DG produced in H; through the Center E draw the Semiaxis EQ, and the Diameter K*k* conjugate to D*d*; and to the longer Axis AB, the Ordinates DM and KN; and in the Hyperbola produce EK and HD till they meet in R. Then by the known Property of the Ellipſis and Hyberbola De L'Hoſpit. Liv. 2. Prop. 2. and

Liv. 3. Prop. 2.	1	$EQ^q\ CA^q :: DM^q : AM \times MB$
Simſon's Conic. Sect. Lib. 2. Prop. 19. and Lib. 3. Prop. 43.	2	$AM \times MB = EN^q$
Comparing the 1ſt and 2d Steps together, and extracting their Roots	3	$EQ : EA :: DM : EN$

The Diameter K*k* being conjugate to D*d* is by the Definition of ſuch a Diameter, parallel to the Tangent PF, and conſequently DH being perpendicular to that Tangent, the Triangle GER is a right-angled one, that Triangle therefore and the right-angled one KEN are ſimilar as having in the Ellipſe, their Angles at E vertical; in the Hyperbola, common: The rightangled Triangles GER and DGM are alſo ſimilar, as having their Angles at G in the Ellipſe, vertical; in the Hyperbola, common: And conſequently the Triangles KEN and DGM are ſimilar in each Figure.

Therefore	4	$DM : EN :: DG : EK$
Comparing the 3d and 4th Steps together	5	$EQ : EA :: DG : EK.$
The rightangled Triangles DER and DHI in the Ellipſe are ſimilar as having their Angles at D common, in the Hyperbola as having their Angles at D vertical, conſequently	6	$DE : DR :: DH : DI.$

By the Definition of the Parameter DI, for the Ellipſe De L'Hoſpit. Lib. 2. Definit. 13. for the Hyperbola Lib. 3. Definit. 15.	7	$DE : EK :: 2\,EK : DI$
The Extremes of the two laſt Steps being the ſame, the Product of the Means of the one muſt be equal to the Product of the Means of the other, that is	8	$2\,EK^q = DH \times DR$
Multiplying the 8th Step by EK	9	$2\,EK^3 = DH \times DR \times EK$
By Simſon's Con. Sect. Lib. 2. Prop. 20. for the Ellipſe, Lib. 3. Prop. 45. for the Hyperbola	10	$DR \times EK = EA \times EQ$
From the 9 and 10.	11	$2\,EK^3 = DH \times EA \times EQ$
Dividing the 11 by DH	12	$\frac{2\,EK^3}{DH} = EA \times EQ$
By the Definit. of a principal Parameter	13	$\frac{1}{2}L : EQ :: EQ : EA$
Wherefore from the laſt by Definit. 10. Elem. 5.	14	$\frac{1}{2}L : EA :: EQ^q : EA^q$
Multiplying the two laſt together	15	$\frac{LL}{4} : EQ \times EA :: EQ^3 : EA^3$
Comparing the 5th and 15th together, we have	16	$\frac{LL}{4} : EQ \times EA :: DG^3 : EK^3$
Multiplying the two laſt Terms of the 16th Step by $\frac{2}{DH}$, it becomes	17	$\frac{LL}{4} : EQ \times EA :: \frac{2\,DG^3}{DH} : \frac{2\,EK^3}{DH}$

Comparing the 12th Step and the 17th together | 18 | $\frac{LL}{4} : EQ \times EA :: \frac{2\,DG^3}{DH} : EA \times EQ$

The Conſequents in the 18th Step being the ſame, the Antecedents are equal, that is | 19 | $\frac{2\,DG^3}{DH} = \frac{LL}{4}$

Multiplying by DH | 20 | $2\,DG^3 = \frac{LL \times DH}{4}.$

But the laſt Step here is the ſame with the 10th in the Demonſtration for the Parabola, from whence therefore the ſame Concluſion is to be drawn here as there: for the Propoſition in Miln. Conic. Sect. referred to at the End of that Demonſtration, relates alſo to both the Ellipſe and the Hyperbola.

Demonſtration of the Propoſition above, to which this Note refers.

Let ADX, Fig. 28. be a Portion of the Figure the revolving Body deſcribes, whether it be a *Parabola*, an *Ellipſe*, or an *Hyperbola*; and let L be the Latus Rectum or Parameter of the Figure; let the Seat of the retaining Power be at S, one of the Foci thereof, and let the revolving Body be ſuppoſed at D: draw the Line PD a Tangent to the Curve at that Point, and let fall the Perpendicular SP. Perpendicular to the Tangent draw the Line DC, which let us ſuppoſe to be the Radius of a Circle as MDN, of the ſame Curvature, that the Figure ADX is at the Point D. Then it is obvious, that the ſame Force which placed at S would retain the revolving Body in the Circumference of the Circle when it came at D, will retain it in the other Figure, at that Point, they being at that Place one and the ſame Curve. But by Lemma 6, ſuch Force is as $\frac{DS}{SP^3 \times R}$. Calling then the retaining Power C; we ſhall have this Proportion, *viz.* | 1 | $C : \frac{DS}{SP^3 \times R}$

And ſince the Circle MDN is of the ſame Curvature and coincident with the Figure ADX, its Radius DC is by Lemma 9, equal to the Quantity $\frac{L \times SD^3}{2\,SP^3}$, putting then R for the Radius as before we ſhall have this Equation | 2 | $R = \frac{L \times SD^3}{2\,SP^3}$

Comparing the 1ſt and 2d Step together, we ſhall have this Proportion | 3 | $C : \frac{DS}{SP^3 \times \frac{L \times SD^3}{2\,SP^3}}$

Expunging SP³ as being both Multiplicator and Diviſor | 4 | $C : \frac{DS}{\frac{1}{2} L \times SD^3}$

Dividing

That a Body ſhall deſcribe Figures ſo different, when the Power that directs its Courſe is the ſame, and acts by one and the ſame Law, is owing to the Degree of Velocity the Body moves with. Thus ſuppoſing the central Power placed at S (in Fig. 34.) and that the revolving Body ſets off at D in the Direction DB perpendicular to the Line DS, with ſuch Velocity as it would obtain in falling by Virtue of that central Power only, half way from D to S, it will then, by Lemma 2, deſcribe a Circle as DKM, whoſe Center is the Point S. If it ſets off from D in the ſame Direction as before, but with a leſs Velocity, it then revolves within that Circle deſcribing an Ellipſe, as DLN, one of whoſe *Foci* is at S, the Seat of the retaining Power, and the other between that and the Point D, as at F. If it ſets off from D with ſomewhat greater Velocity than what it would acquire by the above-mentioned Fall, it will ſtill deſcribe an Ellipſe, as DEO, one of whoſe *Foci* ſhall be at S as before, but

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Dividing all by DS	5	$C : \frac{1}{\frac{1}{2} L \times DSq}$
But $\frac{1}{2}$ L is the ſame, whatever Point of the Figure deſcribed the revolving Body is at, and may therefore be conſidered as Unity, conſequently	6	$C : \frac{1}{DSq}$

That is, the retaining Power, if it be inverſely as the Square of the Diſtance, and ſeated in the *Focus* of the Figure, whether it be a *Parabola*, an *Ellipſe* or an *Hyperbola*, will direct the Courſe of the revolving Body in ſuch Manner, as that it ſhall deſcribe the ſaid Figure. Q. E. D.

the other ſhall be beyond it as at G. If it ſets off with a Velocity which is greater than that it would acquire by the above-mentioned Fall, in the Proportion of $\sqrt{2}$ to 1, that is, the Square Root of 2 to 1, it ſtill deſcribes a Figure, one of whoſe *Foci* is S as before, but the other, as G, goes off to an infinite Diſtance; that is, the Figure deſcribed becomes a Parabola, as DPQ, whoſe Focus is S. If it ſets off with a greater Velocity than this laſt, the Seat of the retaining Power remains ſtill at S, but the other *Focus* G goes yet farther off; that is, it comes on on the other Side the Point D as at H, and the Figure deſcribed becomes an Hyperbola, as DRT, whoſe Foci are S and H.

From hence it is obſervable, that of the four Figures there are but two wherein an exact Adjuſtment is requiſite between the retaining Power and the projectile Force, *viz.* the Circle and the Parabola. In the former of which, the Velocity muſt be ſuch as the Body would acquire by falling half way to S, in the latter it muſt exceed that in the Proportion of the Square Root of 2 to 1; that is, in the Proportion of ſomewhat leſs than $1\frac{1}{2}$ to 1. Thus for Inſtance, if one Degree of Velocity is requiſite to cauſe a Body to revolve in a Circle about the Point S; it will require about one Degree and an half to make it deſcribe a Parabola by Virtue of the ſame retaining Power ſeated in the ſame Point S.

Again, the ſame Figures may be deſcribed, though the Seat of the retaining Power be in the other Foci, F, G, or H; for Inſtance, if it be at F, in order that the Body may deſcribe the Ellipſe DLN, it muſt ſet off at D with the ſame Velocity it had in the former Caſe when it came to N: If it be at G, it will deſcribe the Ellipſe DEO, ſetting off at D with the Velocity it had before at O: And if G, the Seat of the retaining Power be at an infinite Diſtance, or in other Words (becauſe to ſay a determinate Point is at an infinite Diſtance, is a contradiction in Terms) if the retaining Power act in ſuch manner as it would do if the Seat of it was at an infinite Diſtance; that is, if it act upon the revolving Body in Lines parallel to DG; and with the ſame Degree of Force, whether the Body be at D, at P, or at Q, &c. it will deſcribe the Parabola DPQ with whatever Velocity it ſets off with from D. For to deſcribe a Circle round the Point G at an infinite Diſtance, it muſt ſet off with an infinite Velocity, becauſe, as obſerved above, it muſt ſet off with the Velocity it would acquire by falling half way to that Point; now a Body will deſcribe an Ellipſe round a given Point with any Degree of Velocity leſs then it will deſcribe a Circle with round the ſame, as it was obſerved that the Ellipſe DLN was deſcribed with leſs Velocity than the Circle DKM, but a Parabola is no other than an Ellipſe, one of whoſe *Foci* is at an infinite

Diſtance, the Body will therefore deſcribe a Parabola when the Seat of the retaining Power is at an infinite Diſtance, with any Degree of Velocity leſs than an infinite one.

This is the Reaſon, that Projectiles upon the Surface of the Earth are ſaid to deſcribe Parabola's, becauſe on Account of the great Diſtance of the Center of the Earth, compared with the Height Bodies can be thrown to, Gravity does as to Senſe, act upon them uniformly and in parallel Lines.

Again, when the Point G the Seat of the retaining Power goes off at an infinite Diſtance one way; it immediately, or indeed ſtrictly ſpeaking, at the ſame Inſtant, becomes the Point H at an infinite Diſtance the other way; ſo that thoſe two conſidered as at an infinite Diſtance, are as it were one and the ſame Point; and conſequently it is the ſame Thing whether the retaining Power be at G an infinite Diſtance to the *Right*, or at H an infinite Diſtance to the *Left*, and therefore the Figure will ſtill be a Parabola, and may alſo be deſcribed with any Degree of Velocity; only if it be at H, an infinite Diſtance to the left, the Power muſt be negative in reſpect of what it was before; that is, it muſt be of the *repulſive* Kind, as it is obvious it ought to be, to cauſe the revolving Body to deſcribe a Figure whoſe Convexity is turned towards the Seat of the retaining Power.

If the Seat of the retaining Power H comes nearer, ſo as to be at a finite Diſtance from the other Focus S, the Power muſt ſtill be negative, or repulſive; and the Figure deſcribed will be an Hyperbola, whoſe Foci are H and S. And the Hyperbola in this Caſe may be deſcribed whatever be the Velocity the Body ſets off with; for the central Force being of the repulſive kind, the Figure deſcribed will neceſſarily be convex towards the Seat thereof, that is, towards one of its Foci; but none of the Sections of the Cone have one of their Foci on the convex Side of the Figure, and at a finite Diſtance from its Vertex, except the Hyperbola (*b*).

Hence

(*b*) In order to evince the Truth of what is here affirmed, the following Lemma will be of Uſe.

LEMMA X.

Let the Curve Line DPQ in Fig. 35. repreſent a conic Section, one of whoſe *Foci* is S, and let the Circle DKR be of the ſame Curvature therewith at its Vertex D, then if the Curve be a Parabola, DR the Diameter of the Circle will be equal to, if it be an Ellipſe, it will be leſs, if an Hyperbola it will be greater than four times the Diſtance DS.

Demonſtrat. of the Lemma. (By Milnes Conic. Sect. Part III. Prop. 2. Coroll. 5.) DR the Diameter of the Circle is equal to the Parameter of the Axis of the Curve, whether it be a Parabola, Ellipſe or Hyperbola, which let us call, L.

Therefore	1	L = DR
1. But in the Parabola, by the Definition of the Parameter, (De L'Hoſpit. Con. Sect. Livr. I. Def. 5.)	2	L = 4 DS
Comparing 1ſt and 2d Step	3	DR = 4 DS. Q. E. D.

2. If

2. If the Curve be an Ellipſe, let F be its other Focus, then will DF + DS be equal to its longer Axis, De L'Hoſp. Con. Sect. Liv. 2. Cor. 2. of the Definitions; call its ſhorter Axis, X. Then from the Definition of the Parameter (De L' Hoſp. Conic. Sect. Liv. II. Def. 8.) we ſhall have this Proportion	4	$L : X :: X : DS + DF$
Turning the laſt Step into an Equation	5	$L = \frac{X^q}{DS + DF}$
(By De L' Hoſpit. Conic. Sect. Liv. 2. Corol. 4. of the Definitions) the Square of half X is equal to DS × DF, that is	6	$\frac{X^q}{4} = DS \times DF$
Conſequently	7	$X^q = 4\,DS \times DF$
Comparing the fifth and ſeventh Step	8	$L = \frac{4\,DS \times DF}{DS + DF}$
Turning this Equation into a Proportion	9	$L : 4\,DS :: DF : DF + DS$
Comparing the 1ſt and the 9th Step	10	$DR : 4\,DS :: DF : DF + DS$

But DF is leſs than DF + DS, therefore DR is leſs than 4 DS. Q. E. D.

3. If the Curve be an Hyperbola, let H be its other Focus, then will DH — DS be equal to its longer Axis, by De L'Hoſpit. Con. Sect. Liv: 3. Corol. 2. of the Definitions: And call as before its ſhorter Axis X; then from the Definition of the Parameter (De L'Hoſpit. Conic. Sect. Liv. 3. Defin. 8.) we ſhall

have this Proportion	11	$L : X :: X : DH - DS$
Turning the laſt Step into an Equation	12	$L = \frac{X^q}{DH - DS}$
(By De L' Hoſpit. Conic. Sect. Liv. 3. Coroll. 4. of the Definitions) the Square of half X is equal to DS × DH, that is	13	$\frac{X^q}{4} = DS \times DH$
Therefore	14	$X^q = 4\,DS \times DH$

Comparing

Comparing the 11th and 13th Steps	14	$L = \frac{4\,DS \times DH}{DH - DS}$
Turning this into a Proportion	15	$L : 4\,DS :: DH : DH - DS$
Comparing the 1ſt and 14th Steps	16	$DR : 4\,DS :: DH : DH - DS$

But DH is greater than DH—DS, therefore DR is greater alſo then 4 DS. Q. E. D. This being premiſed, we may proceed to confirm what was laid down in the Text above.

1. Let the Curve DPQ be a Parabola, and the Circle DKR being of the ſame Curvature with the Parabola at the Point D, as ſuppoſed above, the Velocity a Body ought to ſet off with from D, to deſcribe the Circle, is the ſame with that with which it ought to ſet off, to deſcribe the Parabola: but the Velocity it ought to ſet off with to deſcribe the Circle, is by Lemma 2. ſuch as it would acquire by falling to S, becauſe DS being (by Lemma 10.) a Quarter of DR, is equal to half the Radius of the Circle; call this Space 2: but the Velocity it ought to ſet off with to deſcribe a Circle, whoſe Center is S, is ſuch as it would acquire by falling through half DS, which Space (becauſe the whole Line DS is called 2) muſt be called one. Now the Velocities Bodies acquire by falling through certain Spaces are to each other as the Square Roots of thoſe Spaces (by Part I. Chap. 5. § 6.) the abovementioned Velocities therefore are to each other as $\sqrt{2}$ to 1. That is, the Velocity a Body ought to ſet off with from D (the Vertex of the Figure, and in a Direction perpendicular to the Axis DS, which I would always have ſuppoſed) in order to revolve in a Parabola whoſe Focus is S, is to that it ought to have at the ſame Place (and in the ſame Direction) to deſcribe a Circle whoſe Center is S, as $\sqrt{2}$ to 1.

2. Had the Figure DPQ been an Ellipſe, a Quarter of DR the Diameter of a Circle of the ſame Curvature therewith, had been (by Lemma 10.) leſs than DS; and ſo the Velocity the revolving Body muſt have had to deſcribe that Circle, or (which is the ſame Thing, becauſe of their equal Curvatures) the Ellipſe, would have been acquirable by falling through a leſs Space than DS, and therefore would have been a leſs Velocity. That is, it would have exceeded the Velocity requiſite for a Circle, whoſe Center is S the Seat of the retaining Power, in a leſs Proportion than that of $\sqrt{2}$ to 1.

3. If the Figure DPQ be an Hyperbola, a Quarter of DR is bigger than DS (by Lemma 10.) and ſo the Space a Body muſt fall through to get a competent Velocity for that Curve, will be greater than what it muſt fall through to obtain a competent Velocity for the Parabola: The Velocity therefore requiſite for

Hence we may obſerve, that, ſuppoſing the Motion of a revolving Body be ſo adjuſted to the Power by which it is retained, that it ſhall deſcribe a *circular* Orb, and its Velocity be afterwards altered by ſome Accident, and thereby the Adjuſtment thereof to the central Force deſtroyed, that Body ſhall not immediately fall to the Center: the Form of its Orbit only will be altered, and from being circular, will become elliptical, parabolical, or hyperbolical, according to the Alteration made in its Velocity. Let its Velocity be diminiſhed in any Degree whatever, or let it be increaſed, ſo it be in a leſs Proportion than as $\surd 2$ to 1, its Orbit becomes an Ellipſe; if it be increaſed exactly in that Proportion, a Parabola; if in a greater, an Hyperbola. Hence we have a probable Reaſon why the Orbits of the Planets are now elliptical; for ſuppoſing them to have been circular at firſt, as it is not unlikely they were, yet upon the firſt Diſturbance in their Motions, whether from their mutual Action upon each other, the Reſiſtance of the Medium they revolve in, the Acceſs of a Comet, or any other Cauſe whatever; and whether their Motion be increaſed or decreaſed thereby, provided it be not increaſed in a Degree

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the Hyperbola exceeds the Velocity required for a Circle whoſe Center is S, in a greater Proportion than that of $\surd 2$ to 1. From which all that is aſſerted above in the Text to which this Note refers, except what is there otherwiſe accounted for, may be collected.

gree beyond the Proportion above laid down, their Orbits would immediately become elliptical. Should the Motion of any of them be increaſed in or above the ſaid Proportion, its Orbit would accordingly be changed into a Parabola, or an Hyperbola, and the Planet would go off, never to return.

Prop. VIII. If the Force of the central Power decreaſes as the Square of the Diſtance increaſes, and ſeveral Bodies revolving about the ſame, deſcribe Orbits that are *elliptical*, the Squares of the periodical Times of thoſe Bodies will be to each other as the Cubes of their middle Diſtances from the Seat of that Power (*k*).

(*k*) To demonſtrate this Propoſition, let the following Lemma be premiſed.

LEMMA XI.

The Area a revolving Body deſcribes, is as its Velocity multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is.

Dem. of the Lemma. Let AB in Fig. 36. be the Curve deſcribed, DC a Portion of the Curve deſcribed in the leaſt Time poſſible, then will DC repreſent the Velocity of the Body; produce DC to P, then will DP be a Tangent to the Curve: From S the Seat of the retaining Power let fall the Perpendicular SP, draw DS and CS; then will DCS be the Area the Body deſcribes, but DC by reaſon of its ſhortneſs may be conſidered as a ſtraight Line, the Area DCS is therefore a Triangle, which is had by multiplying the Baſe DC into half the Perpendicular SP, therefore it is proportionable to DC multiplied by the whole Line SP, that is, to the Velocity of the revolving Body multiplied by a Perpendicular let fall from the Seat of the retaining Power to a Tangent at that Point of the Curve where the Body is. Q.E.D.

Dem. of the Propoſition. Let ADB and *adb* in Fig. 37 and 38. repreſent two Ellipſes, whoſe principal Parameters are L and L, and let S be one of the Foci of the one, and *s* one of thoſe of the other, and let the Ellipſes be ſuppoſed to lie in ſuch manner upon one another, that the Points S and *s* may be one and the ſame Point, which let us ſuppoſe to be the Seat of the retaining Power; and let the revolving Bodies be at D and *d*, through which draw the Tangents PD and *pd*, the Perpendiculars PS and DC, and *ps* and *dc*, and the Lines DS and *ds*; and let DC and *dc* be Radii of Curvature to the Points D and *d*; and let the Force of the retaining Power at the Diſtance SD be called F, and at the Diſtance *sd* be called *f*, and let the Velocities of the revolving Bodies when at the Points D and *d* be called V and *v*. Then becauſe by the Propoſition the Force of the central or retaining Power is ſuppoſed to decreaſe as the Squares of the Diſtances SD and *sd* increaſe,- that is, to be reciprocally as the Squares of thoſe Diſtances, we ſhall have for the firſt Step of the following Proceſs, this Analogy, *viz.* $\Big|\,1\,\Big|\; F : f :: \frac{1}{SDq} : \frac{1}{sdq}$ Parallel to the Tangents draw the Lines SH and *sh*, and let the Force of the retaining Power which acts upon the Body at D in the Direction DS be reſolved into two others, *viz.* DP and DH, the former along the Tangent DP, the latter along the Radius of Curvature DC, then it is obvious that it is by this latter Force only that the revolving Body is retained in the Circle whoſe Radius is DC, that is, in the Ellipſe at the Point D, the other Force along the Tangent only accelerates the Body when moving towards A, and retards it if moving towards B. To compute the Quantity of Force in the Direction DC, ſay as DS is to DH ſo is the oblique Force F, or inſtead thereof $\frac{1}{SDq}$ (ſee Step the 1ſt) to a fourth Number, which fourth Number by the Golden Rule will be $\frac{DH}{SD^3}$, or (becauſe SP is equal to DH) $\frac{SP}{SD^3}$; and in like manner we ſhall have $\frac{sp}{sd^3}$ for the Force retaining the other Body in the Circle whoſe Center is *c*; but by Lemma 3. the Force neceſſary to retain a Body in a Circle whoſe Radius is CD is expreſſible by the Square of its Velocity divided by the Radius of the Circle it deſcribes, that is, in the preſent Caſe by $\frac{Vq}{CD}$. And in like manner the Force retaining the other Body in its Circle, will be $\frac{vq}{cd}$, conſequently $\Big|\,2\,\Big|\; \frac{SP}{SD^3} : \frac{sp}{sd^3} :: \frac{Vq}{CD} : \frac{vq}{cd}$

Multiplying

Multiplying the laſt Step by CD : *cd*	3	$\frac{SP}{SD^3} \times CD : \frac{sp}{sd^3} \times cd :: V^q : v^q$
Since CD is ſuppoſed to be the Radius of a Circle of equal Curvature with the Figure, we ſhall have by Lemma 9. theſe Equations, *viz.*	4	$CD = \frac{L \times SD^3}{2\,SP^3}$, and $cd = \frac{l \times sd^3}{2\,sp^3}$
Comparing the 3d and 4th Steps	5	$\frac{SP}{SD^3} \times \frac{L \times SD^3}{2\,SP^3} : \frac{sp}{sd^3} \times \frac{l \times sd^3}{2\,sp^3} :: V^q : v^q$
That is, by reducing the laſt Step to lower Terms	6	$\frac{L}{SP^q} : \frac{l}{sp^q} :: V^q : v^q$
Extracting the Square Root of each Term	7	$\frac{\sqrt{L}}{SP} : \frac{\sqrt{l}}{sp} :: V : v$
Multiplying by SP : *sp*	8	$\sqrt{L} : \sqrt{l} :: V \times SP : v \times sp$
But by Lemma 2. the Area one Body deſcribes is to that which the other deſcribes in the ſame time, as V × SP to *v* × *sp*, which Areas call A and *a*, and we ſhall have this Proportion, *viz.*	9	$A : a :: V \times SP : v \times sp$
Comparing the 8th and 9th Steps	10	$A : a :: \sqrt{L} : \sqrt{l}$

Now the Area of the whole Ellipſe ADB is to that of the other *adb*, as AB the longer Axis multiplied by QN the ſhorter, to *ab* × *qn*, thoſe Quantities therefore may be put for the Areas themſelves: If then we call the Time in which the Areas A and *a* are deſcribed in, one, (as one Hour, or one Minute ſuppoſe) and the Times the whole Ellipſes are deſcribed in, T and *t*; we ſhall have (ſince the Area a revolving Body deſcribes is proportional to the Time it is deſcribed in) the following Analogies, *Viz.*

Viz.	11	$1 : T :: A : AB \times QN$
And	12	$1 : t :: a : ab \times qn$

Comparing

Comparing the 3 laſt Steps together we have the two following ones, *viz.*	13	$1 : T :: \sqrt{L} : AB \times QN$
	14	$1 : t :: \sqrt{l} : ab \times qn$
By De L'Hoſpital Conic. Sect. Liv. 2. Def. 8.	15	$AB : QN :: QN : L$
Turning the laſt Proportion into an Equation.	16	$AB \times L = QNq$
Extracting the Square Roots of each Side of the Equation	17	$\sqrt{AB} \times \sqrt{L} = QN$
And in like manner	18	$\sqrt{ab} \times \sqrt{l} = qn$
Comparing the 13th and 17th Steps together	19	$1 : T :: \sqrt{L} : AB \times \sqrt{AB} \times \sqrt{L}$
And comparing the 14th and 18th together	20	$1 : t :: \sqrt{l} : ab \times \sqrt{ab} \times \sqrt{l}$
Dividing the latter Part of the 19th Proportion by $\sqrt{L}$	21	$1 : T :: 1 : AB \times \sqrt{AB}$
Dividing the latter Part of the 20th Proportion by $\sqrt{l}$	22	$1 : t :: 1 : ab \times \sqrt{ab}$
Comparing the 21ſt and 22d	23	$T : t :: AB \times \sqrt{AB} : ab \times \sqrt{ab}$
Squaring each Term	24	$TT : tt :: ABq \times AB : abq \times ab$
That is	25	$TT : tt :: AB^3 : ab^3$

But half the longer Axis AB is equal to the middle Diſtance SN, (De L'Hoſpital Conic. Sect. Liv. 2. Corol. 3. of the Definitions) therefore ſince Halves are proportionable to their Wholes 26 $TT : tt :: SN^3 : sn^3$. Q. E. D.

Prop. IX. If the retaining Power decreaſes ſomething faſter as you go from the Seat thereof (or which is the ſame thing, increaſes ſomething faſter, as you come towards it) than in the Proportion mention'd in the laſt Propoſition, and the Figure the revolving Body deſcribes be not a Circle, the Axis of that Figure will turn the ſame Way the Body revolves; but if the ſaid Power decreaſes (or increaſes) ſomewhat ſlower than in that Proportion, the Axis of the Figure will turn the contrary Way. Thus, if a revolving Body as D, Fig. 39. paſſing from A towards B deſcribes the Figure ADB, whoſe Axis AB at preſent points towards M and N, and the Power whereby it is retained decreaſes faſter than the Square of the Diſtance increaſes, after a Number of Revolutions the Axis of the Figure will point towards O and P, and after that towards Q and R, &c. revolving round the ſame Way with the Body, and if the retaining Power decreaſes ſlower than in that Proportion, the Axis will turn the other Way. (*i*)

(*i*) *Dem.* Let S in Fig. 40. repreſent the Seat of the retaining Power, and let a Body as D, deſcribe the Orb ADB paſſing from A towards B; and let another as N deſcribe a revolving Orb as MNO equal and ſimilar to the former; and let both the Orbits be deſcribed in the ſame Time; and let the Velocity wherewith the Orb MNO revolves be ſuch, that the Angle ASN ſhall always be proportionable to the Angle ASD. To explain this a little farther, ſuppoſe that when the Bodies N and D ſet out from M and A, thoſe two Points were then together at A; or that the Orbits did then coincide; but that N advances forwards in ſuch Manner, as that if the Angle ASN is at any time double, treble, or in any other Proportion of the Angle ASD, it ſhall always be ſo: That is, that the Line SN ſhall always move faſter or ſlower than the Line SD in ſome certain Proportion; and that the Point M ſhall follow or go from the Body N, ſo as that the Arch MN ſhall always be equal to the Arch AD. And let it farther be ſuppoſed that the Line SN is always equal to SD.

The firſt Inquiry which ariſes here is, whether this be a poſſible Caſe; that is, whether the Body N moving with the above-mentioned Reſtrictions, the Line SN can deſcribe Areas proportionable to the Times, becauſe as was ſhewn, Part. I. Chap. 8. that is always requiſite where Bodies revolve round a central Force, and are retained in their Orbits thereby. Let it then be conſidered in the firſt Place, that the Body D revolving in the quieſcent Orb ADB may do ſo, its Motion being clogged with none of thoſe Suppoſitions; ſecondly, that the Line NS is always equal to DS, and the Velocity wherewith its Extremity N moves forwards, is proportionable to that which D the Extremity of the other moves with; and conſequently the Areas which one deſcribes, (for the Areas depend ſolely on the Length and Velocity of the Lines that deſcribe them) will be proportionable to thoſe which the other deſcribes: Since then thoſe of the former may be proportionable to the Times they are deſcribed in, it is poſſible thoſe of the latter may be ſo too; the Suppoſition therefore above laid down is not abſurd.

The next Inquiry is, by what Law the Action of the central Force at S muſt decreaſe as we recede from it, that a Body as N may deſcribe the revolving Orb MNO in the manner abovementioned. Or, which will anſwer our Purpoſe as well, how the Force neceſſary to retain the Body N deſcribing the revolving Orb MNO, muſt differ from that which is requiſite to retain the other Body D deſcribing the quieſcent Orb ADB equal and ſimilar thereto.

In order to this, let DR, in Fig. 41, repreſent a Portion of the quieſcent Orb deſcribed in the leaſt Time poſſible, this then may be conſidered as a ſtrait Line; and let the Motion of the Body along this Line be reſolved into two, the one towards S along the Line DT, and another along the Line TR at right Angles therewith; then will DT repreſent the Velocity wherewith the Body deſcends towards S, and conſequently the Action of the central Force whereby it is retained in that Arch; and TR will repreſent that by which it advances forwards in the mean time. Let it now be ſuppoſed that the Orb ADB advances forwards, while the Arch DR is deſcribing; but not by Virtue of any additional Force in the Center, but by ſome extrinſic one, no matter what: That is,

is, ſuppoſe that the Line SD turns round the Point S faſter than it would otherwiſe have done, carrying the Body with it ſo much the faſter; but that the Body deſcends towards S along that Line juſt as it did before: That is, in other Words, that the Line DT which repreſents the Deſcent of the Body, is of the ſame Length as before; but that the other Line TR which repreſents the Progreſs of the Body forwards, is longer than it was in the other Caſe. Let it then become TQ; in which Caſe, the Body moving over the Line DT and the Line TQ in the ſame Time, will in reality get to the Point Q: But obſerve, that the Point Q is farther from S than the Point R is, becauſe the Angle at T is a right one; whereas the Law which we laid down above for the Motion of a Body deſcribing a revolving Orb was ſuch, that at the End of the Time in which DR would be deſcribed in the quieſcent Orb, the Body in the revolving Orb, ſhould be at the ſame Diſtance from S as if its Orb had not revolved; if then round the Point S, we deſcribe a Circle as RFG, and thro' S draw the Line QG, N muſt be the Point the Body muſt come to. Since then the Body in deſcribing the revolving Orb without any Addition to the central Force above what would cauſe it to deſcribe the quieſcent one, would come to Q; but with ſuch an Addition as is neceſſary to make it revolve in the manner above laid down, it comes to N, it is obvious that the Line QN being the Diſtance between thoſe two Points will aptly repreſent that Addition. It remains then to get the Meaſure of that Line, which may be done in the following manner.

Produce QT to F, then (by 35. El. 3.) will the Rectangle QN × QG be equal to QR × QF. Therefore

$$1 \quad QN \times QG = QR \times QF$$

Dividing by QG

$$2 \quad QN = \frac{QR \times QF}{QG}$$

Let it be obſerved then in the firſt Place, that as the Motion of revolving Bodies is ſuch that they deſcribe equal Areas in equal Times, the longer the Line DS is (that is the nearer D is to A, ſuppoſe) the ſhorter the Line TR muſt be, and for the ſame Reaſon alſo the Line TQ: That is, both TR and TQ are reciprocally as SD, which let us put thus

$$3 \quad TR : \frac{1}{SD}$$

And

$$4 \quad TQ : \frac{1}{SD}$$

Obſerve ſecondly, that ſince RF is a Chord of a Circle, and ST perpendicular to it

$$5 \quad RT = TF$$

From the Figure	6	$QR = TQ - TR$
Becauſe RT and TF are equal	7	$QF = QT + TR$

Since then by the 3d and 4th Steps, TR and TQ are both reciprocally as SD. and by the 6th and 7th Steps, QF is equal to the Sum of thoſe Lines, and QR is equal to their Difference, it is obvious that QF and QR are alſo each reciprocally as SD,

which put thus	8	$QF : \frac{1}{SD}$
And	9	$QR : \frac{1}{SD}$
Multiplying the 8th and 9th Steps together	10	$QR \times QF : \frac{1}{SD^q}$

Obſerve alſo that when the Points D, T, R, Q and N coincide, which muſt ever be ſuppoſed, becauſe we are all this Time conſidering only what is done at one and the ſame Point of the Curve ADB, QN is nothing with reſpect to NG, therefore QG and NG are the ſame Line; therefore QG may be ſaid to be as NG, or as half of it, *viz.* RS, or which will be the ſame Thing, DS; therefore inſtead of QG in the 2d Step, we may put DS,

and then it will ſtand thus	11	$QN = \frac{QR \times QF}{DS}$
The 11th Step without altering the Value of it, may be ſet thus	12	$QN = QR \times QF \times \frac{1}{DS}$
Comparing the 10th and 12th Steps together we have	13	$QN = \frac{1}{DS^q} \times \frac{1}{DS}$
That is	14	$QN = \frac{1}{DS^3}$.

That is, the Line QN, or the additional central Force requiſite to cauſe a Body to move in a revolving Orb muſt be reciprocally as the Cube of the Diſtance of the revolving Body from the Seat of the retaining Power. Whereas then, when the centripetal Force decreaſes as the Square of the Diſtance increaſes, the revolving Body deſcribes a Parabola, an Ellipſe, or an Hyperbola; if to that centripetal Force be ſuperadded another, that ſhall decreaſe as the Cube of the ſaid Diſtance increaſes; thoſe two Forces acting conjointly upon a revolving Body ſhall cauſe it to deſcribe the ſame Figure as before, but the Axis thereof ſhall revolve the ſame Way that the Body does.

But obſerve here, that if a Force decreaſes faſter in any Degree whatever than the Square of the Diſtance increaſes, but ſlower than the Cube, that Force is the Sum of two Forces, one of which decreaſes as the Square, the other as the Cube of the

Thus it is the heavenly Bodies, *viz.* the Planets both primary and ſecondary, and alſo the Comets, perform their reſpective Revolutions. The Figures in which the primary Planets and the Comets revolve, are Ellipſes, one of whoſe Foci are at the Sun. The Areas they deſcribe by Lines drawn to the Center of the Sun, are in each proportionable to the Times in which they are deſcribed. The Squares of their periodical Times are to each other as the Cubes of their middle Diſtances from the Sun. The ſecondary Planets deſcribe alſo Circles or Ellipſes,

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Diſtance increaſes. Thus for Inſtance, let the Diſtances be as 2 to 1, the Squares of them will be 4 to 1, their Cubes 8 to 1; and let the Forces be as 6 to 1; I ſay, this Proportion will ariſe from adding 4 to 1 to 8 to 1; for 4 to 1 added to 8 to 1, is 12 to 2, that is 6 to 1. Again, let the Forces be as 5 to 1; I ſay, this alſo will ariſe from adding 4 to 1 to 8 to 1; for inſtead of 4 to 1 let us take 12 to 3 which is the ſame Thing, this ſuperadded to 8 to 1 makes 20 to 4, that is, 5 to 1. Again, let the Forces be as 7 to 1; this alſo is the Sum of 4 to 1 added to 8 to 1; for inſtead of 8 to 1 we may take 24 to 3, which added to 4 to 1 gives 28 to 4, or 7 to 1. And the ſame for Fractions: ſo that let a Proportion be what it will between 4 to 1 and 8 to 1, it may be conſidered as ariſing from 8 to 1 added to 4 to 1. When the retaining Power therefore by which a revolving Body is kept in its Orbit, decreaſes faſter than the Square of the Diſtance increaſes, but not ſo faſt as the Cube thereof does; it is a Power decreaſing as the Square, having another Power decreaſing as the Cube of the Diſtance ſuperadded to it; and therefore as the Propoſition in the Text aſſerts, if the retaining Power decreaſes ſomething faſter, *&c.* Q. E. D. As to the Figures turning the contrary Way, when the centripetal Forces decreaſes ſomewhat ſlower than as the Square of the Diſtance increaſes; that is but the Converſe of this, and therefore needs no particular Demonſtration.

Ellipſes, one of whoſe Foci are in the Center of their Primary ones. The Area that each deſcribes by Lines drawn to the Center of its Primary, is proportionable to the Time it is deſcribed in. The Squares of the periodical Times of the Secondaries belonging to the ſame Primary, are to each other as the Cubes of their Diſtances from their Primary. And above all, that very Force by which Stones or heavy Bodies fall to the Surface of the Earth with us, is at the Diſtance of the Moon, juſt ſufficient to retain her in her Orbit, ſuppoſing it to decreaſe as we go from the Center of the Earth in that Proportion, in which the Cauſe, that retains her and the reſt of the Planets in their Orbs, whatever it be, muſt do to make them elliptical *(l)*. And farther, which alſo is a

neceſſary

(l) To find out whether this be ſo or not, let us calculate what Velocity the Moon would acquire, were ſhe to fall half way to the Center of the Earth by Virtue of that Gravity, whereby heavy Bodies tend to the Earth; and compare it with the Velocity ſhe moves with in her Orbit; becauſe by Lemma 2 of this Chapter, if ſhe be retained in her Orb by that Gravity, thoſe Velocities ought to be the ſame. The mean Diſtance of the Moon from the Earth in round Numbers is 60 Semidiameters of the Earth, therefore the Force of Gravity at the Diſtance of the Moon is the Square of 60 times leſs than it is at the Surface of the Earth; therefore the Fall of a Body at that Diſtance in a Second of Time, will be ſo many times leſs than it is here; but the Fall of a Body here in a Second, is about 15 *Paris* Feet, conſequently at the Moon it is but ,004166 Feet. The Space therefore it would move over in a Second, with the Velocity acquired by that Fall, is, by Part I. Chap. 5. § 7. twice that Quantity, *viz.* ,008332 Feet, which Number therefore may be put to expreſs its Velocity. Now becauſe the Velocities Bodies

acquire

neceſſary Conſequence of what we are about to lay down, the Motion of the Planet *Saturn* is obſerved to be diſturbed by *Jupiter*; and the Secondaries of *Jupiter* to be diſturbed in their Motions upon the nearer Approach of *Saturn*; and the Courſe of the Moon is inceſſantly altered by the Action of the Sun, in ſuch manner as to cauſe all thoſe Irregularities mentioned in Chapter the 8th of this Part. All which, together with the Preceſſion of the Equinoctial Points, the Nutation of the Poles of the Earth, and the Phænomena of the Tides, which naturally flow from it, make it extremely probable that there is a Virtue diffuſed about the Sun and Planets, not unlike that of Attraction, which decreaſes as the Squares of the Diſtances from the Centers of thoſe Bodies increaſe, and

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acquire by falling, are as the ſquare Roots of the Spaces they fall through, Part I. Chap. 5. § 6. ſay, as the Square Root of ,904166, is to the Square Root of 590866170, which is half the Semidiameter of the Moon's Orbit, (ſuppoſing the Semidiameter of the Earth to be 19695539 Feet, which is *Caſſini*'s Meaſure, and the Diſtance of the Moon from the Earth to be 60 Semidiameters as above) ſo is ,008332 to 3135 Feet, which is therefore the Space the Moon would move over in a Second with the Velocity ſhe would acquire by falling half way to the Center of the Earth. But this comes within an hundredth Part of her Velocity in her Orbit, as may eaſily be calculated from the Time ſhe revolves in, *viz.* 27 Days, 7 Hours and 43 Minutes, and the Semidiameter of her Orbit, which according to the Meaſures we have taken above, is 1181732340. That Force therefore by which Bodies fall to the Ground with us, is at the Diſtance of the Moon ſuch as is requiſite to direct her Courſe.

N. B. The Proportion of a *Paris* Foot to that of *England*, is as 367196 to 343800.

acts upon Bodies in Proportion to the Quantity of Matter they contain.

This being allowed, it will follow, that as the Sun attracts the Planets, and thereby retains them in their Orbs, they in like manner attract the Sun, though with Forces proportionable only to the Quantities of Matter they contain; ſo that ſtrictly ſpeaking, each primary Planet revolves not about the Center of the Sun, but about a Point which is the Center of Gravity between the Sun and that; and that the Sun moves alſo round that Point, and is always oppoſite to the Planet with reſpect thereto: And likewiſe that the Center of the ſolar Syſtem is not in any one Body, but in the common Center of Gravity of all the Bodies of which it conſiſts. But then the Sun is ſo immenſely large in reſpect of any one, or all of thoſe Bodies put together, that that Center is very near the Center of the Sun. In like manner, the Moon does not revolve about the Center of the Earth as a Point at reſt, but the Earth and Moon revolve each about the Center of Gravity common to them both, which Center of Gravity it is, and not the Center of the Earth, that deſcribes the Orbit the Earth is commonly ſaid to revolve in. And ſo of the other Planets which are attended by Secondaries.

The Irregularities in the Motion of the Planet *Saturn*, and thoſe of the Secondaries of *Jupiter*, owing to the Cauſe abovementioned, are ſo exceeding ſmall, that it ſhall ſuffice to have juſt mentioned them.

The more remarkable Effects of the diſturbing Force of the Sun, are the lunar Irregularities, the Preceſſion of the Equinoctial Points, the Nutation of the Poles of the Earth, and the ebbing and flowing of the Sea, which ſhall be particularly conſidered in the following Chapter.

CHAP. XIX.

Of the Lunar Irregularities, *the* Preceſſion *of the* Equinoctial Points, *the* Nutation *of the* Poles *of the* Earth, *and the* ebbing *and* flowing *of the* Sea.

I. TO account for the lunar Irregularities, let S in Fig. 42. repreſent the Sun, T the Earth, and LMNO the Orbit of the Moon, and let the Moon be in one of its Quadratures at L, and let the Lines LS and TS be drawn. It is obvious, that the Tendency the Moon has towards the Sun is along the Line LS, and that which the Earth has, is along the Line TS: Let then the former of theſe be reſolved into two others, the one along LA parallel and equal to TS, the other from L to T along the Line LT. The former of theſe Tendencies being parallel and equal to that whereby the Earth tends along the Line TS, alters not the Situation of the two Bodies L and T with reſpect to each other; that is, it diſturbs not the Motion of the Body L; but the other along LT increaſes its Tendency towards T.

And this Increaſe will be to the Tendency the Moon has to A, which is the ſame the Earth has to S, as the Diſtance LT to LA, or TS. Or in other Words, the Gravity of the Moon towards the Earth in the Quadratures is augmented by the Action of the Sun; and that Augmentation is to the Tendency the Earth has to the Sun, as the Length of the Line LT, or the Diſtance of the Moon from the Earth, to TS the Diſtance of the Earth from the Sun.

So that the greater the Moon's Diſtance is from the Earth, the Diſtance of the Sun remaining the ſame, the greater will this increaſe of the Moon's Gravity towards the Earth be. But if the Diſtance of the Moon from the Earth remains the ſame, and the Diſtance of the Sun be augmented, this additional Increaſe will be the leſs in Proportion to the Cube of that Diſtance (*a*).

Let

(*a*) For, if TS be increaſed, while LT remains the ſame, LT will be ſo much the leſs with reſpect to TS, that is the Increaſe will be diminiſhed in Proportion to the Sun's Diſtance: But

Let now the Moon be in one of its Syzygies at M, then will the Tendency ſhe has to the Sun over and above what the Earth has, which is farther off at T, be to that which the Earth has, as the Difference of the Squares of SM and ST is to the Square of SM: but the Difference between the Squares of SM and ST bears ſuch Proportion to the Square of SM as twice MT, that is MO, does to SM; becauſe the Difference between the Squares of two Numbers that are nearly equal (as SM and ST are on account of the great Diſtance of S) bears double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number (m). The Tendency therefore the

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But when TS the Diſtance of the Sun is increaſed, the abſolute Force of the Sun, and therewith the abovementioned Increaſe, will be diminiſhed alſo in proportion to the Square of that Diſtance, conſequently taking in both the Accounts, it will upon the whole, be diminiſhed in Proportion to the Cube of that Diſtance.

(m) *Dem.* Let a be the leſſer Number, and $a+b$ the larger, and let their Difference b bear no ſenſible Proportion to the leſſer a.

Then the Square of the leſſer is —— aa

The Square of the larger is —— $aa+2ab+bb$

The Difference between theſe is —— $2ab+bb$

And bb being rejected as inconſiderable, the Difference is only } $2ab$

Now the Proportion of $2ab$ to aa is (taking a out of each Term) } $2b$ to a.

But $2b$ to a, is double the Proportion that once b has to a, and therefore the Difference between the Squares of the two Numbers bears double the Proportion to the Square of the leſſer Number, that the Difference between the Numbers themſelves bears to the leſſer Number. Q. E. D.

Moon when at M, has to the Sun, over and above what the Earth has, is to that which the Earth has, as MO, or twice LT, to SM, or becauſe of the Sun's great Diſtance, as twice LT to TS. Her Tendency therefore to the Earth is now diminiſh'd in that Proportion: But as was ſhewn above, it was augmented in the Quadratures in the Proportion only of LT to TS. The Diminution here is therefore double of the Augmentation there.

And whereas that Augmentation, when the Diſtance of the Sun remains the ſame, was ſhewn to increaſe with the Diſtance of the Moon; but when the Diſtance of the Moon remains the ſame, to decreaſe with the Cube of the Sun's Diſtance; this Diminution being always double of that, will do the ſame.

When the Moon is in the other Syzygy at O, ſhe is attracted towards the Sun leſs than the Earth is by the Difference of the Squares of SO and ST; which as to the Effect, is the ſame Thing as though the Earth was not attracted at all towards S, and the Moon were attracted the contrary Way, ſo that her Tendency to the Earth is here alſo diminiſhed, as well as when ſhe was at M, and almoſt in the ſame Degree; for on Account of the Sun's great Diſtance, the Difference between the Spuares of SO and ST is nearly the ſame as between ST and SM.

Or becauſe this way of accounting for the Diminution of the Gravity of the Moon towards the Earth in the Syzygies may not be ſufficiently clear, it may be conſidered otherwiſe, thus. The annual Courſe of the Moon round the Sun being performed in the ſame Time that the Earth's is, ſhe ought to be retained in that Courſe by the ſame Force that the Earth is, whereas when ſhe comes to M, the Action of the Sun upon her is greater than it is upon the Earth, by the Difference of the Squares of SM and ST; and when ſhe is at O, it is leſs than it is upon the Earth by the Difference between the Squares of ST and SO: So that in the former Caſe ſhe is drawn too much towards the Sun, and in the latter too little; and therefore in both Caſes her Tendency towards the Earth is diminiſhed; and almoſt in the ſame Degree; becauſe, as was obſerved above, the Difference of the abovementioned Squares is nearly the ſame in either Caſe.

Let the Moon be in a Point of her Orbit between the Quadrature and the Syzygy, as at L in Fig. 43. Then being nearer the Sun than the Earth is, ſhe will be attracted with a ſtronger Force: Let it be expreſſed by LS produced to D 'till LD is of ſuch Length, that TS being put to expreſs the Action of the Sun upon the Earth, LD may be long enough to expreſs the ſtronger Force of the Sun upon the Moon: And let LD be reſolved into two others, one of which let be LA equal and parallel to TS, then will the other be AD, or its equal and parallel LG. This LG is the only diſturbing Force upon the Moon at L, the other LA being parallel and equal to TS, affects the Moon juſt as the Sun does the Earth; and ſo alters not their Situations with reſpect to each other. Let then, to avoid a Confuſion of Lines, this Figure with the Line LG, be removed to the 44th. This Force LG may be reſolved into LI and LH, the one a Tangent to the Orbit of the Moon, and the other perpendicular thereto: The former accelerates the Motion of the Moon when going from the Quadrature at Q to the Syzygy at B; and will retard it when going from B to R. The other when H falls upon TL produced, as in this Figure, diminiſhes the Tendency of the Moon towards the Earth, and when it falls between L and T, it augments it.

When the Moon is at L between the Quadrature R and the oppoſite Syzygy O, in Fig. 45. the Tendency of the Moon towards the Sun is leſs than that of the Earth; it may then be expreſſed by LD a Line ſhorter than TS, and is reſolvable into LA a Line equal and parallel to TS, and DA, or its equal and parallel LG. Which LG is the only diſturbing Force, and may, as in Fig. 44. be reſolved into two others, one of which ſhall draw the Moon towards O, the other to or from T, as the Caſe may happen. So that in the firſt Place, the nearer the Moon is to its Syzygies, the greater will be its Velocity; and the nearer it is to the Quadratures, the ſlower it will move;

becauſe one of the Forces into which LG is reſolvable (as LI in Fig. 44.) accelerates its Motion from the Quadratures to the Syzygies; and retards it as much from thence to the Quadratures. *Which is the firſt Irregularity* (*n*).

2. When the Moon is in the Quadratures as at L or N, or in the Syzygies as at M or O, ſee Fig. 42. the diſturbing Force is directed to or from the Center of the Earth; and therefore when the Moon is paſſing thoſe Points it is no Impediment to her deſcribing Areas proportionable to the Times; but when ſhe is at L in Fig. 44, or 45, where it is expreſſed by LG, only one of the Lines into which it is reſolved, as LH, points to or from the Center of the Earth, the other, as LI, pointing another Way, prevents her deſcribing Areas proportionable to the Times. So that it is only in the Quadratures and Conjunctions, that the Areas are proportionable to the Times. *Which is the ſecond Irregularity.*

3. The Motion of the Moon being accelerated during her Progreſs from the Quadratures to the Syzygies, and retarded from thence to the Quadratures, her Motion in the Syzygies is too quick, in the Quadratures too ſlow: add to this, her Tendency to the Earth is in the former Situation too ſmall, in the latter too large: Both which conſpire to render her Orbit more curved in the Quadratures than in the Syzygies, ſo that ſhe runs off farther from

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(*n*) See the Lunar Irregularities enumerated in Chap. VIII.

the Earth in the Quadratures, and comes nearer in the Syzygies than ſhe would otherwiſe do, deſcribing an Orbit, one of whoſe Axes, *viz.* that which paſſes through the Quadratures, is longer, than that which paſſes through the Syzygies. *Which is a third Irregularity.*

4. The Gravity of the Moon towards the Earth in the Syzygies, being twice as much diminiſhed by the Action of the Sun, as it is augmented in the Quadratures; if we take a whole Revolution together, it may be conſidered as diminiſhed only. In the Perihelion therefore, at which Time the Earth and Moon are neareſt the Sun, it will be diminiſhed the moſt of all; that Diminution (as ſhewn above) being inverſely as the Cube of the Sun's Diſtance, and ſo the Gravity or Tendency of the Moon to the Earth will be the leaſt. On which Account ſhe will run out into a greater Orbit; and ſo her periodical Time will be greater, than when the Earth is in its Aphelion. *Which is a fourth Irregularity.*

5. When the Moon is in the Quadratures, the Action of the Sun (as ſhewn above) increaſes the Tendency of the Moon to the Earth in Proportion to her Diſtance from thence, this Force ſuperadded to the Action of the Earth upon the Moon, which decreaſes as the Square of the Diſtance increaſes, occaſions that Force to decreaſe as you go from the Earth, *ſlower* than it would otherwiſe do; or which is the ſame Thing to increaſe ſlower as you go the

other way. But by Prop. 9. of the foregoing Chapter, when a retaining Power decreaſes or increaſes ſlower than the Square of the Diſtance increaſes or decreaſes, and the Planet deſcribes an Ellipſe, the *Linea Apſidum* of that Planet will go backwards; the *Linea Apſidum* therefore of the Lunar Orbit, when ſhe is in the Quadratures, goes backwards: When the Moon is in the Syzygies the Action of the Sun diminiſhes her Tendency towards the Earth, and thereby make it decreaſe as you go from, or increaſe as you go to the Earth, too faſt; and ſo the *Linea Apſidum* at that Time goes forwards. But the Diminution in the Syzygies exceeds the Augmentation in the Quadratures, and ſo the *Linea Apſidum* goes farther forwards than backwards every Time, till at length it revolves quite round according to the Order of the Signs. *Which is a fifth Irregularity.*

6. When the Gravity by which a Planet is retained in an Orbit that is excentrical, decreaſes or increaſes *too faſt*, the Planet when going off from the Seat of the retaining Power, that is, towards its upper *Apſis*, will go off too far; and when it is coming to its lower *Apſis*, it will approach too near; and ſo the Excentricity of its Orbit will be increaſed. When its Gravity decreaſes or increaſes *too ſlow*, the Planet will not in the former Caſe go off far enough, nor come ſo near to the Seat of the retaining Power in the latter, as it ought to do; in this Caſe therefore, the Excentricity of its

Orbit will be diminiſhed. But the Tendency of the Moon to the Earth when in the Syzygies, decreaſes or increaſes too faſt, the Excentricity of her Orbit is therefore at that time the greateſt; and on the contrary it is the leaſt when ſhe is in the Quadratures. And if we compare ſeveral Revolutions of the Moon together, we ſhall find, then when the *Linea Apſidum* is in the Quadratures, the Excentricity of the lunar Orbit will be the leaſt of all; becauſe in that Situation of the *Linea Apſidum*, the Difference between the Tendency the Moon has to the Earth in one of the Apſes, and that which it has in the oppoſite one, is the leaſt of all: Whereas when the *Linea Apſidum* is in the Syzygies, that Difference will be the greateſt; and therefore the lunar Excentricity will be ſo too. *Which is a ſixth Irregularity.*

7. We have hitherto been conſidering ſuch Irregularities in the Courſe of the Moon as would happen if its Orbit were coincident with the Plane of the Ecliptic: But as it is not ſo, there will ariſe others; in order to account for which, it will be proper to premiſe the following Conſiderations.

Firſt, That when the Line of the Nodes is in the Syzygies, the Plane of the Moon's Orbit paſſes through the Center of the Sun as well as through that of the Earth, and ſo the Moon in that Situation of the Nodes, is not

drawn out of the Plane of her Orbit by the Sun.

Secondly, That when the Line of the Nodes is in any other Situation, and the Moon not in one of the Nodes, ſhe is continually drawn out of the Plane of her own Orbit, on that Side on which the Sun lies. For inſtance, if the Plane of her Orbit produced paſſes above the Sun, the Sun draws her downwards; if on the contrary, the Plane of her Orbit produced paſſes below the Sun, it draws her upwards.

From theſe two Conſiderations it follows, that when the Line of the Nodes is not in the Syzygies, and the Moon having paſſed either of the Nodes, has got out of the Plane of the Ecliptic on either Side, the Action of the Sun occaſions the Moon to return back to the Plane of the Ecliptic ſooner than ſhe otherwiſe would do; but where the Moon enters that Plane, there is the next Node; ſo that each Node does as it were come towards the Moon, meeting her part of the Way. And the nearer the Line of the Nodes is to the Quadratures, the greater is this Effect, becauſe in that Caſe the Sun is the fartheſt of all from the Plane of the lunar Orbit produced. So that the Line of the Nodes goes backwards the faſteſt of all, when it is in the Quadratures; and not at all in the Syzygies. *Which is the ſeventh Irregularity.*

8. Again, when the Nodes are in the Quadratures, and the Moon has lately paſſed one

of them, and is approaching that Syzygy that is next the Sun, the Action of the Sun upon the Moon prevents her aſcending ſo far; that is, departing ſo far from the Ecliptic as ſhe otherwiſe would do; and ſo diminiſhes the Inclination of her Orbit to the Ecliptic: And as ſhe goes on to the next Quadrature, by haſtening her Deſcent thither, it occaſions the Moon to croſs it in a larger Angle than ſhe would otherwiſe do; and ſo increaſes the Inclination of the Orbit as much as it diminiſhed it before. And, for the ſame Reaſon, while the Moon paſſes from that Quadrature to the oppoſite Syzygy, the Action of the Sun decreaſes the Inclination of her Orbit, and increaſes it again in her Paſſage from thence to the next Quadrature. All which needs no farther Illuſtration, unleſs the following Inſtance may be of Uſe. If you toſs a Stone up into the Air, the Action of the Earth upon the Stone prevents it from riſing ſo high as it would otherwiſe do; and if a Stone be thrown down obliquely, the ſame Action by bending its Courſe towards the Earth all the Way, makes it ſtrike the Earth in a larger Angle than it would otherwiſe do.

When the Nodes are in the Syzygies, the Inclination of the lunar Orbit to the Plane of the Ecliptic is neither increaſed nor diminiſhed: The Sun being then in the Plane of the Moon's Orbit produced.

But while the Nodes are paſſing from the Syzygies to the Quadratures, the Inclination of the Moon's Orbit to the Plane of the Ecliptic, is diminiſhed in every Revolution of the Moon; and while they are paſſing from thence to the Syzygies, it is continually increaſing. So that the Inclination of the lunar Orbit is the greateſt of all when the Nodes are in the Syzygies, and leaſt when they are in the Quadratures. *Which is an eighth Irregularity.* But this requires a particular Explanation.

Let then S in Fig. 46. repreſent the Sun, NFDG the Plane of the Ecliptic, QR the Quadratures, and MO the Syzygies. And let NLDI be the Orbit of the Moon; and ſuppoſe the Nodes at N and D in the middle between the Syzygies and the Quadratures. Farther, let there be a Point H in the Ecliptic oppoſite to the Point S, and let the Orbit NLDI be ſo inclined to the Plane of the Ecliptic, that if it were extended every Way, it would paſs above S and below H. Then becauſe when the Moon is nearer the Sun than the Earth is, ſhe is attracted towards the Sun more than the Earth is; and when ſhe is farther off, the Earth is attracted more than ſhe is, in which caſe ſhe is therefore as it were attracted the other way; let us imagine a Sun at S, and another at H; and let it be remembered that the Orbit of the Moon produced, paſſes above S and below H: And let the Moon be paſſing from N towards L. Then the Attraction here being towards S, and the Orbit produced being above S, it is obvious that the Moon will not paſs to L but to A, a Point between L and F, deſcribing the Curve NA; ſo that the Inclination of the lunar Orbit is perpetually diminiſhed, while the Moon is paſſing over 90 Degrees from the Node N, it being a quarter of a Circle from N to A. To avoid Confuſion in the Figure, let us ſuppoſe that the Moon came to L. In going from thence to the next Quadrature, at B, which is 45 Degrees, the Attraction of S prevails ſtill, becauſe the Moon is as yet nearer to S than the Earth is; and therefore as the Orbit produced is above S, and the Moon going downwards, the Attraction of S haſtens her Deſcent, and ſo makes her deſcribe the Curve LC inſtead of LB, which if produced would make with the Plane of the Ecliptic, a larger Angle than her Orbit LB does at D; in going over this 45 Degrees therefore the Inclination of her Orbit is increaſed. Let us now ſuppoſe her going from B towards D; the Attraction here lies towards H, becauſe ſhe is now paſt the Quadrature; and ſhe is tending to a Point below H; H therefore attracts her upwards, making her deſcribe the Curve BE, inſtead of BD, which is about 45 Degrees more, and makes a leſs Angle with the Plane of the Ecliptic, than BD does: Not that E falls beyond D, it only happens ſo in the Figure, becauſe we ſuppoſed the Moon to move from L and B inſtead of A and C. Upon the whole therefore, while ſhe moves from N to D, the Inclination of her Orbit is diminiſhed during three Parts out of four of her Paſſage. In like manner it is diminiſhed by the Attraction of H, while ſhe goes from D to I, and augmented by the ſame Attraction in going from thence to K, and diminiſhed again between K and N. Add to this, that while the Moon moves from N to L, or from D to I, the diſturbing Force, whether of S or H, is much more conſiderable than it is when ſhe is between L and D, or I and N; becauſe in the former Caſe, the Difference between her Diſtance from the Sun, and that of the Earth from the ſame, is greater than it is in the latter. While the Nodes therefore are between M and R, and O and Q; that is, while they are paſſing from the Syzygies to the Quadratures, the Inclination of the lunar Orbit is continually diminiſhing.

For though we have ſuppoſed the Nodes equally diſtant from the Quadratures and Syzygies, the like Effects will happen, though different in Degree, when they are nearer to the one than to the other; as is eaſy to imagine.

Let now the Nodes be in the mid Way betwen R and O, and Q and M, as in Fig. 47. and let the lunar Orbit produced paſs above S and below H as before, and let the Moon be coming from the Node D towards B. She being there in the Power of H, and moving in a Plane which if produced paſſes below H, will be attracted upwards thereby, ſo as to deſcribe the Line DC inſtead of DB; by which means the Inclination of her Orbit will be increaſed. In going from thence to I, ſhe is in the Power of S, which lies below her Orbit produced, and ſo her Aſcent will be diminiſhed, and ſhe will go to A, inſtead of I; whereby the Inclination of her Orbit will be leſſened; and afterwards as ſhe goes towards N ſhe will be attracted downwards all the Way coming to E inſtead of N; by which means the Inclination of her Orbit is again increaſed. So that upon the whole, it is increaſed three Parts out of four of her Paſſage from Node to Node; for the like will hold in her Paſſage through the other Part of her Orbit, and as well when the Nodes are not in the middle between the Quadratures and the Syzygies, as when they are, except in Point of Degree. And for the Reaſon mentioned in the other Caſe, the Force which augments the Inclination of the Orbit, is ſuperior for the Time being, to that which diminiſhes it. While the Nodes therefore are paſſing from the Quadratures to the Syzygies, the Inclination of the Moon's Orbit to the Plane of the Ecliptic is continually increaſing. Which is what remained to be made out.

All theſe Irregularities are greater when the Earth is in its Perihelion, than when it is in its Aphelion, becauſe as was obſerved above, the Effect of the Sun's Action whereby they are produced, is inverſely as the Cube of its Diſtance from the Earth. They are alſo greater when the Moon is in Conjunction with the

Sun, than in Oppoſition for the ſame Reaſon; for the Earth and Moon taken together, are nearer the Sun in the former Situation of the Moon, than they are in the latter.

II. Let us now imagine that the Circle NLDI is a ſolid Ring like that of *Saturn*, and that it moves round its Center T the ſame Way the Moon does round the Earth; it is obvious that every Point of this Ring, will endeavour to put on the ſame Motion that we have ſhewn the Moon to do: That is, that every Point in its Paſſage from N to L, will endeavour to move in the Line NA, (ſee Fig. 46.) every Point between L and B, will endeavour to deſcribe the Curve LC, and every one between B and D, the Curve BE; and the like for Fig. 47. So that the Ring as to the Motion of its Nodes, and its Inclination to the Plane in which its Center moves, will be affected in the ſame manner that the Orbit of the Moon is; and therefore its Nodes, when in the Syzygies, will ſtand ſtill, and its Inclination to the Plane of the Ecliptic will be the greateſt: In all other Situations the Nodes will go backwards, and faſteſt of all when in the Quadratures, at which Time the Inclination of the Ring will be the leaſt.

Let us now ſuppoſe that there is a Redundancy of Matter ſurrounding the Earth in the equatoreal Parts thereof; or in other Words, that the Earth is an oblate Spheroid, having its equatoreal Diameters longer than that which paſſes through the Poles, as we ſhall ſhew it to be in the next Chapter. This Redundancy of Matter, will, like a Ring ſurrounding the Earth and fixed to it, endeavour to put on the abovementioned Motions, and thereby communicate them to the Earth itſelf. The equinoctial Points therefore which anſwer to the Nodes of the Ring, when they are in the Syzygies, that is, at the Equinoxes, will ſtand ſtill, and the Inclination of the Equator to the Plane of the Ecliptic will be the greateſt; in all other Situations they will go backwards, and faſteſt of all at the Summer and Winter Solſtices, at which Times they are as it were in the Quadratures with the Sun; and then the Inclination of the abovementioned Plane will be the leaſt.

III. From hence it follows, that the Axis of the Earth, being perpendicular to the Plane of the Equator, changes therewith its Inclination to the Plane of the Ecliptic twice in every Revolution of the Earth about the Sun. For inſtance, it increaſes while the Earth is moving from the Solſtitial to the Equinoctial, and diminiſhes as much in its Paſſage from the Equinoctial to the Solſtitial Points. Which Phænomenon is called the Nutation of the Poles.

IV. Another Phænomenon and of the ſame kind with the lunar Irregularities, is the ebbing and flowing of the Sea; only, as thoſe ariſe from

from the Action of the Sun upon the Moon, this is owing to the Influence both of the Sun and Moon upon the Waters of the Ocean; and is to be accounted for upon the ſame Principles, after the following Manner.

Let the Point L in Fig. 48. repreſent the Moon, MNOP, the Earth, whoſe Center is C. And let G be the common Center of Gravity of the Moon and the Earth. Which latter let us ſuppoſe ſurrounded with Water to a great Depth. Then, according to what was obſerved at the End of the laſt Chapter, theſe two Bodies continually revolve about the Point G, the Point M deſcribing the Circle AB; the Point C, the Circle NP; and the Point O, the Circle EF; and all in the ſame periodical Time; conſequently by Propoſition the third of the foregoing Chapter, the Forces they require to retain them in thoſe Circles, ought to be to each other as their Diſtances from the Point G: That is, as GM, GC and GO. Conſequently the Point O, which for Diſtinction ſake we will call the *Nadir*, requires a greater Force than the Center C; and the Center, a greater Force than the Point M, which we will call the *Zenith*. Now theſe Points are retained in thoſe Circles by the Moon at L, conſequently the *Nadir* which requires the moſt, is attracted the leaſt, as being fartheſt off; and the *Zenith* which requires the leaſt, being the neareſt, is attracted the moſt; that is, the *Nadir* is attracted too little, and the *Zenith* too much:

The obvious Conſequence of which is, that the Water both in the *Zenith* and *Nadir*, will endeavour to leave the Center C; or, in other Words, will loſe part of its Weight. But the Water at N and P will have its Weight augmented, juſt as the Tendency of the Moon at L (in Fig. 42.) towards T was ſhewn to be augmented by the Action of the Sun at S. So that the Water at N and P will be heavier than an equal Quantity at M or O. And conſequently the Surface of the Waters at N and P will ſubſide, and that at M and O will riſe, 'till the Equilibrium be reſtored. On which Account, the Form of the Earth, or rather the Sea, will become an oblong Spheroid or Oval, as repreſented by NKMH, in Fig. 49. whoſe longer Axis produced paſſes through the Moon at L. As therefore the Moon turns round the Earth once a Day, this Oval of Waters turns with her, occaſioning thereby the two Floods and Ebbs obſervable in each 25 Hours: or to ſpeak more accurately, the Oval of Waters keeps pace with the Moon in her Monthly Courſe, while the Earth in the mean Time, by its Rotation about its Axis, carries each part of its Surface from Ebb to Flood, and from Flood to Ebb continually.

And as the Moon thus raiſes and depreſſes the Water, the Sun does the ſame; but in a much leſs Degree, on Account of the ſmall Proportion the Semidiameter of the Earth bears to the vaſt Diſtance of the Sun; for, as was ſhewn of the Moon, the Force of the Sun whereby it diſturbs her Motion, was proportionable to the Relation the Diſtance of the Moon from the Earth bears to that of the Sun from the ſame, which in the Caſe before us, is the Relation the Semidiameter of the Earth bears to the Diſtance of the Sun, which Relation is very ſmall.

When the Moon is in Conjunction or Oppoſition with the Sun, the Tides which each of them endeavours to raiſe are in the ſame Place, which is the Reaſon they are ſo large at thoſe Times. Whereas when the Moon is in the firſt or laſt Quarter, the Sun being in the Meridian when the Moon is in the Horizon, depreſſes the Water where the Moon raiſes it, on which Account, the Tides are then (*cæteris paribus*) the leaſt of all.

On the full and new Moons, which happen about the Equinoxes, at which Time the Luminaries are both in the Equator or near it, the Tides are the greateſt of all, on the three following Accounts; in the firſt Place, the two Eminences of Water are at the greateſt Diſtance from the Poles, and ſo the Difference between Ebb and Flood is more ſenſible; for if thoſe Eminences were at the Poles, its obvious we ſhould not perceive any Tide at all: Secondly, the equatoreal Diameter of the Earth produced paſſes through the Moon, which Diameter is longer than others, and ſo there is a greater Diſproportion between the Diſtances of the

Zenith, Center and Nadir, from the Center of Gravity of the Earth and Moon, than at other Times: Thirdly, the Water riſing higher in the open Seas, it ruſhes to the Shores with greater Force, where being ſtopped, it riſes higher ſtill; for it not only riſes at the Shores in Proportion to the Height it riſes to in the open Seas, but alſo according to the Velocity it flows with from thence againſt the Shore. The Reaſon why the Spring Tides, which happen a little before the vernal and after the autumnal Equinox, *viz.* in *February* and *October* are the greateſt, is becauſe the Sun is nearer the Earth in the Winter than in the Summer, and ſo the Tides, which otherwiſe would be the greateſt at the Equinoxes, are ſo, a little before the former, and as much after the latter.

We have hitherto conſidered the Tides in general; we muſt now ſee what happens as to Places of different Latitudes. All which will be eaſily underſtood by the help of the 49th Figure: In which let AFD repreſent the Earth whoſe Center is T, the Poles P and O, this the South, the other the North Pole, EQ the Equator, and the Circles FH and KD two Parallels of it, the laſt on the North Side of it, and the firſt on the South Side of it. Let the Fluid ſurrounding the Earth, form itſelf into an oblong Spheroid, whoſe longer Axis HK produced, paſſes through the Moon at L. The right Lines TK or TH, reckoning from the

Center, will repreſent the greateſt Height of the Water, and ſuppoſing NM perpendicular to KH, TN or TM will denote the leaſt, and will repreſent the Height of the Water in all Parts of the Globe through which that Circle NM paſſes. The right Lines TE, TF, TQ and TD, ſuppoſing them drawn, will ſhew the Height of the Water in the reſpective Places E, F, Q and D. Let us now conſider a Place, which by the diurnal Motion of the Earth, deſcribes the Parallel KD: When this Place is at K, the Height of the Water TK, is the greateſt; that is, it is high Tide or Flood when the Moon L is in the Meridian; but afterwards in the ſame Place, the Height of the Water is the leaſt, when the Place is come to X, and again it is Flood when the Place is come to D. But becauſe TK is greater than TD, in the preſent Caſe when the Moon is on the North Side the Equator, the Height of the Sea will be greater, when the Moon is in that Part of the Meridian which is above the Horizon, than when it is in that which is below it. Likewiſe TH is greater than TF, and therefore in a Place which lies under the Parallel FH, or on the other Side the *Line*, the greateſt Height of Water that happens when the Moon is on that Side, is alſo when ſhe is in that Part of the Meridian that is above the Horizon of that Place. Hence it is, that the Moon in the Northern Signs makes the greateſt Tides on our Side the *Line*, when ſhe is above the Earth;

and in the Southern Signs when ſhe is below it. But on Account of that libratory Motion of the Waters, by which they thus riſe and fall alternately, and which would continue ſome Time, although the Sun and Moon ſhould ceaſe to act, the Difference between thoſe Tides which happen when the Moon is above and below the Horizon, is not ſo great as it would otherwiſe be; and the higheſt Spring Tides are not exactly in the new and full Moons, but happen generally three or four Tides after them, and ſometimes later: Becauſe when the Luminaries come to act more forcibly than ordinary, as being in Conjunction or Oppoſition, the Waters will librate backwards and forwards ſeveral Times, before they arrive at their greateſt Height.

Things would be thus, if the Globe of the Earth were covered with Water of a ſufficient Depth, but the Continents which ſtop the Tide; the Straights between them and the Iſlands, and the Shoalneſs of the Sea in ſome Places, which are Impediments to the free Courſe of the Water, cauſe many Exceptions to what has been laid down; and in particular, that even in the open Ocean the Time of high Water is not, when the Moon comes to the Meridian, but always ſome Hours after it. But to be particular in ſuch Circumſtances is not the Deſign of this Treatiſe. See more in *Philoſoph. Tranſact.* No. 226. However, there is one Thing, which becauſe I don't find it taken

Notice of by others, I ſhall juſt mention; and that is, that at both the Summer and Winter Solſtices, there ought to be but one Tide in 24 Hours within the arctic and antarctic Circles; to ſhew this, let KD be the Tropic of Cancer, and let the Sun and Moon be perpendicular over a Point of that Tropic, as at L: Then as obſerved before, there will be an Ebb at N, but becauſe NPK is a quarter of a Circle, N will be a Point of the arctic Circle, and the high Water under that Circle will be at A, the Point oppoſite to N; ſo there will be but one Flood and one Ebb in 24 Hours Time: And as is obvious enough, the ſame will happen in all Places within that Circle, except at the Pole itſelf, where there is no Tide at all. The ſame Things will alſo happen at the other Solſtice, when the Luminaries are in the Tropic of Capricorn. And in thoſe Parallels which lie between the abovementioned Circles and the Equator, each Ebb will be nearer the Time of the leſſer Flood, than it will be to that of the greater. Thus, in the Parallel KD, the Ebbs will be at X, and in the Parallel FH at Y, which Points are nearer to F and D the leſſer Heights of the Water, than to H and K the greater ones. The Reaſon it does not happen exactly in this Manner, at leaſt not in Point of Degree, is that Riſing and Falling or Libration of Waters mentioned above, by which the Ebbs and Floods of each Place are rendered leſs unequal than they ought, according to the Theory, to be.

CHAP. XX.

Of the Figures *of the* Heavenly Bodies.

WHEREAS the Heavenly Bodies do not conſiſt wholly of ſolid Matter, but are in all probability partly fluid and partly ſolid, like our Earth, or at leaſt were ſo at firſt; thoſe which have no Motion about their Axes, if ſuch there be, will, from the mutual Gravity and Attraction of their Parts among themſelves, ſettle into a ſpherical Form. But as to ſuch as revolve about their Axes, all their Parts will endeavour to receed from the Axis of their Motion, and thereby the equatoreal Parts where the Motion is the quickeſt, will tend leſs towards the Center than the reſt; their Endeavour to fly off from the Axis about which they revolve, taking off part of their Tendency that Way; ſo that thoſe Parts will become lighter than ſuch are nearer the Poles. The polar Parts therefore will preſs in towards the Center, and raiſe the equatoreal Parts, till the Quantity of Matter in the latter is ſo far increaſed, as to compenſate for its Lightneſs, and an Equilibrium be reſtored. On which Account, the Form

Form they will aſſume, will be that of an oblate Spheroid, whoſe ſhorter Axis paſſes through the Poles. And other Circumſtances remaining the ſame, the faſter the Bodies revolve, the more oblate or flat will their Form be; accordingly the Axis of the Planet *Jupiter*, which Planet turns round its Axis in leſs than 10 Hours, is, as appears from the Obſervations of Mr. *Flamſtead* and Monſ. *Caſſini*, no greater with reſpect to the Diameter of its Equator, than in the Proportion of ſeven to eight.

By Virtue only of the Rotation of the Earth about its Axis, the Weight of Bodies at the Equator is leſs than at the Poles, in the Proportion of 288 to 289; that is, a Body which at the Poles would weigh 289 Pounds, would, if carried to the Equator, loſe by Virtue of the centrifugal Force alone, one Pound of its Weight, and ſo weigh but 288 Pounds. From hence ariſes, as obſerved above, a ſpheroidical Form of the Earth, and from that ſpheroidical Form ariſes another Diminution of Gravity at the Equator, by which, if the Earth were homogenious throughout, Bodies at the Equator would loſe one Pound in 1121, and ſo on both Accounts taken together, the Gravity of Bodies at the Poles would be to the ſame at the Equator as 230 to 229 (*n*). From whence, if we ſuppoſe

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(*n*) To calculate this, let us ſuppoſe the Semidiameter of the Earth to be 19695539 *Paris* Feet; which on Account of the vaſt Diſtance on the Surface of the Earth *Caſſini* took in meaſuring it, Sir *Iſaac Newton* makes uſe of. Since then the Earth turns round its Axis in 23 Hours 56 Minutes and 4 Seconds, a Body at the Equator moves through 1436.223 Feet in a Second: But the centrifugal Force of a Body revolving in a Circle is equal to the centripetal Force, which would be requiſite to retain it in that Circle, and the Space a Body would fall through in a Second, by Virtue of that centripetal Force, is by Chap. XVIII. Lemma 1. equal to the Square of the Arch deſcribed in a Second, divided by the Diameter of the Circle, that is, in the preſent Caſe to ,0523 Feet, or 7,54064 Lines. Now the Space a Body falls through at *Paris*, by Virtue of the Gravity there, is 15 Feet 1 Inch and $2\frac{1}{18}$ Lines; or 2174,055 Lines. Therefore the centrifugal Force at the Equator is to the Gravity at *Paris* as 7,54064 to 2174,055. But Bodies do not fall at *Paris*, by the whole Force of Gravity, or as they would do at the Poles of the Earth, becauſe they are in ſome Meaſure prevented by the centrifugal Force there. We muſt therefore compute how much that centrifugal Force is, and add it to their Gravity at *Paris*, to find what it is at the Poles. Which may thus be performed.

Let EPQ in Fig. 50. repreſent the Earth, PP its Poles, EQ its equatoreal Diameter. Let A repreſent any Point between the Equator and the Poles, and parallel to the equatoreal Diameter EQ draw the Line AH, then will the centrifugal Force at E be to that at A as the Radius EC, or which is very nearly the ſame, AC, to AH, Chap. XVIII. Prop. 3. for AH is the Radius of the Circle the Point A deſcribes, while the Earth revolves about the Axis PP. But a Body at A is not thrown off by Virtue of this centrifugal Force directly from the Center of the Earth, but from H along the Line HA produced. Let then AB repreſent that Force, which becauſe it does not tend directly from the Center, let it be reſolved into two others, *viz.* BN and AN; the former perpendicular to the Radius, the latter coincident therewith: It is by this latter Force only that the Gravity of a Body at A is diminiſhed. But the centrifugal Force at the Equator was ſhewn to be to that other Force along the Line AB, as AC to AH, and that other is to the Force along AN as AB to AN, which becauſe of the ſimilar Triangles ABN and ACH is alſo as AC to AH, therefore the centripetal Force at the Equator is to that at A, ſo far as it diminiſhes the Gravity of Bodies there, as AC^q to AH^q; that is, becauſe AH is the Coſine of Latitude of the Place A, as the Square of the Radius to the Square of the Co-ſine of the Latitude of the Place where the Body is. Now the Latitude of *Paris* is 48 Degrees and 50 Minutes, the centrifugal Force therefore at the Equator is to that ſuppoſe the Gravity of Bodies within the Earth to be directly as their Diſtance from the Center, as it was ſhewn to be in Part I. Chap. 3. § 8. thoſe Numbers will alſo expreſs the Relation between its polar and equatoreal Diameter (*o*). This

at *Paris*, as the Square of the Radius to the Square of the Coſine of that Latitude, that is, as 7,540 64 Lines to 3,267. Add this to 2174,055 the Gravity by which Bodies deſcend at *Paris*, and we have 2177,322 for the Gravity they fall with at the Poles: Which Number is to 7,540 the centrifugal Force at the Equator, as 289 to 1. So that the Gravity at the Poles is to the Gravity at the Equator, ſo far as it is diminiſhed in this latter Place by the Rotation of the Earth about its Axis, in the Proportion of 289 to 288. But this is not all: For it may be gathered, from what Sir *Iſaac Newton* has demonſtrated in his *Principia*, Book I. Sect. 13. where he treats of the attractive Forces of Bodies not ſpherical, (though by a tedious and intricate Calculation too long to be inſerted here) that ſuppoſing the Earth to be an oblate Spheroid, ſuch as we ſhall determine by and by, and homogeneous throughout, a Body at the Poles even when the Earth is at Reſt, would be heavier than the ſame at the Equator in the Proportion of about 1121 to 1120. The Weight of a Body therefore at the Poles, when the Earth revolves about its Axis, is to the Weight of the ſame at the Equator, in a Proportion compounded of 289 to 288, and 1121 to 1120; that is, in the Proportion of 230 to 229.

N. B. A *Line* is the 12th Part of an Inch.

(*o*) To ſhew this, call an equatoreal Column extended from the Surface to the Center, r; and a polar Column, x; and call the Force of Gravity at the Equator, p: then from what was juſt obſerved in the foregoing Note, ſuppoſing the Earth to be at Reſt, the Force of Gravity at the Poles will be $\frac{1121p}{1120}$, becauſe as $1120 : 1121 :: p : \frac{1121p}{1120}$: and becauſe the Gravity of a Column if it be equally heavy in all Parts, is equal to the Force of Gravity multiplied by the Contents of the Column, the Weight of an equatoreal Column of Matter, if as heavy in all Parts as at the Surface, would, ſuppoſing the Earth to be at Reſt, be equal to p multiplied by r, that is, pr; and the Gravity of a polar

This is upon a Suppoſition that the Earth was at firſt fluid, or a Chaos, having its ſolid and fluid Parts confuſedly mixed together; but if we ſuppoſe it at firſt partly fluid and partly dry, as it now is, ſince we find that the Land

polar Column would for the ſame Reaſon be $\frac{1121px}{1120}$; but whereas if we ſuppoſe the Force of Attraction inverſely as the Squares of the Diſtances, the Gravity of Bodies within the Earth will decreaſe as we go to the Center, Part I. Chap. 3. § 8. where it terminates in Nothing, the Weights of the above-mentioned Columns decreaſing uniformly therewith, will be but half what we made them before, that is, $\frac{pr}{2}$ and $\frac{1121px}{1120\times 2}$. Call the centrifugal Force at the Equator, n; then ſince the centrifugal Force decreaſes alſo as we approach the Center, and there terminates in Nothing, the centrifugal Force of a whole Column of Matter will be $\frac{nr}{2}$. Take this away from $\frac{pr}{2}$, the Weight of that Column when the Earth is at Reſt, and the remainder $\frac{pr}{2} - \frac{nr}{2}$ will be the Weight of the ſame when it moves. But to preſerve an Equilibrium of Parts, this Weight muſt be equal to $\frac{1121px}{1120\times 2}$ which was ſhewn to be the Weight of a Column at the Poles.

Which gives us this Equation $\frac{pr}{2} - \frac{nr}{2} = \frac{1121px}{1120\times 2}$

Multiplying by 2, we have $pr - nr = \frac{1121px}{1120}$

Multiplying by 1120, we have $1120pr - 1120nr = 1121px$

Which gives us this Equation $r : x :: 1121p : 1120p - 1120n$.

But as was determined in the foregoing Note, p is to n as 289 to 1. Putting therefore thoſe Numbers for p and n, in the laſt Step, we ſhall have $r : x :: 230 : 229$.

That is, the equatoreal Semidiameter is to the polar one, as 230 : 229. Which was to be ſhewn.

is very nearly of the ſame Figure with the Sea, except raiſed a little to prevent its being overflowed, the Earth muſt ſtill be of the ſame Form; for otherwiſe the major Part of the Water would flow towards the Equator, and ſpread itſelf like an Inundation over all the Land in thoſe Parts.

The ſpheroidical Figure of the Earth is greatly confirmed by Obſervations made with Pendulum Clocks, at different Diſtances from the Equator. Firſt of all Monſ. *Richer* in the Year 1672, when at the Iſland *Cayenne*, found that his Clock, which at *Paris* kept true Time, now loſt 2 Minutes and 28 Seconds every Day. Dr. *Halley* going to the Iſland St. *Helena* in the Year 1677, was obliged to ſhorten the Pendulum of his Clock one eighth part of an Inch. With many others, all which compared together make it appear that a Pendulum at the Equator that ſwings Seconds, ought to be about one ſixth Part of an Inch ſhorter than at *Paris*; from whence it appears that the Difference between the Gravity at the Poles and at the Equator, is almoſt twice as great as what ariſes from the Rotation of the Earth about its Axis, and its ſpheroidical Form put together *(p)*. And therefore the Difference between the equatoreal and polar Diameter muſt be ſo too.

From

(p) For the Length of a Pendulum, ſwinging Seconds is proportionable to the Space a Body would fall through in a Second; as being equal to an eighth Part of that Space, Part I. Chap. 6. Prop. 10. And the Space a Body would fall through in a Second, is proportionable to the Force by which it falls; conſequently

From hence it is probable, that the Parts of the Earth which lie near the Center, may be denſer than ſuch as lie nearer the Surface. For on this Suppoſition, and not otherwiſe, is it accountable, that the Gravity at the Equator diſcoverable by Pendulums, and that which ariſes from the above-mentioned Cauſes, ſhould be ſo different (*q*). So

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quently the Length of a Pendulum ſwinging Seconds under the Equator ought to be to the Length of one that ſhall ſwing Seconds under the Pole, as the Gravities in thoſe Places are to each other: That is, as 229 to 230. The Length of a Pendulum therefore under the Equator, being 438,5 Lines, (as being two Lines ſhorter than one at *Paris*) the Length of one at the Poles ſhould be 440,4 Lines; that is, it ought to be but 1,9 Line longer than the former. Whereas it is found by the Obſervations abovementioned, that the Difference between a Pendulum in the Latitude of *Paris*, and one under the Equator, is as much as that, or rather more; and conſequently the Difference between one at the Equator and one at the Poles would be almoſt twice as much; there being almoſt as much Difference between a Pendulum at the Poles and at *Paris*, as between one at *Paris* and the Equator: And conſequently the Diminution of Gravity is almoſt twice as much as that which ariſes from the Rotation of the Earth about its Axis, and its ſpheroidical Form put together.

(*q*) If the Earth be more denſe at the Center than near the Surface, we may conſider that Redundancy of Matter near the Center over and above what there would be there, was the Earth of uniform Denſity, apart, and as a ſeparate Body from the reſt, then upon Account of the ſpheroidical Form of the Earth, a Body at the Poles is nearer to this redundant Matter than at the Equator, therefore in removing a Body from the Poles to the Equator, its Gravity ſo far as it depends on the Attraction of this redundant Matter, that is, that Part of its Gravity which is owing to that Attraction, is diminiſhed: The whole Gravity of the Body is leſs therefore at the Equator than at the Poles. As therefore the Pendulum diſcovers a greater Difference of Gravity than ariſes from the Rotation of the Earth about its Axis, and the ſpheroidical Form of it conſidered as homogeneous, put together, it is very probable that the Earth is more denſe at the Center than at the Surface.

So that upon the whole, Bodies are lighter at the Equator than at the Poles, on a threefold Account. Firſt, by the centrifugal Force there, in the Proportion of 289 to 288: Secondly, on Account of the ſpheroidical Form of the Earth, or its Flatneſs at the Poles, in the Proportion of 1121 to 1120; and laſtly on Account of the greater Denſity of the Earth at the Center, but in a Proportion not yet ſufficiently determined, for want of knowing the exact Length of a Pendulum that ſwings Seconds at the Poles.

The greater warmth of the Air near the Equator, increaſes the Length of a Pendulum by Rarefaction, on which Account alone, it would be neceſſary to ſhorten it at the Equator; but this, as may be gathered from the Obſervations of *Pitcairn* and *De la Hire*, cannot be between this Latitude and the Equator above one fortieth part of an Inch; for as *De la Hire* obſerved, there was but two third Parts of a Line Difference between an Iron Rod ſix Feet long, expoſed to the Froſt and the Summer's Sun. Beſides, this is allowed for above, in ſaying that a Pendulum ought to be ſhortned but one ſixth part of an Inch.

It follows from hence, that the Tendency of heavy Bodies upon the Surface of the Earth, except at the Equator and the Poles, is not towards the Center of the Earth, but towards a Point between that and the Equator. Thus, let EPQ in Fig. 50. repreſent the Earth, PP its Poles, EQ its equatoreal Diameter. The Tendency of a Body as A, upon the Surface of the Earth, between E and P, will not be towards C, but along the Line AD which croſſes the equatoreal Diameter in a Point nearer the Body than the Point C is (*r*).

Another Conſequence of the ſpheroidical Form of the Earth is, that as you go from the polar Parts to the equatoreal, the Degrees upon the Surface of the Earth grow leſs and leſs;

(*r*) To determine the Point D, ſay as EC is to AH; that is, as the Radius is to the Co-ſine of the Angle of the Latitude of the Place A, ſo is the centrifugal Force at E, to a fourth Number, which will expreſs the centrifugal Force at A in the Direction AB. Produce therefore the Line HA to B, till AB be of ſuch Length, that it may bear the ſame Proportion to AC, that the Quantity laſt found has to Gravity upon the Surface of the Earth. Compleat the Parallelogram ABCD, and D will be the Point ſought, and the Tendency of an heavy Body will be along the Line AD. Thus ſuppoſe it required to find the Direction in which heavy Bodies tend towards the Earth in at the Latitude of 51 Degrees and 46 Minutes. Since the centrifugal Force at the Equator, as ſhewn above, bears that Proportion to the Force of Gravity, which 1 does to 289, let us take thoſe Numbers (or which will do as well, any other two that are proportionable to them) the former to expreſs the centrifugal Force of a Body at the Equator, and the latter the Force of Gravity. Then ſay, as the Radius is to the Co-ſine of 51 Degrees 46 Minutes, ſo is 1 to a fourth Number, which will be found ,618. This Quantity is therefore to Gravity as ,618 to 289. Therefore AB or its equal CD is to AC in that Proportion. But the Proportion between CD and AC being known, and the Angle at C the Latitude of the Place, *viz.* 51 Degrees 46 Minutes being given, the Angle CAD will be found about 5 Minutes, which is the Meaſure of the Deviation of the Line of Direction of heavy Bodies in that Latitude, from a Line drawn to the Center of the Earth.

that is, a Degree meaſured upon the Meridian near the Poles, contains more Miles than the ſame meaſured near the Equator. The Reaſon of this is, that, as appears by Inſpection of the Figure, an Arch near the Poles comes nearer to a ſtraight Line; that is, it is leſs curved than one near the Equator; the former is therefore an Arch of a larger Circle than the latter; but the larger the Circle, the larger are the Degrees meaſured upon its Circumference. For in going from P towards E, we are not to imagine ourſelves upon the Circumference of a Circle whoſe Center is C, but to paſs continually from one Arch to another, which Arches are Portions of different Circles, the Centers of which approach, as the Arches themſelves become more curved.

As the Waters of the Earth by Virtue of the Attraction of the Moon, and the Revolution of the Earth about the common Center of Gravity of that and the Moon, were ſhewn in Chapter the laſt, to put on the Form of an oblong Spheroid, whoſe Axis produced paſſes through the Moon; ſo in like manner, if we conſider the Moon as we have now done the Earth, we ſhall find, that as ſhe turns round her Axis in the ſame Time ſhe turns round the Earth, and therefore has nearly the ſame Side always towards the Earth, her Figure is that of an oblong Spheroid, whoſe longer Axis points to the Earth; thoſe Parts which lie next the Earth being attracted too much, and alſo having too ſmall a centrifugal Force; and thoſe on the contrary, which lie on the oppoſite Side of her being attracted too little, and having a centrifugal Force too great; which for the Reaſons given in the above-mentioned Place, will neceſſarily give her that Form.

Her Revolution about her Axis would, as in the Earth, give her a contrary Figure, but it is ſo very ſlow, that it is without any ſenſible Effect.

Material ERRATA in Part III and IV.

PART III.

Page 164 Line 2 for ſuppoſed, read diſpoſed.

PART IV.

Page	Line	Inſtead of	Read
12	21	ADME	ADBE
22	23	neither the Sun nor the Earth	both the Sun and the Earth, or round neither of them
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45	2	Side	Side of the Equator
48	29	XIV.	XVI.
95	13	can-	cannot
99	26	*(d)*	*(b)*
114	4	Horizon	Heavens
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